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8.02 Electricity and Magnetism, Spring 2002  
Transcript – Lecture 16

I'd like to thank you for your evaluations.

They were very useful to me.

I already sent e-mail to about 50 students and I had some interesting exchanges with some of you.

Many of you are very happy with their recitation instructors.

That's great.

Many are moderately happy.

Maybe that's OK.

But there are quite a few who are very unhappy with their recitation instructors.

If you are very unhappy with your recitation instructor, you are complete idiots if you stay in that recitation.

We have 13 recitation instructors, and I can assure you that it will be very easy to find one that agrees with you, and you can come and see me if that helps.

Some are better than others.

That's the way it goes in life.

Some students would like to see more cut-and-dried problem solving in my lectures.

I think that's really the domain of recitations.

Lectures and recitations are complementary.

In lectures, I prefer to go over the concepts and I always give numerical examples to support the concepts -- in a way that's problem solving -- and I show demonstrations to further support the concept, because seeing, obviously, is believing.

I try to make you see through the dumb equations and admittedly my methods are sometimes somewhat different from what you're used to here at MIT.

I try to inspire you and at times I try to make you wonder and think.

And I want to keep it that way.

I believe that hardcore problem-solving is really the domain of the recitations.

Many of you found the exam too easy, and many of you found the exam too hard.

Some complained it was too hard because it was too easy.

[audience laughter] Quite ironic, isn't it?

They say we want more math, we want more standard problems.

Look, who wants more math?

I'm teaching physics.

I test you physics, I don't test you math abilities.

If you digest the homework, and that's very important that you make the homework part of your culture, that you study the solutions.

The solutions that we put on the web, today, 4:15, solutions to number four will go on the Web.

Believe me, they are truly excellent solutions, not cut and dried.

They give you a lot of background.

If you digest those solutions, then the concepts will sink in.

And now, at your fifty minute test, do you really want problems which are complicated maths?

Clearly, not.

I could try that, during next exam, but then I may have to buy myself a bullet-proof vest to be safe.

Concepts is what matters.

When I gave my exam review here, I highlighted the concept.

Each little problem I did here was extremely simple.

Conceptually, they were not so simple.

But from a math point of view, trivial.

Clearly, I can not cover all the subjects in a fifty minute exam.

I have to make a choice, so your preferred topic may not be there.

Some of you think that the pace of this course is too slow.

Some of you think it's too fast.

The score, the average score, was 3.8.

4.0 would have been ideal.

What do you want me to do?

I can't accommodate all of you.

Those who think it's too slow, and those who think it's too fast.

3.8 is close enough to ideal for me, 4.0.

And so I'll have to leave it the way it is.

Besides that, keep in mind you are now at MIT.

You're no longer in high school.

Now the good news.

There were quite a few students who said the homework is too long.

Not a single person said it was too short.

I can fix that.

I will reduce all future assignments by about 25%, effective tomorrow.

I have already taken off assignment number five, two problems.

You're down now to seven, and I will do that, all assignments that are coming up.

My pleasure.

Today, I'm going to cover with you something that conceptually is the most difficult of all of 8.02.

And you will need time to digest it.

And if you think that what you're going to see is crazy, then you're not alone.

The only good news is that conceptually, it's not going to become more difficult.

Remember that Orsted in 1819 discovered that a steady current produces a steady magnetic field, and that connected electricity with magnetism.

A little later, Faraday therefore suggested that maybe a steady magnetic field produces a steady current.

And he did many experiments to show that.

Turned to not to be so.

And one way he tried that is as follows.

He had here a battery, with a switch, and here he had a solenoid.

He closes the switch.

A current will flow, and that creates a magnetic field in the solenoid, and that magnetic field, maybe it runs like so, depends on the direction of the current.

And so now, he put around this solenoid a loop.

Let's call this loop number two, and it was around the solenoid, and let's call this loop number one, of which the solenoid is part.

Whenever there was a current in number one, he never managed to see a current in number two.

If there is a current going in number one, there is a magnetic field and that magnetic field is seen, of course, by the conductor number two, by that loop.

Never any current.

So he concluded that a steady magnetic field as produced by the solenoid, circuit one, does not produce a steady current in number two.

But then, one day he noticed that as he closed the switch he saw a current in number two, and when he opened the switch again he saw a current in number two, and therefore he now concluded that a changing magnetic field is causing a current.

