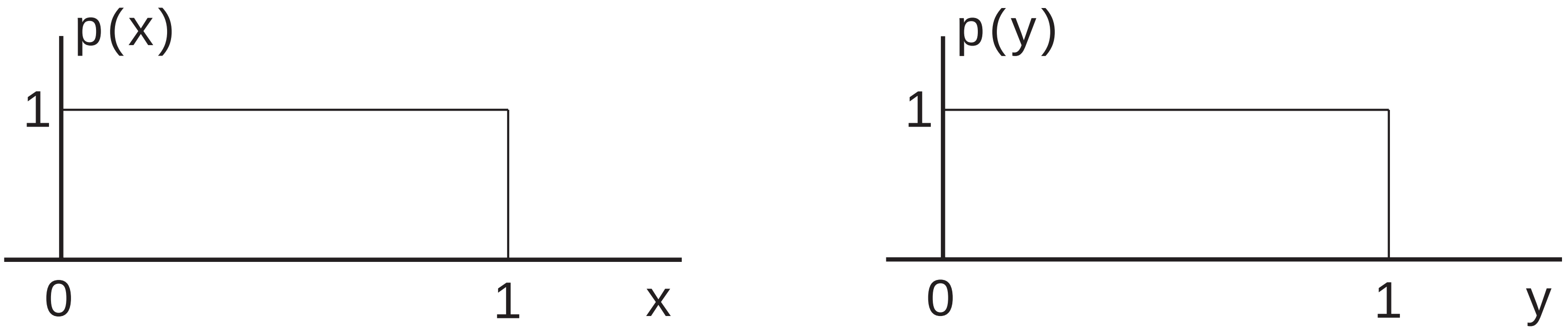
Example Random number generator for programmers



x and y are statistically independent

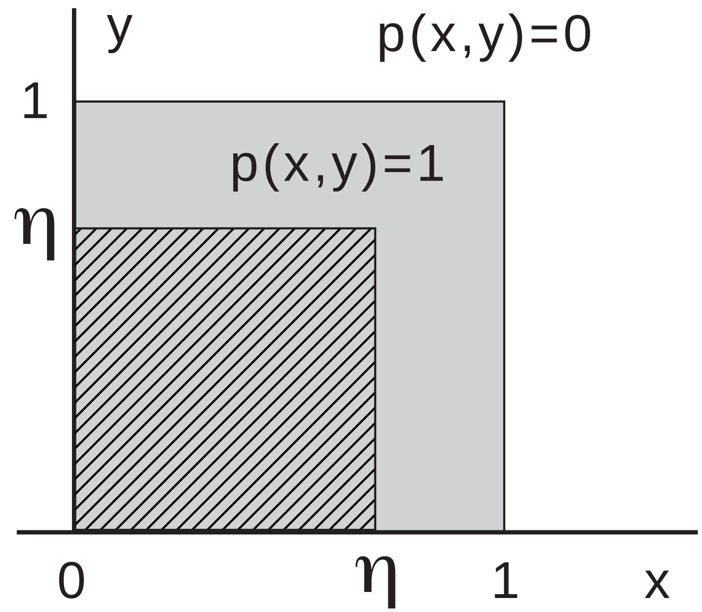
$z \equiv \text{MAX}(x, y)$ Find $p(z)$

$p(x, y) = p(x)\, p(y)$

Where is $\text{MAX}(x, y) = \eta$?

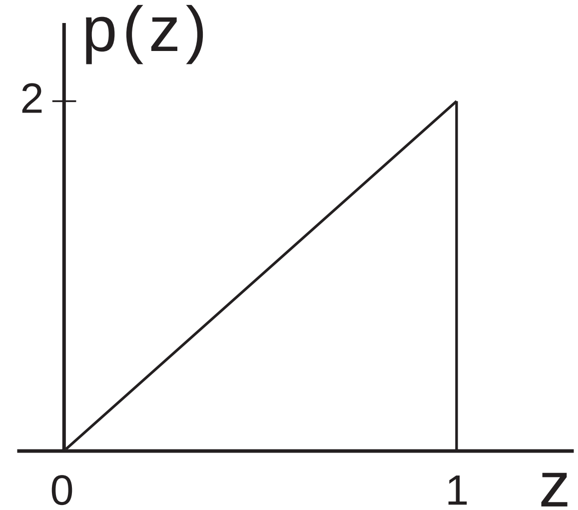
Where is $\text{MAX}(x, y) < \eta$?

A



B $P_z(\eta) = \eta^2$

C $p_z(\eta) = 2\eta \quad 0 \leq \eta \leq 1$



$$< z > \; = \; \int_0^1 2\eta^2 \, d\eta = (2/3) \left[_0^1 \eta^3 = 2/3\right.$$

$$< z^2 > \; = \; \int_0^1 2\eta^3 \, d\eta = (2/4) \left[_0^1 \eta^4 = 1/2\right.$$

$$\text{Var}(z) = \frac{1}{2} - \frac{4}{9} = \frac{1}{18}, \qquad \text{S.D.} = \frac{1}{\sqrt{18}} = 0.24$$

