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8.044

SOLUTIONS

EXAM #2

$$1. \quad dP = \left. \frac{\partial P}{\partial T} \right|_V dT + \left. \frac{\partial P}{\partial V} \right|_T dV$$

$$\left. \frac{\partial P}{\partial V} \right|_T = \frac{1}{\left. \frac{\partial V}{\partial P} \right|_T} = -\frac{1}{V \kappa_T} = -\frac{2c}{V^3}$$

$$\left. \frac{\partial P}{\partial T} \right|_V = \frac{-1}{\left. \frac{\partial T}{\partial V} \right|_P \left. \frac{\partial V}{\partial P} \right|_T} = \frac{1}{V} \frac{\left. \frac{\partial V}{\partial T} \right|_P}{-\frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T} = \frac{\alpha}{\kappa_T} = \frac{5}{2} a T^{3/2} + 3b T^2$$

$$P(T, V) = \int \left. \frac{\partial P}{\partial T} \right|_V dT + f(V) = a T^{5/2} + b T^3 + f(V)$$

$$\left. \frac{\partial P}{\partial V} \right|_T = f'(V) = -\frac{2c}{V^3} \Rightarrow f(V) = +\frac{c}{V^2} + \text{CONSTANT} \quad \begin{array}{l} \nearrow 0 \text{ SINCE } P \rightarrow 0 \\ \text{AS } V \rightarrow \infty \end{array}$$

$$\underline{\underline{P(T, V) = a T^{5/2} + b T^3 + \frac{c}{V^2}}}$$

2. USE P+V AS INDEPENDENT VARIABLES

$$a/ \quad dU = \alpha P^2 dV + 2\alpha P V dP \quad \delta W = -P dV$$

$$dQ = dU - \delta W = 2\alpha P V dP + P(1 + \alpha P) dV$$

$$= 0 \text{ FOR ADIABATIC CHANGES}$$

$$\Rightarrow \underline{\underline{\left. \frac{dP}{dV} \right|_{\delta Q=0} = -\frac{P(1 + \alpha P)}{2\alpha P V}}}$$

$$b/ \quad \left. \frac{dP}{dV} \right|_{\text{ISOTHERMAL}} = \left. \frac{\partial P}{\partial V} \right|_T = \frac{1}{\left. \frac{\partial V}{\partial P} \right|_T} = \frac{-1/V}{-\frac{1}{V} \left. \frac{\partial V}{\partial P} \right|_T} = \underline{\underline{-\frac{1}{\kappa_T V}}}$$

$$c/ \quad \left. \frac{dQ}{dT} \right|_V = 2\alpha P V \left. \frac{\partial P}{\partial T} \right|_V = 2\alpha P V \left(\frac{-1}{\left. \frac{\partial V}{\partial P} \right|_T \left. \frac{\partial P}{\partial T} \right|_T} \right)$$

$$= 2\alpha P V \left(-\frac{\left. \frac{\partial V}{\partial T} \right|_P}{\left. \frac{\partial V}{\partial P} \right|_T} \right) = \underline{\underline{\frac{2\alpha \alpha P V}{\kappa_T}}}$$

3 a/ COMPUTE THE CUMULATIVE VOLUME IN PHASE SPACE FIRST.

$$\phi = \left[\int_0^{2\pi} d\theta \right]^N \int_{\sum_{i=1}^N \frac{1}{2} l_i^2 < E} \{d^N l_i\} = (2\pi)^N \times \text{VOLUME OF N-DIMENSIONAL SPHERE OF RADIUS } \sqrt{2IE}$$

$$= (2\pi)^N \pi^{N/2} (2IE)^{N/2} \frac{1}{(N/2)!}$$

$$\approx (2\pi)^N (2\pi IE)^{N/2} \frac{1}{\left(\frac{N}{2e}\right)^{N/2}} = (2\pi)^N \left(\frac{4\pi e IE}{N}\right)^{N/2}$$

$$\Omega = \Delta \frac{\partial \phi}{\partial E} = \underline{\underline{\left(\frac{N\Delta}{2E}\right) (2\pi)^N \left(\frac{4\pi e IE}{N}\right)^{N/2}}}$$

$$b/ \rho(\theta) = \Omega'(\text{ONE ANGLE FIXED}) / \Omega = \frac{(2\pi)^{N-1}}{(2\pi)^N}$$

$$= \underline{\underline{\frac{1}{2\pi} \quad 0 \leq \theta < 2\pi, \quad = 0 \text{ ELSEWHERE}}}$$

c/ Ω' : SET $N \rightarrow N-1, E \rightarrow E - \frac{l^2}{2I}$ IN ANGULAR MOMENTUM CONTRIBUTION

$$\Omega' = \underline{\underline{\left(\frac{(N-1)\Delta}{2(E - \frac{l^2}{2I})}\right) (2\pi)^{N-1} \left(\frac{4\pi e I (E - \frac{l^2}{2I})}{N-1}\right)^{(N-1)/2}}}$$

E WILL BE REPLACED BY $N \langle l^2 \rangle / 2I$

d/ $S = k \ln \phi$

$$\frac{1}{T} = \left. \frac{\partial S}{\partial E} \right|_N = k \frac{1}{\phi} \left. \frac{\partial \phi}{\partial E} \right|_N = k \frac{1}{\phi} \left(\frac{N}{2} \frac{1}{E} \phi \right) = \frac{Nk}{2E}$$

$$\underline{\underline{E = \frac{1}{2} NkT}}$$