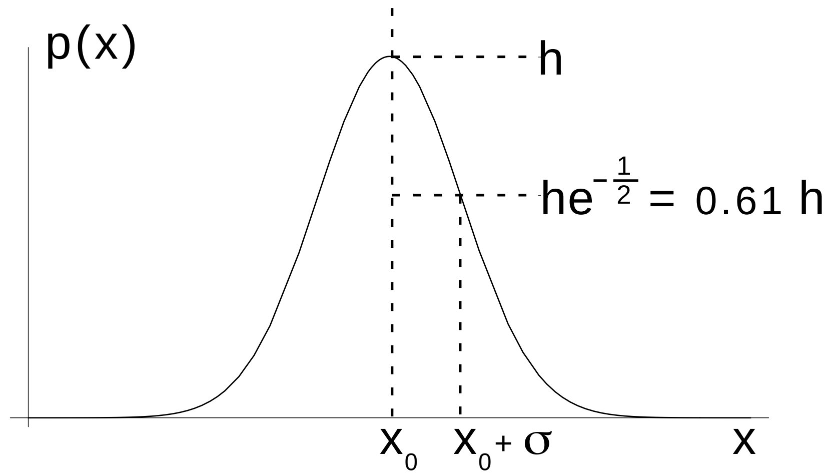
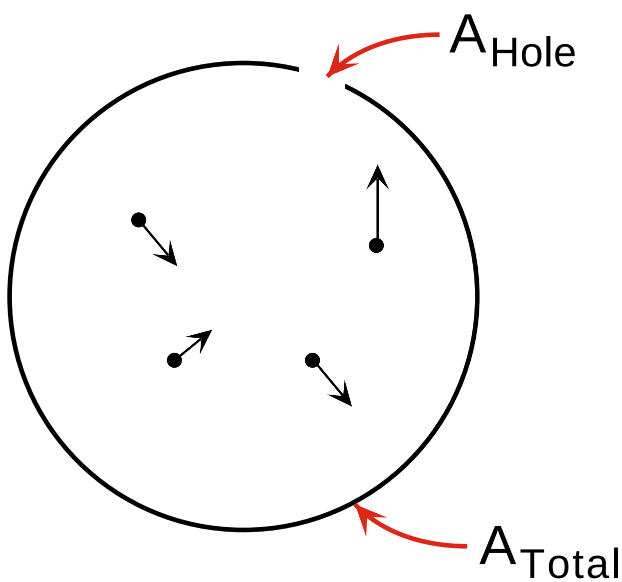
Gaussian density (memorize)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$



2 parameters

Example Atom escaping from a cavity



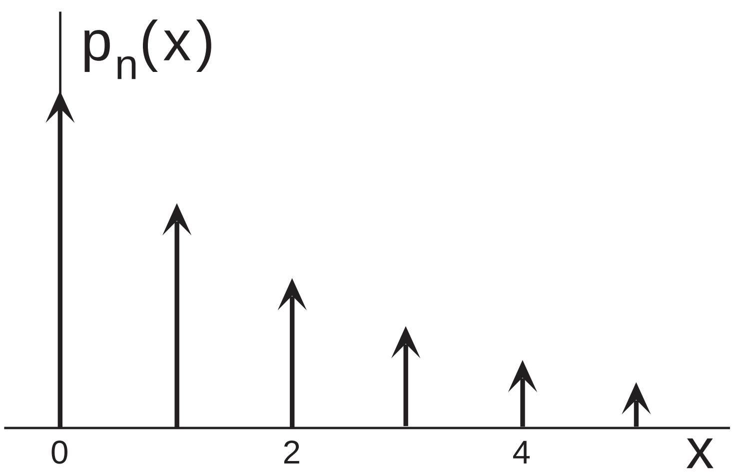
Atom escapes after n^{th} wall encounter

