Thermodynamic Approach

$$u(T) \equiv \int_0^\infty u(\nu, T)\, d\nu$$

Then

$$E(T, V) = u(T)V$$

$$P(T, V) = \tfrac{1}{3}u(T)$$

This is enough to allow us to find $u(T)$.

$$
\begin{aligned}
dE &= TdS - PdV \\
\left(\frac{\partial E}{\partial V}\right)_T &= T\left(\frac{\partial S}{\partial V}\right)_T - P = T\left(\frac{\partial P}{\partial T}\right)_V - P \\
&= \tfrac{1}{3}Tu'(T) - \tfrac{1}{3}u(T) \\
\text{also} &= u(T) \\
\Rightarrow \quad u'(T) &= \frac{4}{T}u(T) \\
u(T) &= AT^4
\end{aligned}
$$

Emissive Power of a Black ($\alpha = 1$) Body

$$e(\nu, T) = \tfrac{1}{4} c\, u(\nu, T) \Rightarrow e(T) = \tfrac{1}{4} c\, u(T) = \tfrac{1}{4} A c T^4$$

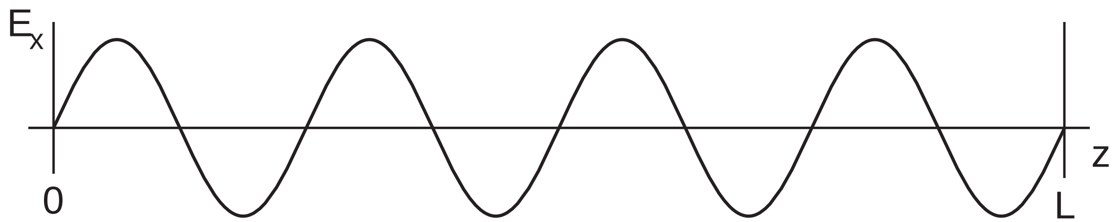
$$e(T) \equiv \sigma T^4$$

This is known as the STEFAN-BOLTZMANN LAW.

$$\sigma = 56.7 \times 10^{-9} \text{ watts}/m^2 K^4$$

Statistical Mechanical Approach

$\mathcal{H}$? Single normal mode (plane standing wave) in a rectangular conducting cavity.



$$\vec{E}_{0,0,n,\vec{1}_x}(\vec{r},t) = E(t)\sin(n\pi z/L)\vec{1}_x$$
$$\vec{B}_{0,0,n,\vec{1}_y}(\vec{r},t) = (n\pi c^2/L)^{-1}\dot{E}(t)\cos(n\pi z/L)\vec{1}_y$$

Energy density $= \frac{1}{2}\epsilon_0 \vec{E}\cdot\vec{E} + \frac{1}{2}\frac{1}{\mu_0}\vec{B}\cdot\vec{B}$ [no $\vec{r}$ or t average]

$$\mathcal{H} = \frac{V}{2}\left[\tfrac{1}{2}\epsilon_0 E^2(t) + \tfrac{1}{2}\frac{1}{\mu_0}(n\pi c^2/L)^{-2}\dot{E}^2(t)\right]$$

$$= \frac{V}{2}\frac{\epsilon_0}{2}\left[\, E^2(t) + (n\pi c/L)^{-2}\,\dot{E}^2(t)\right]$$

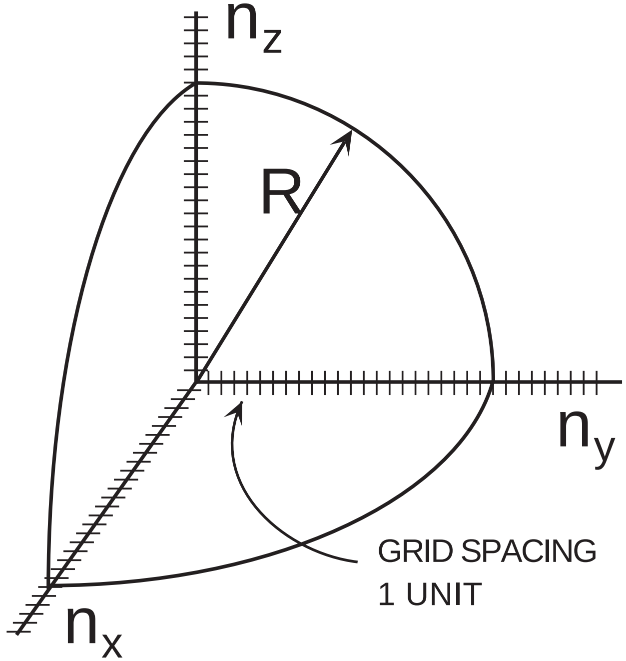
$\Rightarrow$ Each mode corresponds to a harmonic oscillator.

Count the modes.

$$\vec{E}_{n_x,n_y,n_z} = |E|\vec{\epsilon}_j \sin(n_x\pi x/L)\sin(n_y\pi y/L)\sin(n_z\pi z/L)e^{i\omega t}$$

The unit polarization vector $\vec{\epsilon}_j$ has 2 possible orthogonal directions and $n_i = 1, 2, 3\cdots$.

$$\frac{\partial^2\vec{E}}{\partial t^2} - c^2\,\vec{\nabla}^2 E = 0 \quad \Rightarrow \quad \omega^2 = \left(\frac{\pi c}{L}\right)^2 (n_x^2 + n_y^2 + n_z^2)$$



If the radian frequency $< \omega$

$$R = \sqrt{n_x^2 + n_y^2 + n_z^2} = \frac{L}{\pi c}\,\omega$$

modes (freq. $< \omega$)

$$= 2 \times \frac{1}{8} \times \frac{4}{3}\pi R^3$$

$$= \frac{\pi}{3}\left(\frac{L}{\pi c}\right)^3 \omega^3$$

$$D(\omega) = \frac{d\#}{d\omega} = \pi \left(\frac{L}{\pi c}\right)^3 \omega^2 = \underline{\frac{V}{\pi^2 c^3}\,\omega^2}$$