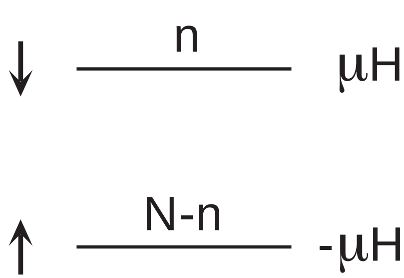
PARAMAGNETS

Spin 1/2 Paramagnet, Microcanonical Ensemble

Assume the moment is parallel to the spin. Take n to be the state variable. μ is the magnetic moment of the particle.



$$\vec{H} = H\hat{z}$$

$$(\uparrow),\ N_{\uparrow} = N - n,\ \vec{\mu}_{\uparrow} = \mu\hat{z},\ \epsilon_{\uparrow} = -\mu H$$

$$(\downarrow),\ N_{\downarrow} = n,\ \vec{\mu}_{\downarrow} = -\mu\hat{z},\ \epsilon_{\downarrow} = \mu H$$

$$
\begin{aligned}
M &= \mu(N_\uparrow - N_\downarrow) = \mu(N - 2n) = M(n) \\
E &\equiv N\langle\epsilon\rangle = N_\uparrow\epsilon_\uparrow + N_\downarrow\epsilon_\downarrow = -\mu H(N - 2n) = \underbrace{-HM(n)}_{H\,\&\,n} \\
S &= k\ln\left(\frac{N!}{n!(N-n)!}\right) = S(n) \\
&\approx k\,[\,N\ln N - n\ln n - (N-n)\ln(N-n)\,]
\end{aligned}
$$

$$\frac{dS}{dn} = k\left[-1 - \ln n + 1 + \ln(N-n)\right] = -k \ln\left(\frac{n}{N-n}\right)$$

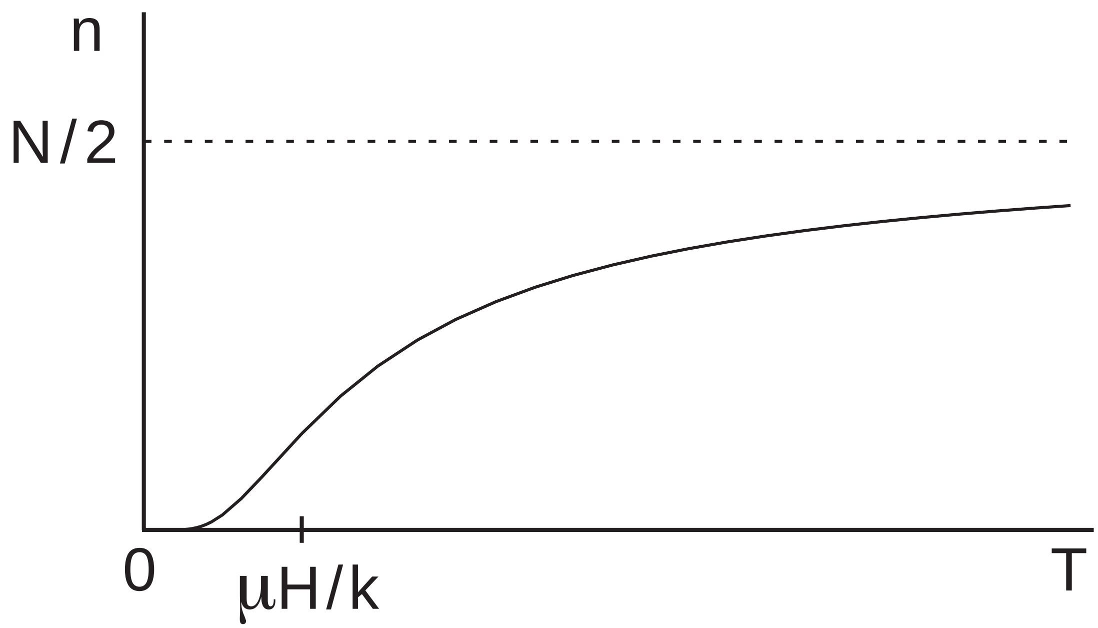
$$S = S(n) \text{ and } M = M(n) \Rightarrow S = S(M)$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_M \text{ does not exist!}$$