Not a steady magnetic field, but a changing magnetic field.

And this was a profound discovery which changed our world and it contributed largely to the technological revolution of the late nineteenth and early twenty century.

The current, therefore an electric field, can be produced by a changing magnetic field, and that phenomenon is called electromagnetic induction, and that phenomenon runs our economy, as you will see in the next few lectures.

I have here a conducting wire, a square.

I could've chosen any other shape.

Try to make you see three dimensionally.

And I approach this conducting wire with a bar magnet.

The bar magnet has a magnetic field running like so.

As I approach that loop, that conducting wire, moving the bar magnet, that's essential.

I can't hold it still.

I have to move it.

If I come down from above and I move it down, you're going to see a current going through this loop.

And that current will go into such a direction that it opposes the change of the magnetic field.

The magnetic field is in down direction and it is increasing as I move the bar magnet in.

Then this current loop will produce a magnetic field which is in this direction, and when you look from below the current will go clockwise, producing a current -- a magnetic field in this direction.

If you move the bar magnetic out, then the magnetic field is going down here, then the current will reverse.

The current wants to oppose the change in the magnetic field, and that's called Lenz's Law.

It is the most human law in physics, because there's inertia in all of us.

We all fight change at some level.

Lenz's Law is extremely powerful in always determining in which direction these induced currents will run.

It is not a quantitative law.

You can not calculate how strong the current will be, but it's very useful as you will see today to know the direction of that current -- that gets you out of all kinds of problems with minus signs.

I now want to do a demonstration which is very much like what you see here.

I have here a loop.

That is the square that you see there except that it's not- not one loop, but it is many of them.

Hundreds, doesn't matter.

And what we're going to show you is an amp meter that is connected, so there is somewhere in this circuit an amp meter.

I have a bar magnet and I'm going to approach this conducting loop with a bar magnet and you're going to see a current running in one direction and when I pull it out it will be running in the opposite direction, and when I hold my hand still so that the magnetic field is not changing, no current.

You're going to see the current meter there, and here is my bar magnet.

I come close to this conducting loop.

Notice you see a current.

I pull back, the current is in the other direction.

Now I will go faster, so that the change of the magnetic field per unit time is stronger.

[whistle] More current.

I go out fast.

[whistle] More current.

So we see it's the change of the magnetic field that matters.

If I come in very slowly, which I do now, very slowly, we almost see nothing.

Right now the entire magnetic field is inside this loop.

The strongest I can have it.

Nothing happens because there is no change in the magnetic field.

It's only when I do this that you see the current.

So an induced current is clearly the result of a driving force.

There must be, just like we had with batteries in the past, there must be an EMF.

There must be an electric field that is produced in this conducting loop.

And so I create now an induced EMF -- we used that word EMF earlier for batteries, so now we have an induced EMF, which is the result of this changing magnetic field, and that therefore is the induced current times the resistance of that entire closed conductor, whatever is in there.

In this case, the total resistance of all these windings, of all the copper wire.

That's Ohm's Law.

So the induced EMF is always the induced current times the resistance.

Faraday did a lot of experiments, and one of the experiments that he did was that he produced a magnetic field, so he ran a current through a loop of some kind, let's say he ran a current going around, creating therefore a magnetic field, and he was switching the current in and out so that he could change the current, and so it produces a magnetic field and this magnetic field changes when you close and open the switch.

And then here, he had his second conducting wire, just like we had there, and he measured in there the current.

And what he found, experimentally, is that the EMF that is generated in here, which I will call EMF generated in my conducting loop number two, is proportional to the magnetic field change produced by number one, so the field goes through number two and this field is changing, so he knows that if the change is faster, as you just saw, you get a higher EMF.

He also noticed that  $\mathcal{E}$  is proportional to this area, so to the area of number two.

And that gave him the idea that the EMF really is the result of the change of the magnetic flux through this surface of number two.

And I want to refresh your memory on the idea of magnetic flux.

We do know, or we remember what electric flux is.

And magnetic flux is very similar.

If this is a surface, and the local vector perpendicular to the surface is like so, of course it could be in a different direction, and the local magnetic field is for instance like so, then a magnetic flux through this surface is defined, we call it  $\Phi_B$ , is the integral over an open surface - this is an open surface - of  $\mathbf{B} \cdot d\mathbf{A}$ .