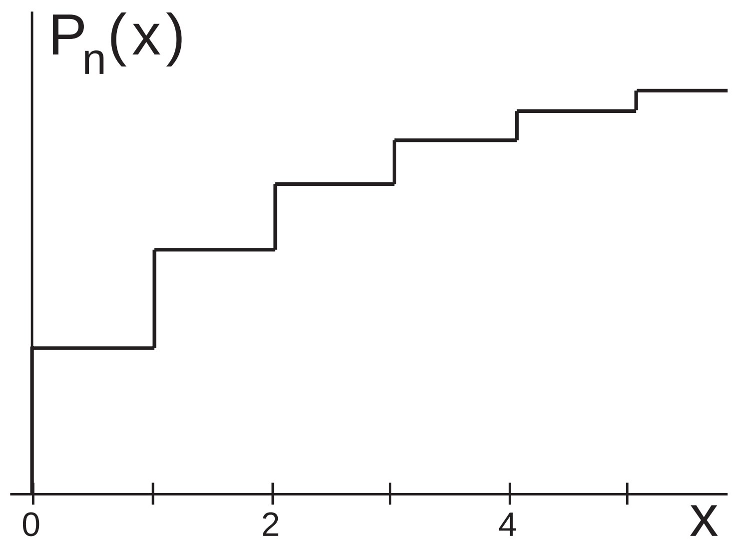
$$p(n) = (\frac{A_H}{A_T})(1 - \frac{A_H}{A_T})^n$$

$$n = 0, 1, 2, \cdots$$

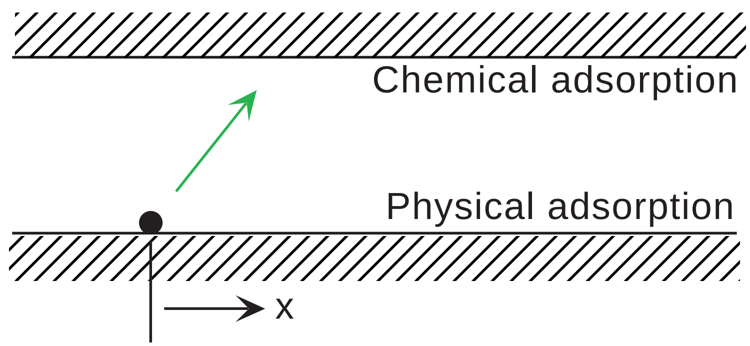
$$p_n(x) = \sum_{n=0}^{\infty} (\frac{A_H}{A_T})(1 - \frac{A_H}{A_T})^n \delta(x-n)$$

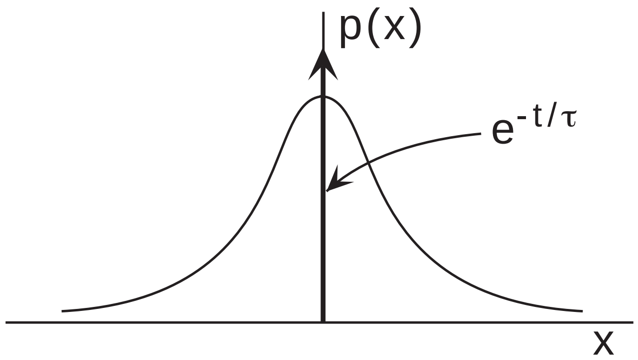
Called a geometric or a Bose-Einstein density