Example Desorbing atom

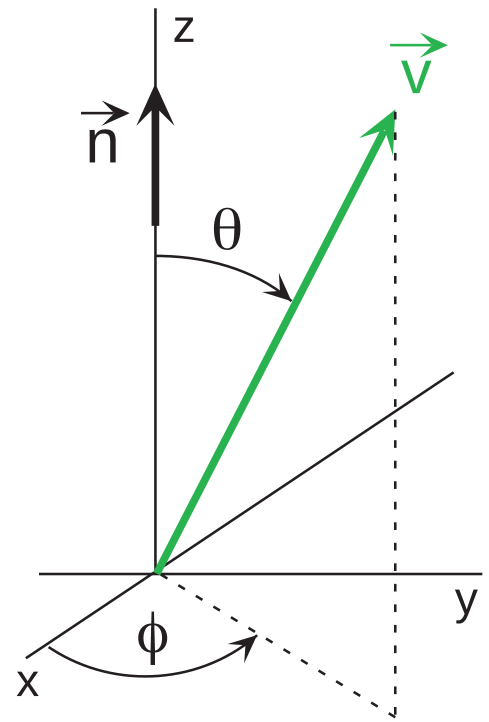
$$p(v,\theta,\phi) = p(v)\,p(\theta)\,p(\phi)$$

$$p(v) = (1/2\sigma^4)\,v^3\,\exp[-v^2/2\sigma^2]$$

$$p(\theta) = 2\sin\theta\,\cos\theta$$

$$p(\phi) = 1/2\pi$$

Find $p(v_z)$



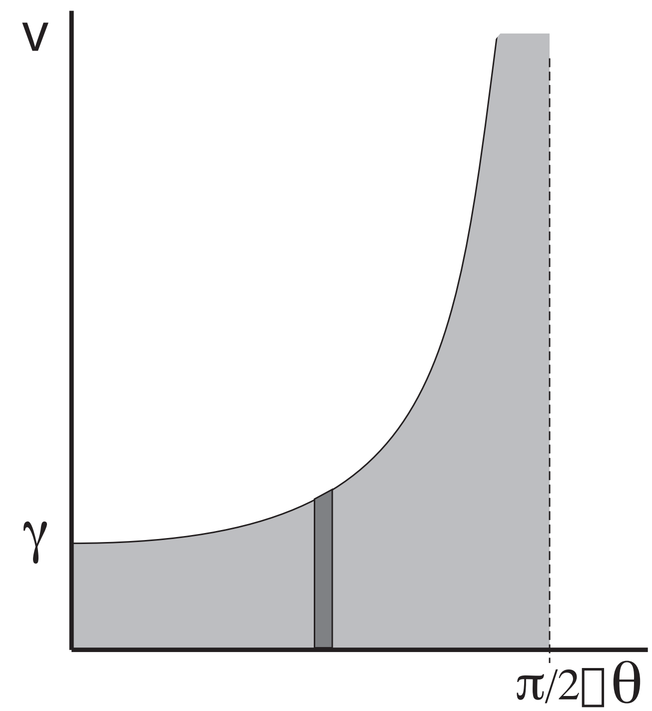
leaving the surface

$$v_z = v \cos\theta$$

$$v \ \cos\theta < \gamma$$

$$\Rightarrow v < \gamma / \cos\theta$$

A



B

$$
\begin{aligned}
P_{v_z}(\gamma) &= \int_0^{\pi/2} \int_0^{\gamma/\cos\eta} p_v(\zeta)\, p_\theta(\eta)\, d\zeta\, d\eta \\
&= \int_0^{\pi/2} p_\theta(\eta) \left[\int_0^{\gamma/\cos\eta} p_v(\zeta)\, d\zeta \right] d\eta
\end{aligned}
$$

C

$$
p_{v_z}(\gamma) = \frac{dP_{v_z}(\gamma)}{d\gamma} = \int_0^{\pi/2} p_\theta(\eta) \left[\frac{1}{\cos\eta}\, p_v\left(\frac{\gamma}{\cos\eta}\right) \right] d\eta
$$

$$p_{v_z}(\gamma) =$$

$$\int_0^{\pi/2} (2 \sin\eta \cos\eta) \left[\frac{1}{\cos\eta} \, \frac{1}{2\sigma^4} \, \left(\frac{\gamma}{\cos\eta} \right)^3 \exp[-\frac{1}{2\sigma^2} \, \frac{\gamma^2}{\cos^2\eta}] \right] d\eta$$

Let $\frac{1}{2\sigma^2} \, \frac{\gamma^2}{\cos^2\eta} \equiv \mathrm{X}$

$$d\mathrm{X} = -\frac{1}{\sigma^2} \, \frac{\gamma^2}{\cos^3\eta} (-\sin\eta) d\eta$$

$\eta = 0 \quad \Rightarrow \quad \mathrm{X} = \gamma^2/2\sigma^2;$ $\qquad \eta = \pi/2 \quad \Rightarrow \quad \mathrm{X} = \infty$

$$p_{v_z}(\gamma) = \frac{\gamma}{\sigma^2}\int_{\gamma^2/2\sigma^2}^{\infty} e^{-X}\,dX = -\frac{\gamma}{\sigma^2}\Big[_{\gamma^2/2\sigma^2}^{\infty} \; e^{-X}$$

$$= \frac{\gamma}{\sigma^2}\,\exp[-\gamma^2/2\sigma^2] \qquad \gamma > 0$$