And electric field we defined in exactly the same way, electric flux, except we had an  $\mathbf{E}$  here.

There was nothing there.

So if this magnetic flux is changing, Faraday concluded that then you have an EMF in this conducting wire.

So essential is the change of the magnetic flux.

If we take some kind of a conducting wire, like so, let's make it in the blackboard for now to make it easy.

And I attach to this wire a surface because the moment that you talk about flux you must always specify your surface.

A flux can only go through a surface, so this is my surface now for simplicity.

And there is a magnetic field coming out of the blackboard at me, and it is growing.

It is increasing.

I will now get an EMF, a current flowing in this direction.

Lenz law.

If the magnetic field is increasing, then the current will be in such a direction that it opposes the change.

It doesn't want that magnetic field to increase, and so it goes around like this, the current, so that it produces a magnetic field that is in the blackboard.

And so it is the flux change of that magnetic field through this flat surface that determines the EMF.

So the EMF is then the flux change,  $d\phi/dt$ , through that surface.

To express Lenz's Law that it is always opposing the change of the magnetic flux, we have a minus sign here.

But minus signs will never bother you, believe me, because you'll always know in which direction the EMF is.

It's clear that the EMF is going to be in this direction.

That's the direction in which it will make the current flow.

But we have to put it there to be mathematically correct.

That's really Lenz's Law.

You're looking at Lenz's Law here.

So you can also write down for this minus the surface integral of  $B \cdot dA$  over that open -- whoo, I hope you didn't see this.

Over this open surface.

That's the [break in tape] Oops, look what I did.

I forgot the  $d/dt$  in front of the integral sign.

Sorry for that.

If you put yourself inside that conductor, and you march around in the direction of the current, you will see everywhere in the wire an electric field, of course.

Otherwise, there would be no current flowing.

And so if you go once around this whole circuit, then that EMF must of course also be  $\int \mathbf{E} \cdot d\mathbf{L}$  over the closed loop.

So you're marching inside the wire, you find everywhere an electric field and these little sections  $I d\mathbf{L}$ .

$\mathbf{E}$  and  $d\mathbf{L}$  are always in the same direction if you stay in the wire, and so this should be the same and this is a closed loop.

So this is all if you want what we call Faraday's Law.

We never see it in so much detail.

I will abbreviate it a little bit on the board there.

But I want you to appreciate that there is no battery in this circuit.

There is only a change in the magnetic flux through a surface that I have attached through the conducting wire, and then I get an induced EMF and the induced EMF will produce a current given by Ohm's Law.

So I want to write down now on that blackboard there, Faraday's law in a somewhat abbreviated way because we have all Maxwell's equations here and so we now have that the closed loop integral, closed loop of  $\mathbf{E} \cdot d\mathbf{L}$  -- that's that induced EMF.

You can take minus  $d\phi/dt$  or the time derivative of the integral  $\mathbf{B} \cdot d\mathbf{A}$ .

That's the one I will take.

Integral of  $\mathbf{B} \cdot d\mathbf{A}$  and this is over an open surface.

And that open surface has to be attached to this loop, and that is Faraday.

We have Gauss's Law, we have Ampere's Law.

We have this one which tells you that magnetic monopoles don't exist.

This would only not be 0 if you had a magnetic monopole and put it in a closed surface.

Come and see me if you find one.

And this now is Faraday's Law, so you think that all four Maxwell's equations are now complete.

Not quite.

We're going to change this one shortly.

So we can't celebrate yet.

We have to wait.

It's a big party.

There's always a little bit of an issue about the direction of  $dA$  and I will explain to you how the convention goes but it really is not so crucial because Lenz's Law always helps you to find the direction of the EMF, but if we are trying to be purist, if this is my conducting loop and if I attach a flat surface to this, if I did that, and if I go around closed loop  $\int E \cdot dL$ , Faraday doesn't tell me which way I have to go.

I can go clockwise.

I can go counterclockwise.

We will then do the same thing that we did before with Ampere's Law, apply the right-hand corkscrew rule, and that is that if you march around clockwise, then  $dA$  will be in the blackboard perpendicular to the blackboard, perpendicular to this surface, and if you go counterclockwise then  $dA$  will come towards you.

The surface doesn't have to be flat.

