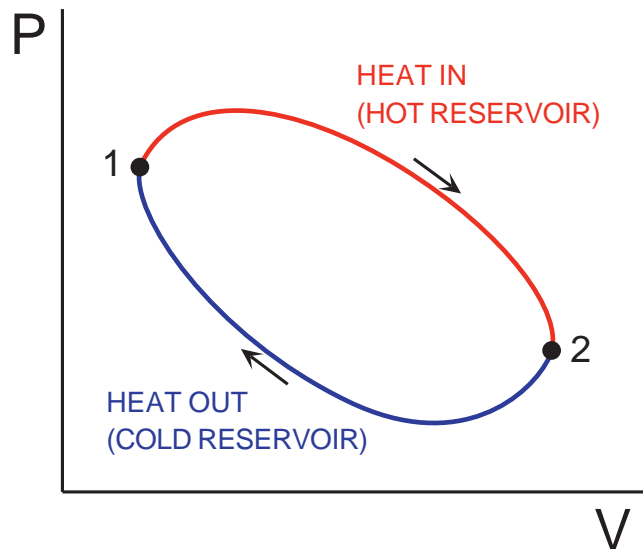


## Heat Engine

- Takes a substance around a closed cycle
- Heat is put into the substance and taken out
- Work is taken out
- Efficiency,  $\eta \equiv (\text{work out}) / (\text{heat in})$

Closed cycle  $\Rightarrow \Delta U = \Delta Q + \Delta W = 0 \Rightarrow \Delta Q = -\Delta W$



$$\Delta Q \equiv \oint dQ$$

$$= \underbrace{\int_1^2 dQ}_{\equiv |Q_H|} + \underbrace{\int_2^1 dQ}_{\equiv -|Q_C|}$$

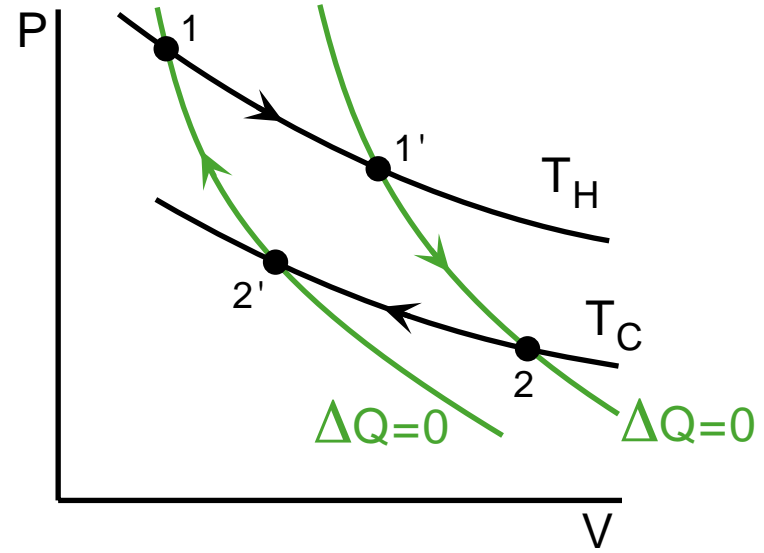
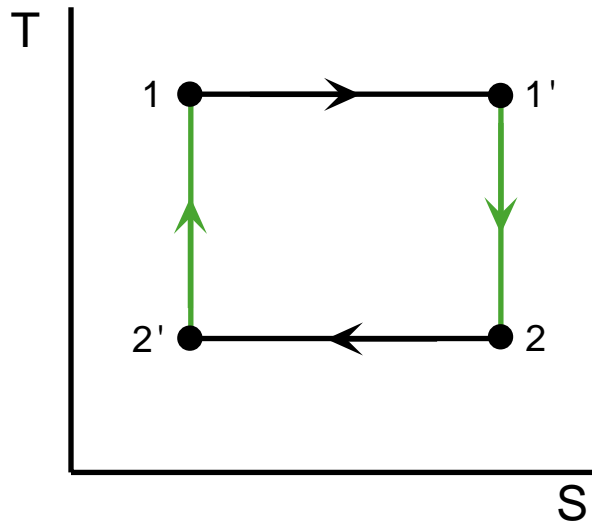
## Most General Case