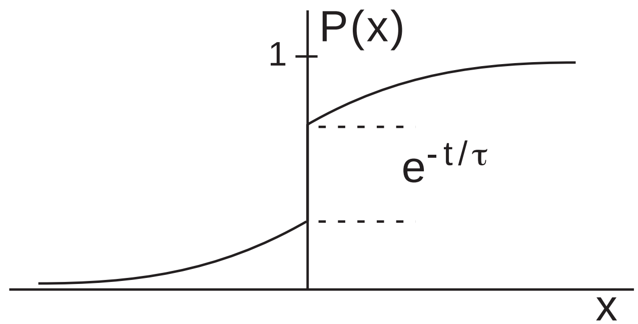
<u>Example</u> Mixed, t dependent RV





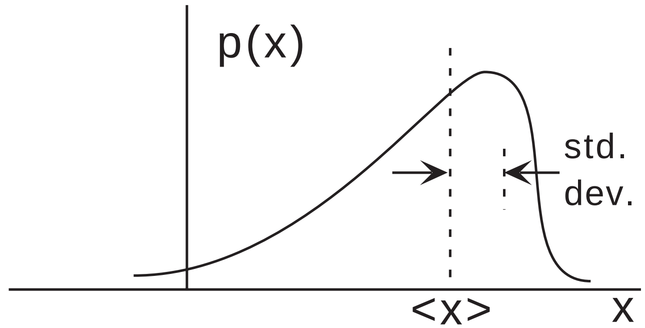
Given: atom on bottom at $t = 0$

$$p(x) = e^{-t/\tau}\,\delta(x) + (1 - e^{-t/\tau})\,f(x)$$



Averages

$$< f(x) > \equiv \int_{-\infty}^{\infty} f(x) p(x) \, dx$$



$< x >$ is the **mean**

$< x^2 >$ is the **mean square**

$$\begin{aligned}
< (x - < x >)^2 > &= < (x^2 - 2x < x > + < x >^2) > \\
&= < x^2 > -2 < x >^2 + < x >^2 \\
&= < x^2 > - < x >^2 \quad \text{is the } \textbf{variance} \\
&\equiv (\textbf{standard deviation})^2
\end{aligned}$$

Gaussian

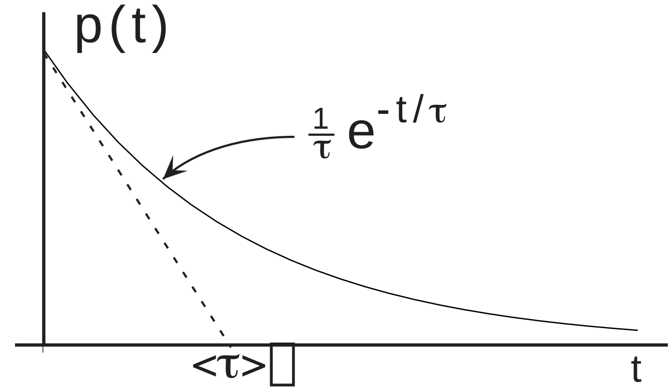
$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-(x-x_0)^2/2\sigma^2}$$

$$< x >= x_0$$

$$\text{Var}(x) = \sigma^2$$

Controlled separately

Exponential

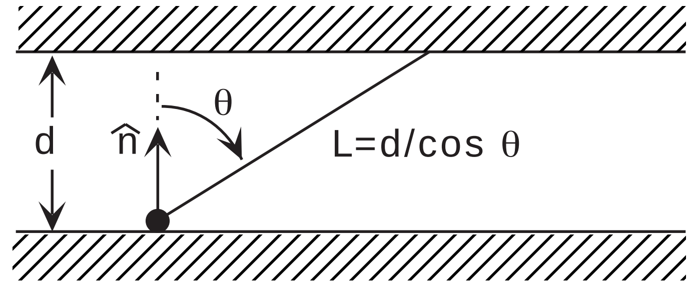


$$< t >= \tau$$

$$\text{Var}(t) = \tau^2$$

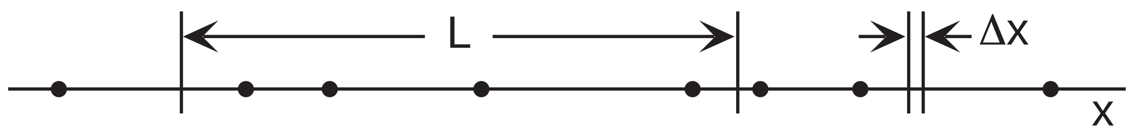
Determined by same parameter

Example Mean free path



$$
\begin{aligned}
< L > &= \int_0^{\pi/2} (d/\cos\theta) p(\theta)\, d\theta \\
&= \int_0^{\pi/2} (d/\cos\theta)\, 2\sin\theta\cos\theta\, d\theta = 2d \int_0^{\pi/2} \sin\theta\, d\theta \\
&= 2d\, [_0^{\pi/2} (-\cos\theta) = 2d
\end{aligned}
$$

Poisson density   Events occur randomly along a line at a rate r per unit length



$p(1) \to r\Delta x$ as $\Delta x \to 0$

Events are statistically independent

$$p(n) = \frac{1}{n!}(rL)^n e^{-rL} = \frac{1}{n!} < n >^n e^{-<n>}$$

Examples of Poisson probability densities