It can be flat.

There's nothing wrong with it.

But there can also be a bag attached to it, as we also had earlier.

I have here a closed conducting wire and I could put a surface right here but I can also make it a hat, like this, perfectly fine.

Nothing wrong with that.

That's an open surface attached to this loop.

That's fine.

You have a choice, and the convention with  $dA$  is then exactly the same, that if you go clockwise then the  $dA$  would be in this direction using the right-hand corkscrew locally here.

If you went counterclockwise, the  $dA$  would flip over.

So what is now the recipe that you have to follow?

You have a circuit, electric circuit, that determines then your loops, of course.

You can take loops anywhere in space, but that's not too meaningful, so you take them into your circuits, and so you define the loop first.

Then you define the direction in which you want to march around that circuit.

You attach an open surface to that closed loop, and you can determine on that entire surface the integral of  $B \cdot dA$ .

Everywhere on that surface locally you know the  $dA$ , locally you know the  $B$ , you do the integration and you get your magnetic flux, and then if you know the time change of that magnetic flux, then you know the EMF.

If you go around in this conducting circuit, and you measure everywhere the electric field, then the integral of  $E \cdot dL$ , if you go

around the loop will give you the same answer, and that connects the two.

The magnetic flux change is connected with the integral of  $\mathbf{E} \cdot d\mathbf{L}$  when you go around.

And you have to take that minus sign into account.

How come it doesn't matter whether you choose a flat surface or whether you choose a bag?

Well, think of magnetic field lines as a flow of water or spaghetti, if you like that, or a flow of air.

It is clear that if there is some kind of a flow of air through this opening, that it's got to come out somewhere, so it always comes out of this surface.

And therefore, you're really free to choose that surface, so you always pick a surface that is the best one for you.

Now, all this looks very complicated.

But in practice, it really isn't, because your loop is always a conducting wire in your circuit, and the minus sign is never an issue because you always know with Lenz's law in which direction the EMF is.

In fact, when I solved these problems, I don't even look at the minus sign.

I ignore it completely.

I def- I calculate the magnetic flux change, and then I always know in which direction the current is, so I don't even look at the minus sign.

Now I want to show you a demonstration which is very much like what Faraday tried to do.

I have here a solenoid.

We've seen this one before.

We can generate quite a strong magnetic field with that.

And we're going to put around this solenoid one loop, like we had here, like Faraday did, and then we're going to close the switch, and so we're going to build up this magnetic field and we're going to see the current in that loop.

And so if we look- if we make a cross-section straight through here, then it will look as follows.

Then you see here the solenoid, so the magnetic field is really confined to the solenoid.

Magnetic field outside the solenoid as we discussed earlier is almost 0, so there's only a magnetic field right here.

Keep that in mind in what follows.

And now we're going to put a wire around it, with an amp meter in there.

If the magnetic field comes out of the board, and is growing, increasing, the current will flow in this direction.

Lenz's Law.

If it is decreasing, the current will go in the opposite direction.

Now keep in mind that the magnetic flux through this surface, that is my surface which I attach to this closed loop, that that magnetic flux remains the same whether I make the loop this big or whether I make the loop very crooked like so, because the magnetic flux is only confined to the inner portion of the solenoid and that's not changing.

And so when I change the shape of this outer loop, you will not see any change in the current.

I hope that doesn't confuse you.

I'm going to purposely change the size of the loop, and so I'm going to do that now.

You're going to see there a very sensitive amp meter and you're going to see here this loop, the big wire, and I'm going to just put it over this solenoid.

Let me first make sure that my amp meter, which is extremely sensitive, I can zero it.

It's sign sensitive.

If the current goes in one direction, you will see the needle go in one direction.

If the current goes in the other direction, you will see the change.

And so now I put this loop around here, crazy shaped, this loop.

So it's around this solenoid once, so the magnetic field is inside the solenoid, and so think of a surface which is attached to this crazy loop, and now I'm going to turn the current on, and only while the current is changing will there be a changing magnetic flux.

Only during that portion will you see a current flow.

Three, two, one, zero.

I will break the current, three, two, one, zero.

Went the other direction.

If I change the size of the loop, I'm making it now different, much smaller.

Makes no difference, for reasons that I explained to you, because the magnetic flux is not determined in this case by the size of my loop but is determined by the solenoid, so if I do it again now, with a very different shape of the loop -- let me zero this again.

