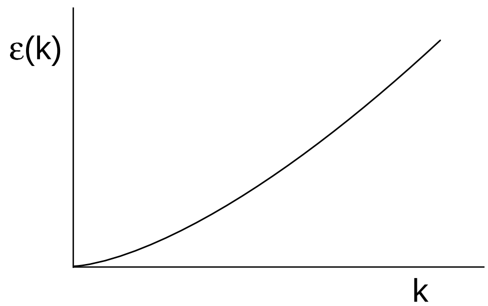
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2003

Practice Exam #4

Problem 1: Ripplons (35 points)



We have seen that the bulk motion of a solid or liquid can be described by harmonic normal modes (that is, normal modes each having a harmonic oscillator Hamiltonian) known as phonons. In a similar manner the two dimensional waves on an interface between a liquid and its vapor can be described by harmonic normal modes known as "ripplons" each having a single direction of polarization perpendicular to the interface. The dispersion curve for these elementary excitations is isotropic and given by $\hbar\omega(k) \equiv \epsilon(k) = bk^{3/2}$ where $k = \sqrt{k_x^2 + k_y^2}$. For a rectangular sample with dimensions L_x and L_y, the wavevectors allowed by periodic boundary conditions are $\vec{k} = (2\pi/L_x)m\hat{x} + (2\pi/L_y)n\hat{y}$ where m and n can take on all positive and negative integer values.

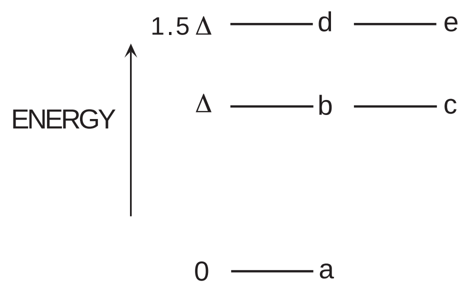
a) What is the density of allowed wavevectors $D(\vec{k})$ such that $D(\vec{k})dk_x dk_y$ gives the number of allowed wavevectors in the area $dk_x dk_y$ around the point $\vec{k}$ in k-space?

b) Find an expression for the density of states as a function of energy $D(\epsilon)$ for the ripplons in terms of the parameter b and the area $A = L_x L_y$. Sketch your result.

c) Find an expression for the ripplon contribution to the constant area heat capacity $C_A(T)$. Leave your result in terms of a dimensionless integral (do not try to evaluate the integral). How does $C_A(T)$ depend on T? Sketch the result.

d) Does the system exhibit energy gap behavior? Explain your reasoning.

Problem 2: Impurity Atom (35 points)



An impurity atom in a solid has 3 electrons (spin 1/2 Fermions) above a filled, inert electronic shell. These electrons have available to them 5 spatial single-particle states, $\psi_a, \psi_b, \psi_c, \psi_d$ and ψ_e with energies $\epsilon_a = 0, \epsilon_b = \epsilon_c = \Delta$, and $\epsilon_d = \epsilon_e = 3\Delta/2$. In what follows, assume that there is no interaction between the electrons. [Note that part g revisits b,c,d and f.]

a) How many 3-particle states are available to the atom? Be sure to take into account both the spin and spatial variables when determining your number.

b) Write down the terms in the partition function $Z(T)$ arising from the states corresponding to the <u>two</u> lowest 3-particle energies.

c) Write down the terms in the partition function $Z(T)$ arising from the states corresponding to the <u>two</u> highest 3-particle energies.

d) What is the entropy at $T = 0$?

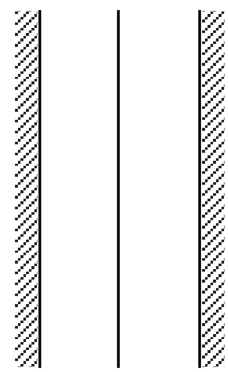
e) What value does the entropy approach asymptotically at very high T?

f) What is the asymptotic value for the heat capacity at very high T?

g) Repeat b), c), d) and f) [but not a) or e)] for the case where the three identical particles are spin 0 Bosons*.

*I will treat to a free dinner in the fall anyone who answers a) for spin 0 Bosons without resorting to brute force.

Problem 3: Realistic Super Insulation (20 points)



Two parallel plates of infinite extent are separated by a vacuum and maintained at temperatures T_H and T_C. The surface of each plate acts as a black body. A thin conducting sheet is suspended in the vacuum as shown in the figure. Heat can be transferred to the sheet only through the vacuum. The sheet has an absorptivity $\alpha < 1$, and a power reflectivity $r = 1 - \alpha$.

a) Find the steady state temperature T of the sheet.

b) Find the heat flow from the hotter plate to the colder plate as a fraction $\mathcal{F}$ of that which would occur in the absence of the sheet.

Problem 4: Adiabatic Demagnetization (10 points)

Consider the extreme situation of an ideal paramagnet in thermal contact with a sample so small that the thermodynamics of the assembly is dominated by that of the paramagnet alone. The assembly is cooled adiabatically by reducing the applied magnetic field from 8 kilogauss to 20 gauss. What is the final temperature if the initial temperature was 1 Kelvin? [This does not require an extensive calculation.]

Work in simple systems

Hydrostatic system	$-PdV$
Surface film	$\mathcal{S}dA$
Linear system	$\mathcal{F}dL$
Dielectric material	$\mathcal{E}d\mathcal{P}$
Magnetic material	HdM

Thermodynamic Potentials when work done on the system is $dW = Xdx$

Energy	E	$dE = TdS + Xdx$
Helmholtz free energy	$F = E - TS$	$dF = -SdT + Xdx$
Gibbs free energy	$G = E - TS - Xx$	$dG = -SdT - xdX$
Enthalpy	$H = E - Xx$	$dH = TdS - xdX$

Statistical Mechanics of a Quantum Harmonic Oscillator

$$\epsilon(n) = (n + \tfrac{1}{2})\hbar\omega \qquad n = 0, 1, 2, \ldots$$

$$p(n) = e^{-(n+\frac{1}{2})\hbar\omega/kT}/Z(T)$$

$$Z(T) = e^{-\frac{1}{2}\hbar\omega/kT}(1 - e^{-\hbar\omega/kT})^{-1}$$

$$< \epsilon(n) >= \tfrac{1}{2}\hbar\omega + \hbar\omega(e^{\hbar\omega/kT} - 1)^{-1}$$

Radiation laws

Kirchoff's law: $e(\omega, T)/\alpha(\omega, T) = \frac{1}{4}c\,u(\omega, T)$ for all materials where $e(\omega, T)$ is the emissive power and $\alpha(\omega, T)$ the absorptivity of the material and $u(\omega, T)$ is the universal blackbody energy density function.

Stefan-Boltzmann law: $e(T) = \sigma T^4$ for a blackbody where $e(T)$ is the emissive power integrated over all frequencies. ($\sigma = 56.9 \times 10^{-9}$ watt-m^{-2}K^{-4})