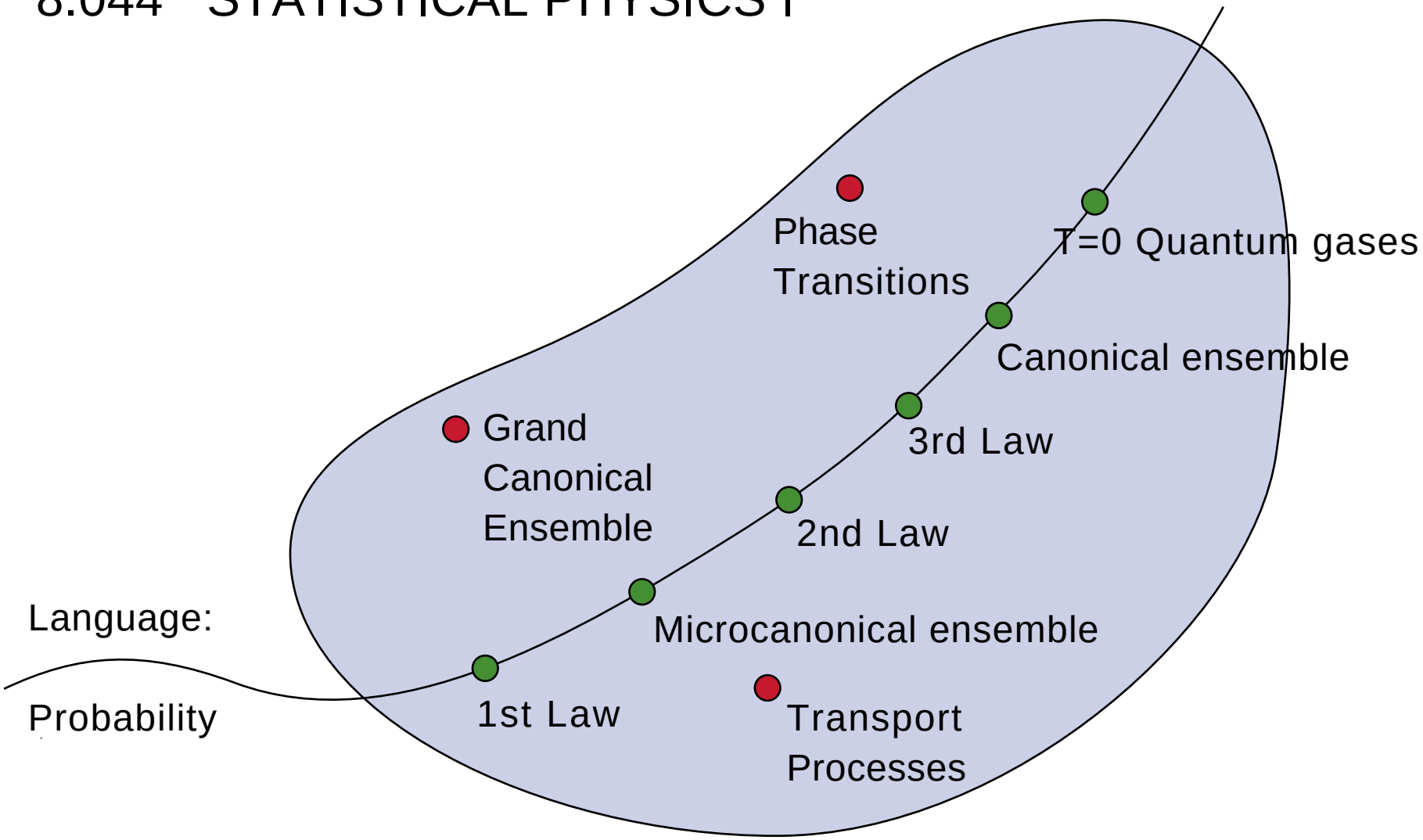
8.044 STATISTICAL PHYSICS I

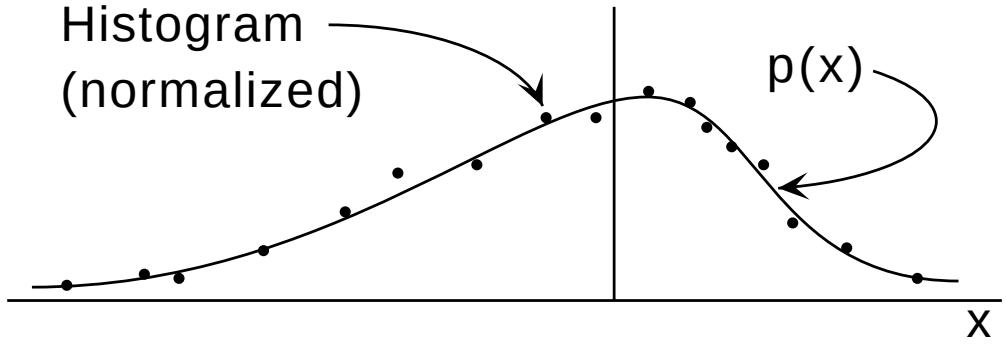


PROBABILITY

Random variable (ignorance and/or QM)

Continuous, discrete, or mixed

Probability density: $p(x) \leftrightarrow p_x(\zeta)$