Three, two, one, zero.

Three, two, one, zero.

No change.

Almost the same which you saw before.

Now comes something that may not be so intuitive to you.

I'm now going to wrap this wire three times around.

And so this outer loop, this outer conducting wire, is now like this.

One, two, three.

Something like that.

Now I have to attach in my head a surface to this closed loop.

My god, what does it look like?

What a ridiculous surface.

Well, that's your problem, not Faraday's problem.

How can you imagine that there is a surface attached to this loop?

Well, take the whole thing and dip it in soap.

Take it out and see what you see.

The soap will attach everywhere on the conducting loop.

And if this loop were like this, going up like a spiral staircase, you're going to get a surface that goes up like this.

But the magnetic field goes through all three of them.

Therefore, the changing magnetic flux will go three times through the surface now, and so Faraday says, fine, that you're going to see three times the EMF that you would see if there were only one loop.

And if you go 1000 times around, you get 1000 times the EMF of one loop.

Not so intuitive.

So I'm around now once.

I go around twice.

And I go around a third time.

I have three loops around it now.

I can zero that, but that's not so important.

Three, two, one, zero, and you saw a much larger current.

It's about three times larger because the EMF is three times larger.

I break the current.

We see it three times larger.

And this is the idea behind transformers.

You can get any EMF in that wire that you want to, by having many, many loops.

You can get it up to thousands of volts, and that's not so intuitive.

So Faraday's law is very non-intuitive.

Kirchhoff's Rule was very intuitive.

Kirchhoff said when you go around a circuit the closed-loop integral of  $\mathbf{E} \cdot d\mathbf{L}$  is always 0.

Not true if you have a changing magnetic flux.

If you have a changing magnetic flux, the electric fields inside the conducting wires now become non-conservative.

Kirchhoff's Rule only holds as long as the electric fields are conservative.

If an electric field is conservative and you go from 0.1 to 0.2, the integral  $\mathbf{E} \cdot d\mathbf{L}$  is independent of the path.

That's the potential difference between two points, that's uniquely defined.

That's no longer the case.

If you go around once with this experiment, you get a certain EMF, you go three times around, you get a different value.

Your path is now different, and that's very non-intuitive, because you're dealing with non-conservative fields for which we have very little feeling.

Now, I'm going to blow your mind.

I'm going to make you see something that you won't believe, and so try to follow step-by-step leading up to this unbelievable and very non-intuitive result.

I have here a battery, and the battery has an EMF of 1 volt.

Here is a resistor,  $R_1$ , which is 100 ohms.

And here is a resistor,  $R_2$ , which is 900 ohms.

And I'm asking you what is the current that is flowing around.

And you will laugh at me.

You will say that's almost an insult.

I wish you had given that problem at the first exam, because  $E$  equals the current that is going to run, divided by  $R_1$  plus  $R_2$ . Oh, my goodness, what did I do.

I forgot Ohm's Law.

$E$  equals  $IR$ , remember, not  $I$  over  $R$ .

So  $R_1$  plus  $R_2$  should go upstairs.

And everything that follows is correct, so you don't have to worry about that.

This was just a big slip of the pen.

And so the current  $I$  is  $10^{-3}$  amperes.

1 milliamperes.

Big deal.

Easy.

Current is going to flow like this.

Fine.

Let's call this point D, and call this point A.

And I ask you what is the potential difference between D and A.

You will be equally insulted.

$V_D$  minus  $V_A$ , you apply Ohm's Law, you say that's this current times  $R_2$ .

Absolutely.

$I$  times  $R_2$ .

So that is +0.9 volts.

Now I say to you, well, suppose you had gone this way, then you would've said, well, I find the same thing, of course.

Kirchhoff's Rule.

So indeed, if you go  $V_D$  minus  $V_A$ , and you go this way, then notice this battery, this point is 1 volt above this point.

But you have in the resistor here, you have a voltage drop according to Ohm's Law, and the current times 100 ohms gives you a one-tenth voltage drop here, so  $V_D$  minus  $V_A$  is the 1 volt from the battery minus  $I$  times  $R_1$ , and that is +0.9 volts.

What a waste of time that we did it twice and we found the same result.

So I connect here a voltmeter.