$$W_{\text{out}} = -\Delta W = \Delta Q = |Q_H| - |Q_C|$$

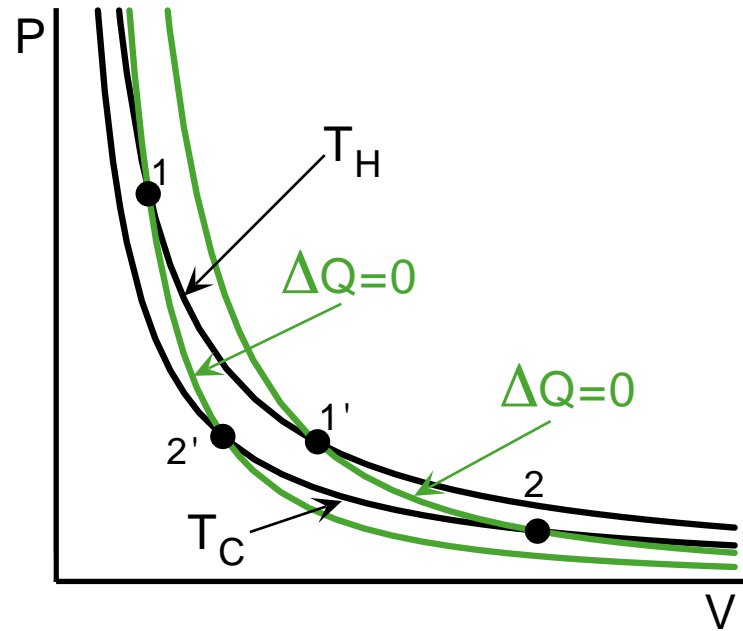
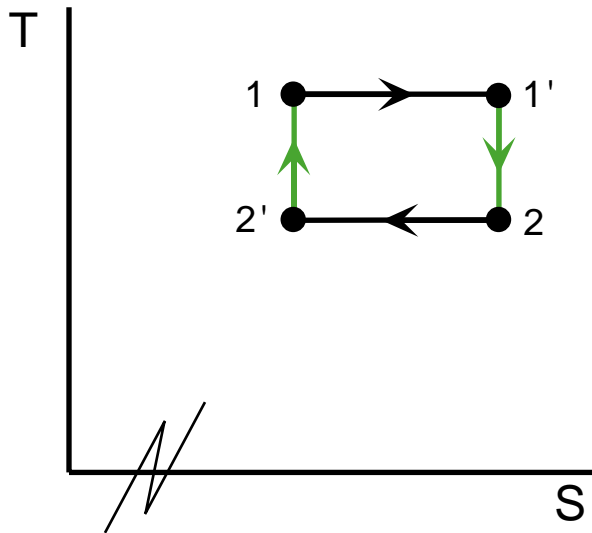
$$\eta \equiv \frac{W_{\text{out}}}{|Q_H|} = \frac{|Q_H| - |Q_C|}{|Q_H|} = 1 - \frac{|Q_C|}{|Q_H|}$$

## Very Special Case      Example: Carnot Cycle

- Any substance
- Isothermal and adiabatic changes



Use the second law:  $dQ \leq TdS$



DRAWN TO SCALE FOR AN IDEAL GAS:  $PV=NkT$

$$T_H = 1.5 T_C \quad S_{\text{HIGH}} - S_{\text{LOW}} = (3/2) Nk \ln 2$$

$$|Q_H| \leq T_H \int_1^{1'} dS$$

$$-|Q_C| \leq T_C \int_2^{2'} dS, \quad \text{use } \int_2^{2'} dS = - \int_1^{1'} dS$$

$$\leq -T_C \int_1^{1'} dS \Rightarrow |Q_C| \geq T_C \int_1^{1'} dS \quad \text{and} \quad \frac{|Q_C|}{|Q_H|} \geq \frac{T_C}{T_H}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \leq 1 - \frac{T_C}{T_H}$$

## Arbitrary Engine Cycle

$dQ \leq TdS$  for each element along the path.

$$\underbrace{\int_1^2 dQ}_{|Q_H|} \leq \int_1^2 TdS \leq T_{\max} \underbrace{\int_1^2 dS}_{\text{positive}}$$

$$\int_2^1 dQ \leq \int_2^1 T dS, \quad \text{both sides are negative}$$

$$|Q_C| \geq \left| \int_2^1 T dS \right| \geq T_{\min} \left| \int_2^1 dS \right|$$

$$\geq T_{\min} \left| \int_1^2 dS \right| \quad \text{since } \oint dS = 0$$

$$\frac{|Q_C|}{|Q_H|} \geq \frac{T_{\min}}{T_{\max}}$$

$$\eta = 1 - \frac{|Q_C|}{|Q_H|} \leq 1 - \frac{T_{\min}}{T_{\max}}$$

## Carnot cycle in a pure thermodynamic approach

- Used to define temp.  $\eta = 1 - \frac{|Q_C|}{|Q_H|} \equiv 1 - \frac{T_C}{T_H}$
- Used to define the entropy

$$\oint \frac{dQ}{T} \leq 0 \Rightarrow \frac{dQ}{T} \text{ is an exact differential}$$