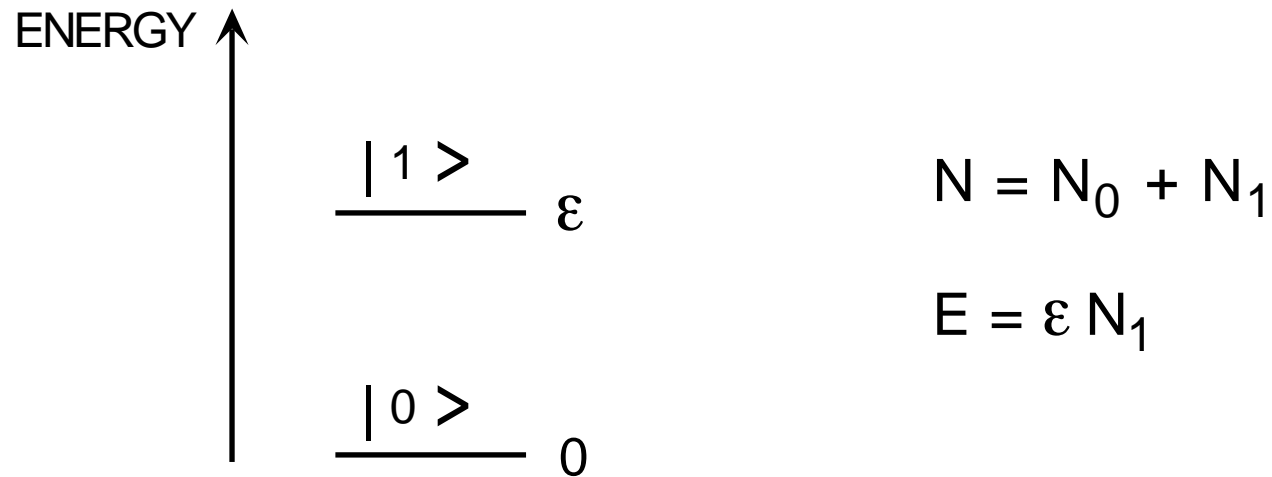
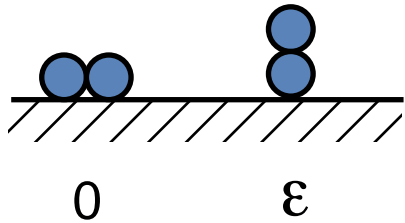


EXAMPLE 2 Level System

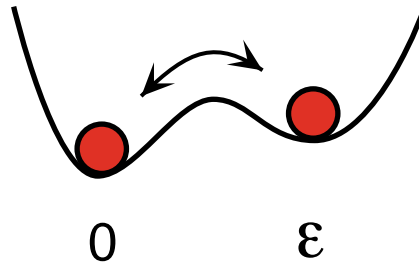
Ensemble of N "independent" systems



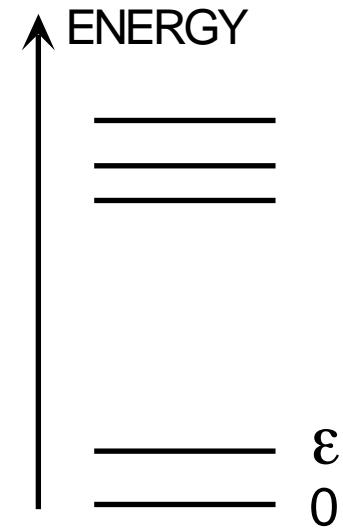
SURFACE MOLECULES



IONS IN A CRYSTAL



LOWEST LYING STATES



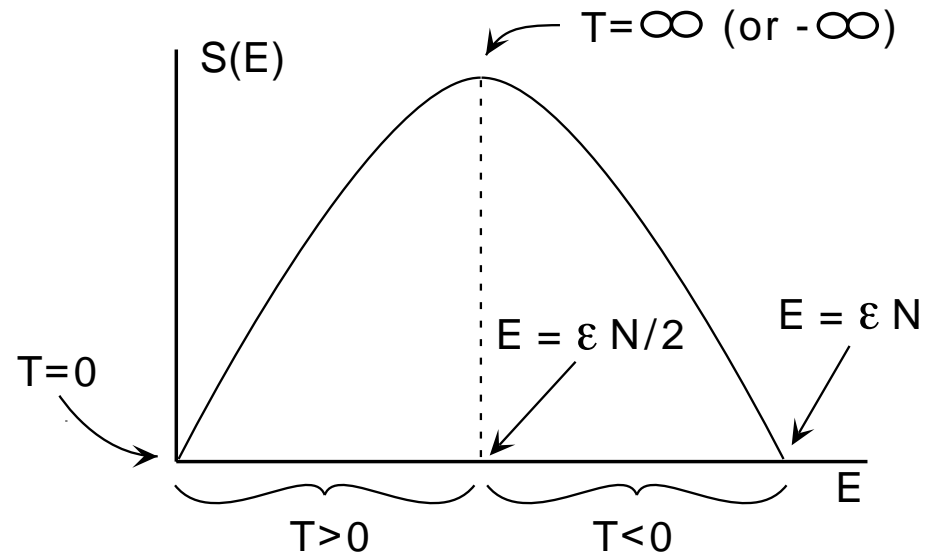
- $E \leftrightarrow N_1$
- NO WORK POSSIBLE (JUST HEAT FLOW)

