

## 8.044 SOLUTIONS EXAM #4

$$1. a) C_V \sim (\text{CLASSICAL RESULT}) \times (\text{FRACTION OF ELECTRONS INFLUENCED}) = \left(\frac{3}{2} Nk\right) \left(\frac{kT}{E_F}\right) \\ \propto T \Rightarrow \underline{\underline{\eta = 1}} \text{ IN } C_V = \gamma V T^\eta$$

$$b) \underline{\underline{\frac{dE}{dT} = C_V(T) = \gamma V T}}$$

$$c) \underline{\underline{\frac{dE}{dt} = -4\pi R^2 \sigma T^4}} \quad \text{FROM STEFAN-BOLTZMANN LAW}$$

$$d) \frac{dT}{dt} = \frac{dT}{dE} \frac{dE}{dt} = -4\pi R^2 \sigma T^4 / \gamma \left(\frac{4}{3}\pi R^3\right) T$$

$$\underline{\underline{\frac{dT}{dt} = -\frac{3\sigma}{\gamma R} T^3}}$$

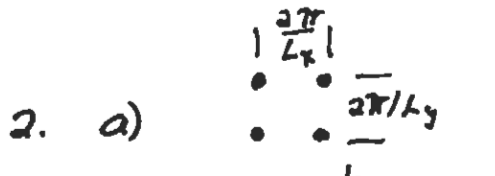
SOLUTION (NOT REQUIRED)

$$-\frac{dT}{T^3} = \frac{3\sigma}{\gamma R} dt$$

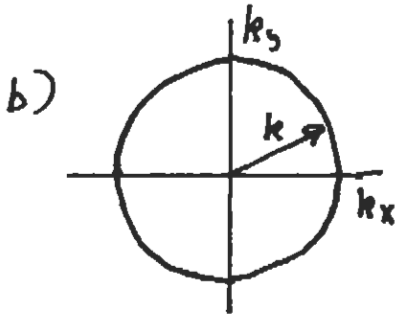
$$-\int_{T_0}^{T(t)} \frac{dT}{T^3} = \frac{3\sigma}{\gamma R} t = \frac{1}{2} (T(t)^{-2} - T_0^{-2})$$

$$T^{-2}(t) = T_0^{-2} + \frac{6\sigma}{\gamma R} t = \frac{1}{T_0^2} \left(1 + \underbrace{\frac{6\sigma T_0^2}{\gamma R}}_{\equiv \tau^{-1}} t\right)$$

$$\underline{\underline{T(t)/T_0 = (1 + t/\tau)^{-1/2}}}$$



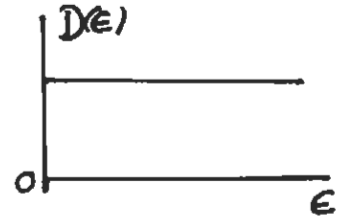
$$D(\vec{k}) = \frac{1}{\frac{2\pi}{L_x} \frac{2\pi}{L_y}} = \frac{A}{(2\pi)^2} \quad \forall \vec{k}$$



SPIN →

$$\# = 2 D(\vec{k}) \pi k^2 = \frac{2A\pi}{(2\pi)^2} \frac{2mE}{\hbar^2}$$

$$D(E) = \frac{d\#}{dE} = \frac{Am}{\pi \hbar^2}$$

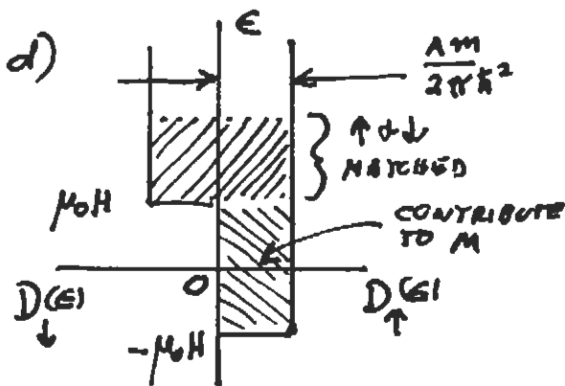


c)

$$N = \int_0^{E_F} D(E) dE = \left( \frac{Am}{\pi \hbar^2} \right) E_F \Rightarrow E_F = \frac{\pi \hbar^2}{m} \frac{N}{A}$$

$$E = \int_0^{E_F} E D(E) dE = \left( \right) \frac{1}{2} E_F^2 = \frac{1}{2} N E_F$$