$$\text{PROB}(\zeta \leq x < \zeta + d\zeta) = p_x(\zeta)d\zeta$$

$\Rightarrow p_x(\zeta) \geq 0,$

$$\int_{-\infty}^{\infty} p_x(\zeta) d\zeta = 1,$$

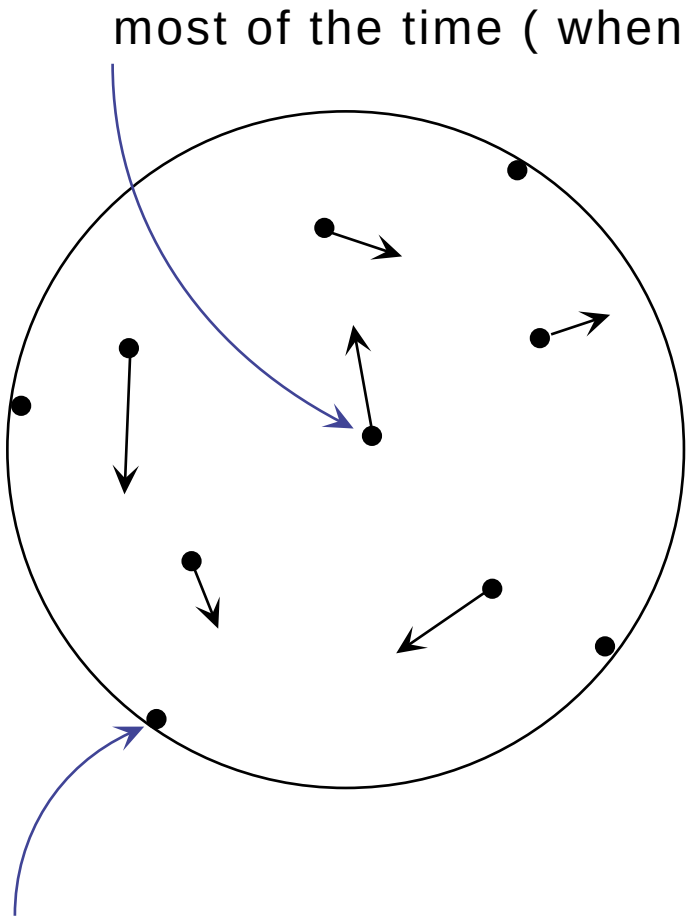
$$\text{PROB}(a \leq x < b) = \int_a^b p_x(\zeta) d\zeta$$

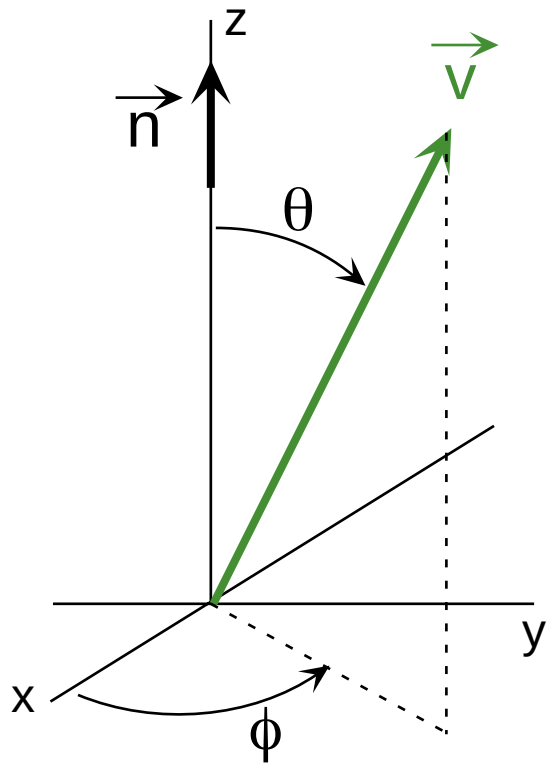
Cumulative probability:

$$P_x(\zeta) \equiv \int_{-\infty}^{\zeta} p_x(\zeta') d\zeta' \quad \Rightarrow \quad p_x(\zeta) = \frac{d}{d\zeta} P_x(\zeta)$$

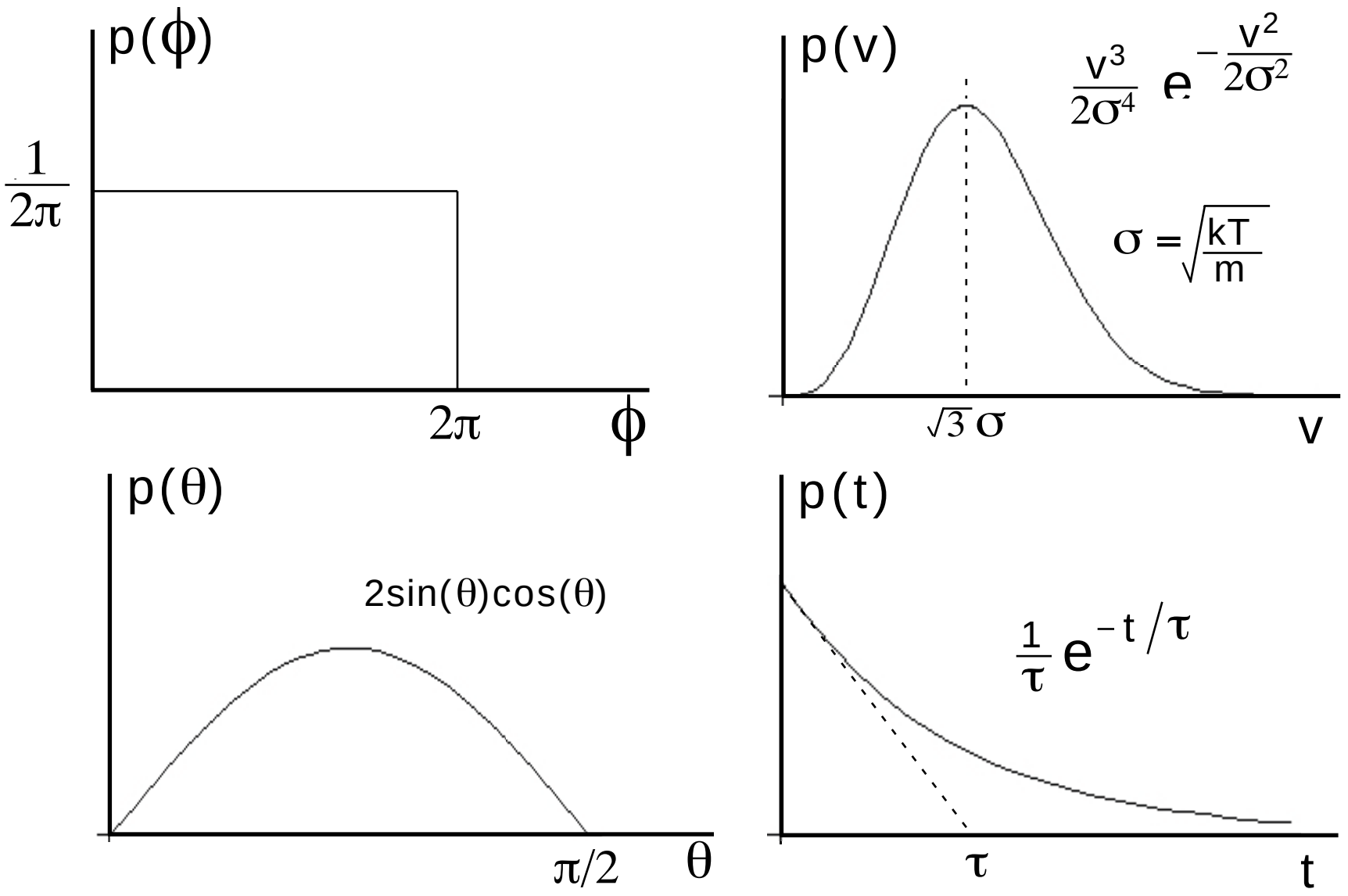
Either $p_x(\zeta)$ or $P_x(\zeta)$ completely specifies the RV x.