$$\Omega(E) = \frac{N!}{N_1!(N-N_1)!}$$

1 when $N_1 = 0$ or N

Maximum when $N_1 = N/2$

$$S(E) = k \ln \Omega(E)$$



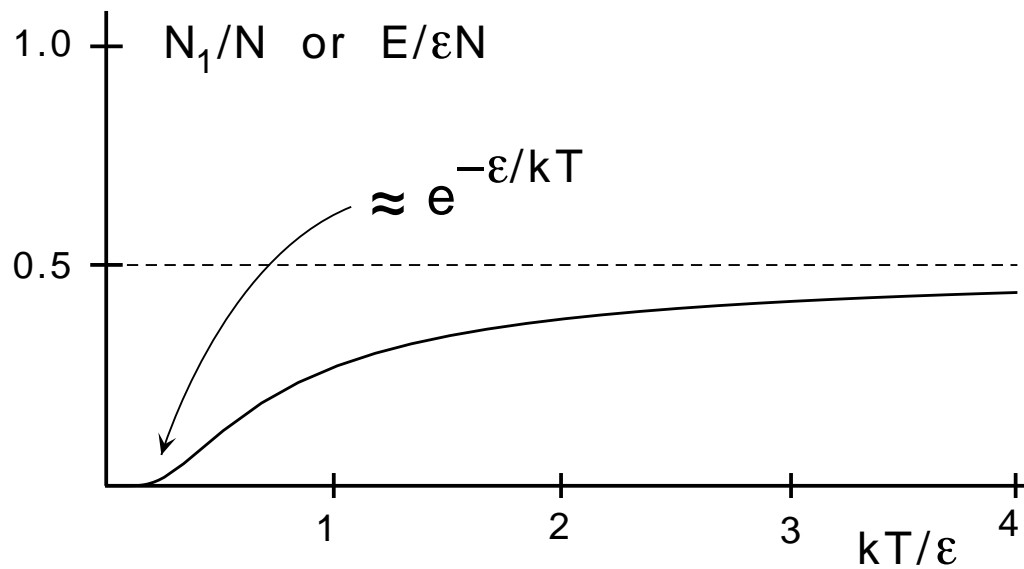
$$\ln N! \approx N \ln N - N$$

$$S(E) = k[N \ln N - N_1 \ln N_1 - (N - N_1) \ln(N - N_1) - N + N_1 + N - N_1]$$

$$\begin{aligned} \frac{1}{T} &= \left(\frac{\partial S}{\partial E} \right)_N = \frac{\partial S}{\partial N_1} \underbrace{\frac{\partial N_1}{\partial E}}_{1/\epsilon} = \frac{k}{\epsilon} [-1 - \ln N_1 + 1 + \ln(N - N_1)] \\ &= \frac{k}{\epsilon} \ln \left(\frac{N - N_1}{N_1} \right) = \frac{k}{\epsilon} \ln \left(\frac{N}{N_1} - 1 \right) \end{aligned}$$

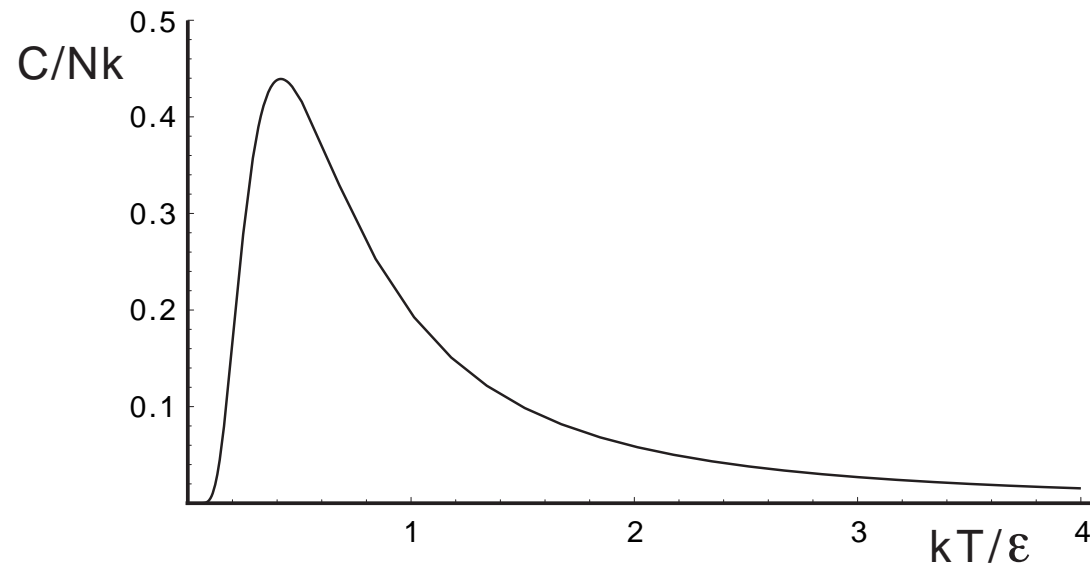
$$\frac{N}{N_1} - 1 = e^{\epsilon/kT} \rightarrow N_1 = \frac{N}{e^{\epsilon/kT} + 1}$$