The voltmeter is connected to point D and to point A.

And I asked you what are you going to see.

The answer is +0.9 volts, and you will provided that the plus side of the voltmeter is connected here and the minus side of the voltmeter there.

Voltmeters are polarity sensitive.

This is fine.

Kirchhoff's Rule works.

The closed-loop integral from  $\oint \mathbf{E} \cdot d\mathbf{L}$  going from B back to D is 0.

So far, so good.

Now hold on to you chairs.

I'm going to take the battery out.

Who needs the battery?

I'm going to replace the battery by a solenoid which you see right here, and this solenoid when I switch it on is creating an increasing magnetic field.

Only here, and let's assume that an increasing magnetic field is coming out of the board, and that it is increasing.

Lenz's Law will immediately tell you in what direction the current is.

If this magnetic field is increasing towards you, the current will be in this direction.

The magnetic flux change,  $d\phi/dt$ , at a particular moment in time, happens to be 1 volt.

An amazing coincidence, isn't it.

$\mathcal{E}$  induced at a moment in time is 1 volt.

Now, I ask you, what is the current?

Well, you'll be surprised that I even have the courage to ask you that, because Ohm's Law holds.

The induced EMF is one volt and  $R_1$  plus  $R_2$  is still a 1000 ohms, so 10 to the -3 amperes.

I really make a nuisance of myself when I say what is  $V_D$  minus  $V_A$ , and you get annoyed at me and you say, look, the current  $I$  through  $R_2$  Ohm's Law,  $V$  equals  $IR$ , +0.9 volts.

And then I say, but now suppose we go the other- the other side, and we want to know now what  $V_D$  minus  $V_A$  is, and now it's not so simple, because there's no battery.

And so now when I go from  $D$  to  $A$ , I don't have this one, and therefore I now find -0.1 volts.

I find a totally different answer.

I attach a voltmeter here.

That voltmeter will show me +0.9 volts.

Now I attach a voltmeter here, the same one.

I flip it over.

It's connected between point  $D$  and point  $A$ .

It will read -0.1 volts.

This voltmeter, which is connected between  $D$  and  $A$ , reads +0.9.

This voltmeter which is connected to  $D$  and  $A$  reads -0.1.

The two values are different, and I placed on the web a lecture supplement which goes through the derivation step-by-step, which will convince you that indeed this is what is happening.

Why we can't digest this so easily is we don't know how to handle non-conservative fields.

If you have a non-conservative field, then if you go from  $A$  to  $D$  of  $\int \mathbf{E} \cdot d\mathbf{L}$  or from  $D$  to  $A$  for that matter, doesn't matter, the answer depends on the path.

It's no longer independent of the path.

And so if here is D, and here is A, and we go this way, you find 0.9 volts, plus, if you go this way you find -0.1 volts.

Faraday has no problems with that.

Kirchhoff has a problem with that, but who cares about Kirchhoff?

Faraday is the law that matters, because Faraday's Law always holds, because if  $d\phi/dt$  is 0, then you get Kirchhoff's.

Kirchhoff's rule is simply a special case of Faraday's Law, and Faraday's Law always holds, so Kirchhoff is for the birds, and Faraday is not.

Suppose you go from D to A and back to D.

Well, we know that  $V_D$  minus  $V_A$ , if we go through this- if we go this way, through  $R_2$ , we know that  $V_D$  minus  $V_A$  is +0.9 volts.

Now we are at A and we go through the left side back to D.

So we now have  $V_A$  minus  $V_D$ .

That of course is now +0.1 volts, because remember, if  $V_D$  minus  $V_A$  is -0.1 then  $V_A$  minus  $V_D$  is plus.

And so we add them up, and we find that  $V_D$  minus  $V_D$  is plus 1 volts.

Kirchoff said, has to be 0, because I'm back at the same potential where I was before.

Faraday says, uh-uh, I'm sorry, you can't do that.

That 1 volt is exactly that EMF of 1 volt.

That is the closed loop integral of  $\mathbf{E} \cdot d\mathbf{L}$  around that loop.

It's no longer 0.

And therefore, whenever you define potential difference, if you do that in the way of the integral of  $\mathbf{E} \cdot d\mathbf{L}$ , keep in mind that with non-conservative fields, it depends on the path, and that is very non-intuitive.

And I'm going to demonstrate this now to you.