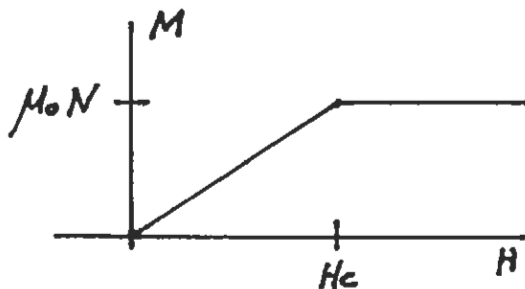
$$E_{\text{Electron Gas}} = \frac{\pi \hbar^2}{2m} \frac{N^2}{4\pi R^2} = \frac{\hbar^2}{8m} \frac{N^2}{R^2}$$



$$M = \mu_0 (2\mu_0 H) \frac{Am}{2\pi \hbar^2}$$

$$= \frac{\mu_0^2 H m}{\pi \hbar^2} 4\pi R^2$$

$$M = \frac{4m\mu_0^2 R^2}{\hbar^2} H \quad M < \mu_0 N$$



$$\frac{4m\mu_0^2 R^2 H_c}{\hbar^2} = \mu_0 N$$

$$H_c = \frac{N \hbar^2}{4m R^2 \mu_0}$$

$$3. a) \langle \mu \rangle = \iint p(\theta, \varphi) [\mu_0 \cos \theta] d\Omega$$

$$\text{BUT } \frac{dz_1}{d\gamma} = \iint \cos \theta e^{\gamma \cos \theta} d\Omega$$

$$= \frac{z_1}{\mu_0} \iint [\mu_0 \cos \theta] p(\theta, \varphi) d\Omega = \frac{z_1}{\mu_0} \langle \mu \rangle$$

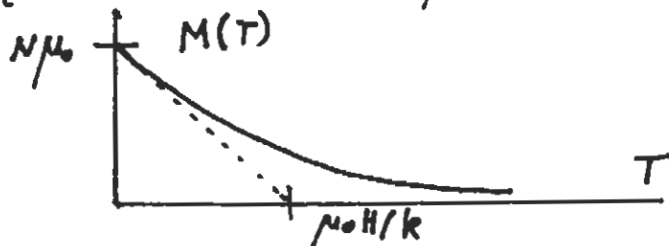
$$\text{THUS } M = N \langle \mu \rangle = \underline{\underline{N \mu_0 \frac{1}{z_1} \frac{dz_1}{d\gamma}}}$$

$$b) M = N \mu_0 \frac{\gamma}{\sinh(\gamma)} \left[ \frac{1}{\gamma} \cosh(\gamma) - \frac{1}{\gamma^2} \sinh(\gamma) \right]$$

$$\underline{\underline{M = N \mu_0 \left[ \coth(\gamma) - \frac{1}{\gamma} \right]}}$$

$$c) \lim_{\gamma \rightarrow 0} M(\gamma) = N \mu_0 \left( \frac{1}{3} \gamma \right) = \underline{\underline{\frac{N \mu_0^2 H}{3} \frac{1}{kT}}} \quad \text{FOR HIGH } T$$

$$\lim_{\gamma \rightarrow \infty} M(\gamma) = \underline{\underline{N \mu_0 \left( 1 - \frac{kT}{\mu_0 H} \right)}} \quad \text{FOR LOW } T$$



- d) CURIE LAW  $\Rightarrow M \propto \frac{H}{T}$  FOR HIGH T YES  
 ENERGY GAP  $\Rightarrow e^{-\Delta/kT}$  LEADING T DEP, AT LOW T  
NO BECAUSE THERE IS NO GAP

$$e) -kT \ln Z = -NkT \ln Z_1(\gamma) = G(T, H)$$

$$S = -\left. \frac{\partial G}{\partial T} \right|_H = Nk \left( \ln Z_1(\gamma) + T \left. \frac{d \ln Z_1}{d\gamma} \frac{\partial \gamma}{\partial T} \right|_H \right) = \underline{\underline{Nk \left( \ln Z_1(\gamma) - \frac{\gamma}{z_1} \frac{dz_1}{d\gamma} \right)}}$$