PROBLEM SET #1 SOLUTION **Fall 2005**

PROBLEM 1:

(a) Let H = 500 MT of 90 w/o U-235 (HEU) and

N = total number of MT of U_{NAT} to blend

Balance total mass and U-235 mass

$$H + N = P$$

$$90H + 0.711N = 5P$$

From which $P = 20.82H = 10{,}409$ MT

$N = 9909$ MT to blend-down.

For the conventional route

$$\frac{F}{P} = \frac{5 - 0.3}{0.711 - 0.3} = 11.436$$

Thus, $F = 119{,}038$ MT otherwise required. And net U_{NAT} displaced is

$$F - N = 109{,}131 \text{ MT } U_{NAT}$$

(b) Assume that the SWU originally required to make the HEU is a sunk cost, then we need to calculate SWU for 10,409 MT @ 5 w/o U-235 for 0.3 w/o tails:

	MT	$V = (1-2X)\ln\frac{1-X}{X}$	Total (MT×V)
P	10,409	2.650	27,584
F	119,038	4.869	579,596
W	108,629	5.771	626,898

$$\text{SWU} = 27584 + 626898 - 579596 = 74{,}886 \text{ MT},$$

i.e., $\frac{S}{P} = 7.194$.

(c) $$\frac{B_d P}{Q_{R,th}} = \frac{60{,}000 \frac{\text{MWd}}{\text{MT}} \times 10409\text{MT}}{3411\text{MW} \times 365.25 \text{ days/year}} = 501 \text{ EFPY}$$

or enough to operate all ~100 US reactors for about 5 years.

(d) Spent on U_{NAT} (conventional route)

$$119{,}038 \times 10^3 \text{ kg} \times \$40/\text{kg} = \$4.76 \times 10^9$$

Spent on conversion (conventional route)

$$119{,}038 \times 10^3 \text{ kg} \times \$6/\text{kg} = \$0.71 \times 10^9$$

Spent on SWU (conventional route)

$$74{,}886 \times 10^3 \text{ kg} \times \$100/\text{kg} = \$7.49 \times 10^9$$

TOTAL $\$12.96 \times 10^9$

which is slightly more than the 12 billion dollars promised to Russia.

However, there are other factors:

(1) The actual transactions will take place over a period of time, during which conventional route costs may rise (or fall);
(2) National/international security concerns justify spending more to get rid of this weapons-usable stockpile even at some added cost.

Note also the following fact: Instead of using natural uranium to blend down the HEU, obviously, using depleted uranium (the tails of former enrichment activities and essentially a waste product available for "free") would be much more favorable.
Ironically, the Russians cannot use this material directly. As it turns out, the HEU contains too much U-234 (and possibly U-232) and does not meet the required American specifications for “commercial grade enriched uranium” as set out in the relevant ASTM standard (C-996-96) when blended with natural or depleted uranium.
The depleted uranium is therefore ENRICHED in a Russian facility to 1.5wt% U-235. Reportedly, 30% of the total SWU currently performed in Russia is used for this purpose. The slightly enriched uranium is finally blended with the HEU. U-234 and U-232 concentrations are now acceptable because less HEU is needed to push enrichment to 4-5 wt% in the product. Estimating the actual costs of the deal for Russia is very difficult and has been subject of various studies.

PROBLEM 2:

(a) Assume data from 1c:

3411 MWth x 365d /yr
= 1.25e6 MWd/yr

1.25e6 MWd/yr / 60 MWD/kgU
= 2.08e4 kgU/yr

Note: this is enriched to 5 w/o.
Feed to product ratio is:

$F/P = (e_p - e_w)/ (e_N - e_w)$
$= (5 - 0.71) / (0.71 - 0.3)$
$= 10.46$

So the 2.08e4 kgU/yr @ 5w/o requires 2.08e4 kgU/yr x 10.46 = 21.8e4 kgU_n/yr for each reactor.

Adding the reserves, 3.56e6 tons = 3.23e9 kgU_n which will support 3.23e9/21.8e4 reactor years of operation = 14,800 reactor years.

(b) The number N of reactors in year t is:

$N(t) = 350\ (1.02)^t$

The cumulative number of reactor years of operation up through year t is:

$Y(t) = 350\ \Sigma(1.02)^t$, where the sum is from 0 to T.
$Y(t)= 350\ (1- 1.02^{(T+1)} / (1-1.02)$
$= 17{,}500(1.02^{(T+1)} - 1)$

Since there are 14,800 reactor-years of uranium reserves available, the stuff runs out when:

$14{,}800 = 17{,}500*(1.02^{(T+1)} - 1)$
which occurs when T ~ 30 years.

(c) This is not well defined. Assume for simplicity that in 2025 all LWRs are magically converted into advanced reactors. Then,

$14{,}800 = 17{,}500*(1.02^{20}- 1) + (17{,}500*33/45)*(1.02^{(T+1)}- 1 - [1.02^{20}- 1])$
{U utilized through 2025} {U between 2025 and T}

So T ~ 33 years.

PROBLEM 3

Part a)

37,000 kg of yellowcake (U_3O_8)

$$37{,}000 \text{ kg} \frac{[3(238.03)]}{[3(238.03)+8(16)]} = 31{,}375.9 kgHM$$

F = 31,375.9 kg

$$\frac{F}{P}=\frac{x_p-x_w}{x_f-x_w}=\frac{31{,}375.9kg}{100kg}=\frac{0.93-x_w}{0.00711-x_w}$$

$x_w=0.0042$
=0.42 w/o

This is a reasonable tails enrichment. The calculation is thus believable.

Part b)

$$SWU=\left[P\cdot V(x_p)+W\cdot V(x_w)-F\cdot V(x_f)\right]$$

$$V(x_p)=V(0.93)=\left[2(0.93)-1\right]\ln\left[\frac{0.93}{1-0.93}\right]=2.224$$

$$V(x_w)=V(0.004159)=\left[2(0.004159)-1\right]\ln\left[\frac{0.004159}{1-0.004159}\right]=5.433$$

$$V(x_f)=V(0.00711)=\left[2(0.00711)-1\right]\ln\left[\frac{0.00711}{1-0.00711}\right]=4.869$$

$$W=F-P=31{,}275.9kg$$

$$SWU=\left[100(2.224)+31{,}275.9(5.433)-31{,}375.9(4.869)\right]=17{,}375kgSWU$$

Part C)

$x_f=0.2$
$x_p=0.93$
$x_w=0.004159$
$P=100kg$

$$\frac{F}{P}=\frac{x_p-x_w}{x_f-x_w}=\frac{F}{100kg}=\frac{0.93-0.004159}{0.2-0.004159}=472.8\text{kg}$$

$$W=F-P=372.8kg$$

$$SWU=\left[P\cdot V(x_p)+W\cdot V(x_w)-F\cdot V(x_f)\right]$$

$$V(x_p) = V(0.93) = [2(0.93) - 1]\ln\left[\frac{0.93}{1 - 0.93}\right] = 2.224$$

$$V(x_w) = V(0.004159) = [2(0.004159) - 1]\ln\left[\frac{0.004159}{1 - 0.004159}\right] = 5.433$$

$$V(x_f) = V(0.2) = [2(0.2) - 1]\ln\left[\frac{0.2}{1 - 0.2}\right] = 0.832$$

$$SWU = [100(2.224) + 372.8(5.433) - 472.8(0.832)] = 1{,}854 kgSWU$$