I have a circuit which is exactly what you have here.

I have 900 ohms in a conducting copper wire here and I have 100 ohms here and here is the solenoid.

We can switch the current on in the solenoid and get a blast of magnetic field coming up, and the system is going to react by driving a current in the direction that you see there.

And I'd like to be even a little bit more quantitative, so that you get a little bit more for your money.

The magnetic field takes a little bit of time to reach the maximum value.

In this course, we will be able to calculate the time that it takes for the magnetic field to build up.

We didn't get to that yet, so forget that part.

It's not so important.

I just want you to appreciate the fact that the magnetic field as a function of time will come up like this and will then reach a maximum.

It's no longer changing.

It's constant.

It's a maximum value.

It's very high, seven, 800 Gauss or so for this unit.

We are not interested in a magnetic field.

We are interested in the change of the magnetic field, so the change of the magnetic field,  $dB/dT$ , is going to be something like this, it's the derivative of this curve.

And that is proportional with the induced EMF and that's in proportion with the current, through Ohm's law.

So if we now plot the voltage as a function of -- let me do that here, the voltage as a function of time, then that voltmeter on the right side, I call that  $V_2$ , will do this.

This is  $V_2$ , which is  $I$  times  $R_2$  at the maximum value.

If those values were correct it would be 0.9 volts, and  $V_1$  would go like this.

$V_1$  equals minus  $I$  times  $R_1$ .

That gives me the -0.1 volts.

So the question now is what is the largest value of  $dB/dT$  that we can expect.

We also have to know the surface area of the solenoid so we can convert it to a flux change.

Well, the change in magnetic fields is roughly at the fastest here is about 100 Gauss in 1 millisecond.

Very roughly.

So that would mean a field change,  $dB/dT$ .

That's the maximum value possible only in the beginning of about 10 Tesla per second.

And the surface area, which is that inner circle there through which the flux is changing, the fact that my surface has to be attached to that loop doesn't change the magnetic flux.

The magnetic flux is only determined, of course, by that inner portion, and so if the inner portion has an area of say 10 square centimeters, which is  $10 \times 10^{-4}$  square meters, then  $d\phi/dt$  will be approximately  $10 \times 10^{-4} \times 100$ , so that's about 0.1, and that's volts.

That's EMF.

I don't care about the direction, because I know Lenz's Law.

So you're going to see an experiment which is almost identical to what I have there, except all values are down by a factor of 10.

But that's all.

And you're going to see that demonstration there.

And a few years ago, when I first did this experiment in 26-100, there were several of my colleagues, professors of both the physics department and EE department in my audience.

And some did not believe what they saw.

In fact, it was so bad that after my lecture they came to me and some accused me for having cheated on the demonstration.

This tells you something about them.

Imagine, professors in physics and professors in electrical engineering department who did not believe what they were seeing.

That tells you how non-intuitive this is.

The simple fact that we had 1 voltmeter connected to point D and A, and another voltmeter connected to the same point, they were unwilling to accept that the 2 voltmeters read a totally different value.

They were not used to non-conservative fields.

Their brains couldn't handle it.

But that's the way it is, and I'm going to show this to you now.

You're going to see it there, and when you see this demonstration, it will be probably the only time in your life that you will ever see this, and I want you to remember this.

You're going to see something that is very strange, and I want you to tell you grandchildren about it, that you have actually seen it with your own eyes.

You're going to see it there on the left side, you're going to see V1, and on the right side you're going to see V2.

The vertical scale is such that very roughly from here to here is about 0.1 volts.

And the horizontal unit is about 5 milliseconds, and the whole voltage pulse lasts about 10 milliseconds, because from here to here is about 10 milliseconds.

And the value that you expect for V2 will be 9 times higher than V1 and the polarities will be reversed.

If you're ready for this big moment in your life, three, two, one, zero.

Look on the left.

There's V1.

Notice, it's negative.

Look on the right.

There's V2.

It's about 9 times larger than V1.

Don't pay any attention to this wiggle.

It has to do with the voltage that we apply, which is not exactly flat.

And notice that the whole pulse goes from here to here, lasts about 10 milliseconds.

The moment that the magnetic field reaches a maximum and remains constant, there is no longer any induced current.

Think about this.

Give this some thought.

This is not easy.

And have a good weekend.