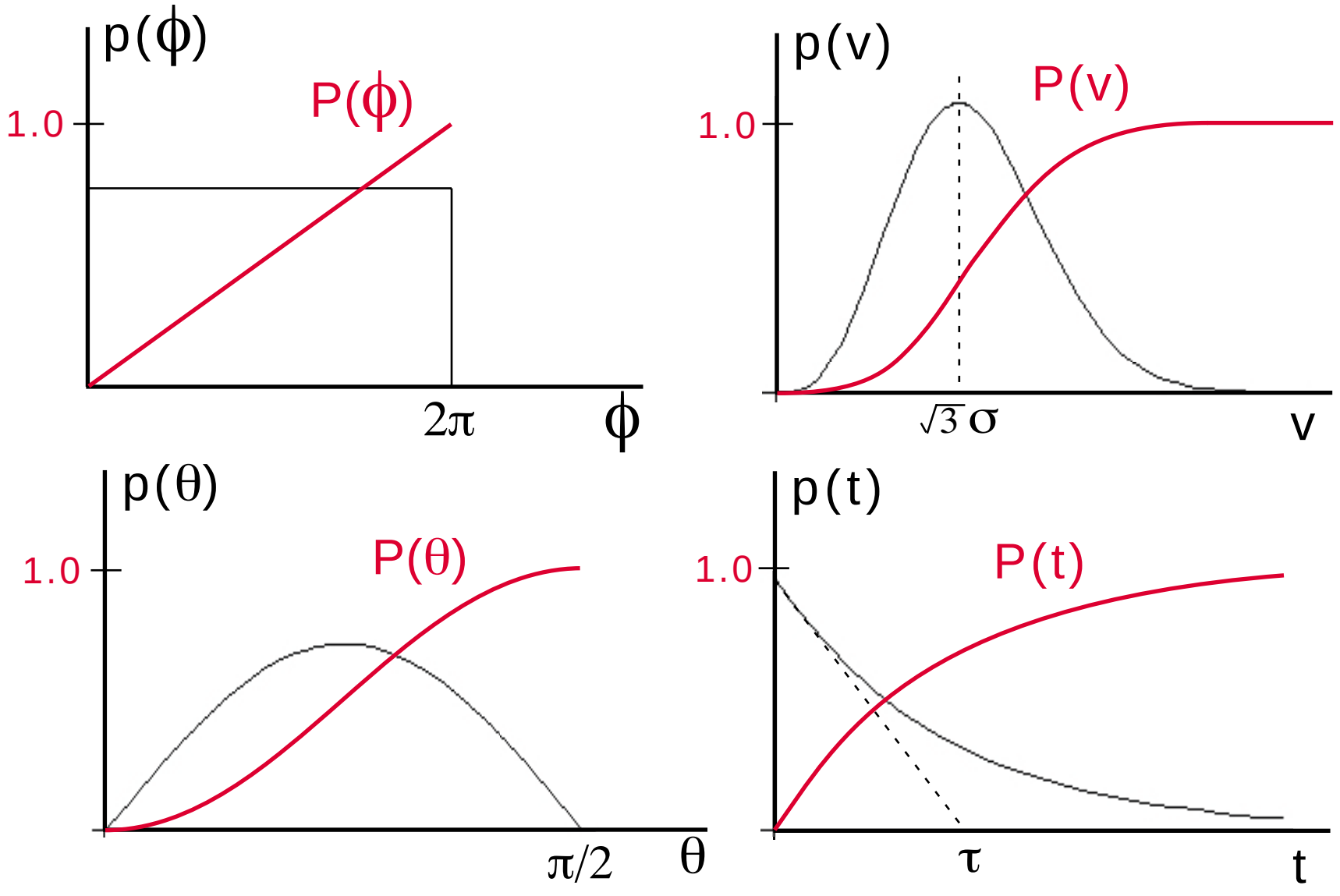
Example Physical adsorption of a gas



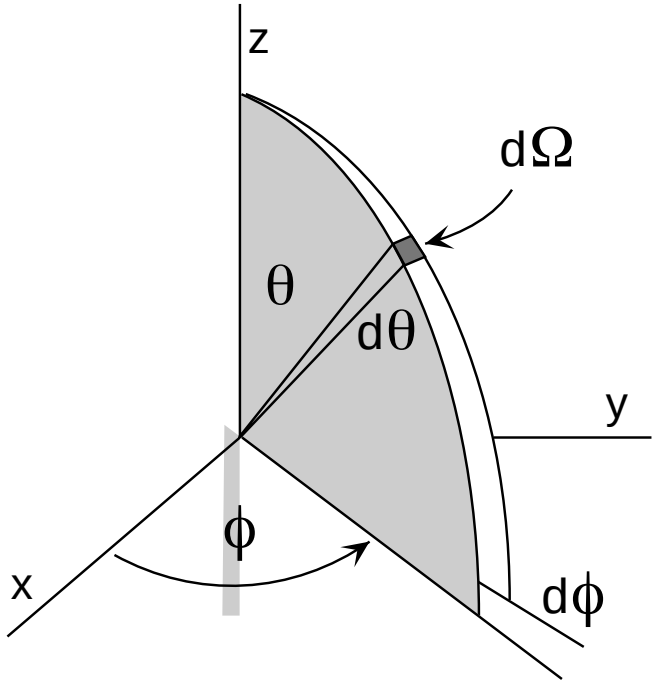


leaving the surface


p(ϕ)
1/2π
2π
ϕ
p(v)
v³/2σ⁴ e^−v²/2σ²
σ = √(kT/m)
√3 σ
v
p(θ)
2sin(θ)cos(θ)
π/2
θ
p(t)
1/τ e^−t/τ
τ
t


p(ϕ)
P(ϕ)
1.0
2π
ϕ
p(v)
P(v)
1.0
√3 σ
v
p(θ)
P(θ)
1.0
π/2
θ
p(t)
P(t)
1.0
τ
t

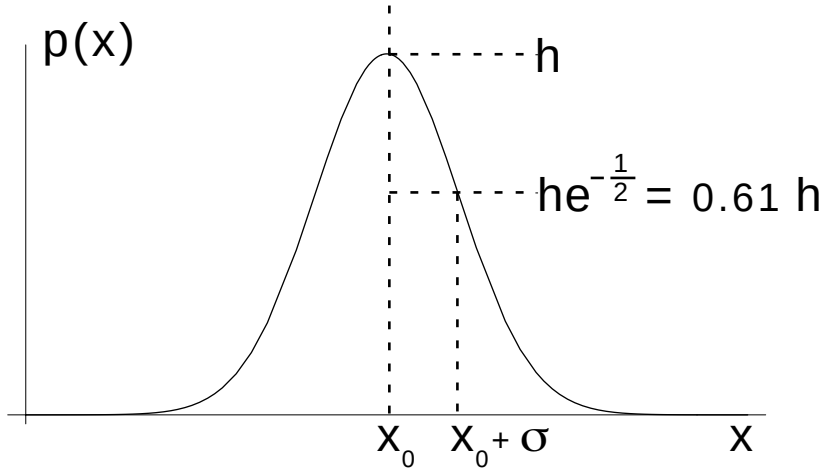
$$\begin{aligned} \text{PROB} &= p(\theta)d\theta\, p(\phi)d\phi \\ &= 2\sin(\theta)\cos(\theta)d\theta(1/2\pi)d\phi \\ d\Omega &= \sin(\theta)d\theta d\phi \end{aligned}$$



$$\text{PROB}/d\Omega = (1/\pi)\cos(\theta)$$

Gaussian density (memorize)

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-x_0)^2}{2\sigma^2}}$$



2 parameters