$$E = \epsilon N_1 = \frac{\epsilon N}{e^{\epsilon/kT} + 1}$$



$$C \equiv \frac{\partial E}{\partial T} = Nk \left(\frac{\epsilon}{kT} \right)^2 \frac{e^{\epsilon/kT}}{(e^{\epsilon/kT} + 1)^2}$$

$$\rightarrow Nk \left(\frac{\epsilon}{kT} \right)^2 e^{-\epsilon/kT} \text{ low } T, \quad \rightarrow \frac{Nk}{4} \left(\frac{\epsilon}{kT} \right)^2 \text{ high } T$$



$$p(n) = ? \quad n = 0, 1 \qquad p(n) = \frac{\Omega'}{\Omega}$$

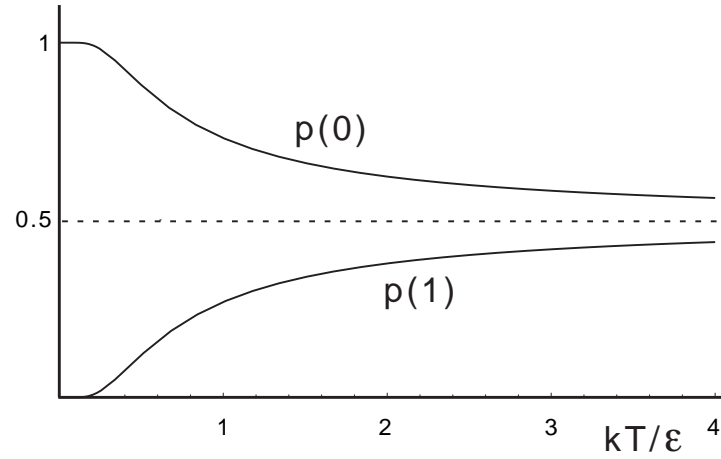
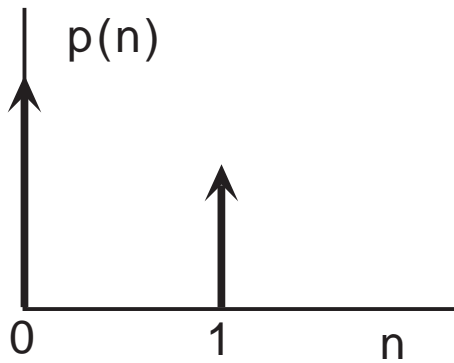
In Ω' $N \rightarrow N - 1$ and $N_1 \rightarrow N_1 - n$

$$p(n) = \frac{\frac{(N-1)!}{(N_1-n)!(N-1-N_1+n)!}}{\frac{N!}{N_1!(N-N_1)!}}$$

$$p(n) = \underbrace{\frac{(N-1)!}{N!}}_{1/N} \underbrace{\frac{N_1!}{(N_1-n)!}}_{1 \quad n=0} \underbrace{\frac{(N-N_1)!}{(N-N_1-1+n)!}}_{N-N_1 \quad n=0}$$

$$N_1 \quad n=1 \qquad \qquad \qquad 1 \quad n=1$$

$$\left. \begin{aligned} p(0) &= \frac{N-N_1}{N} = 1 - \frac{N_1}{N} \\ p(1) &= \frac{N_1}{N} = [e^{\epsilon/kT} + 1]^{-1} \end{aligned} \right\} p(0) + p(1) = 1$$



$$E = (0)N p(0) + (\epsilon)N p(1) = \frac{\epsilon N}{e^{\epsilon/kT} + 1}$$

But we knew E , so we could have worked backwards to find $p(1)$.

MICROCANONICAL ENSEMBLE

MODEL THE SYSTEM



FIND $\Omega(E, N, V \dots)$

THERMODYNAMIC RESULTS

MICROSCOPIC INFORMATION

FIND $S(E, N, V \dots)$

$P(\dots) = \Omega'/\Omega$



$$\left. \frac{\partial S}{\partial E} \right|_{N, V} = \frac{1}{T}$$

$$\left. \frac{\partial S}{\partial V} \right|_{E, N} = \frac{P}{T}$$

etc.

The microcanonical ensemble is the starting point for Statistical Mechanics.

- We will no longer use it to solve problems.
- We will develop our understanding of the 2^{ND} law.
- We will derive the canonical ensemble, the real workhorse of S.M.