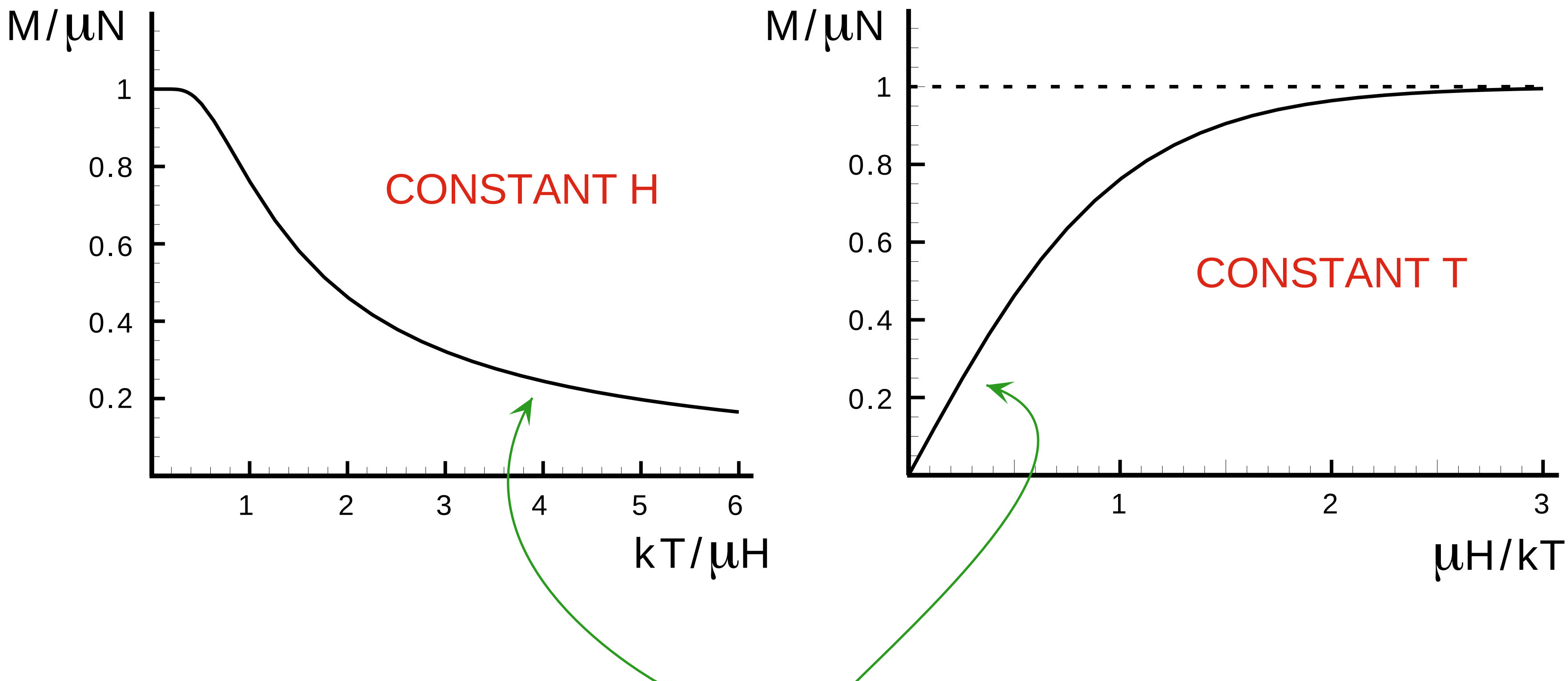
$$-\frac{H}{T} = \left(\frac{\partial S}{\partial M}\right)_E = \frac{dS}{dn}\left(\frac{\partial n}{\partial M}\right)_E = \left(-k \ln \frac{n}{N-n}\right)\left(\frac{-1}{2\mu}\right)$$

$$\frac{n}{N-n} = e^{-2\mu H/kT} \rightarrow \underline{n = N\,\frac{1}{e^{2\mu H/kT}+1}}$$

$$M = \mu(N-2n) = \underline{\mu N \tanh(\mu H/kT)}$$



MAGNETIZATION IN SPIN 1/2 PARAMAGNET



CURIE LAW BEHAVIOIR: $M = \dfrac{N\mu^2 H}{kT} \propto H/T$

Internal Energy

$$dU = đQ + đW = TdS + HdM$$

$dU \equiv$ adiabatic ($đQ = 0$) work

$$đQ = TdS, \quad đQ = 0 \Rightarrow dS = 0 \Rightarrow dM = 0 \Rightarrow dU = 0$$

$dU = 0$ for any change: $U = 0$ for this model

But $E \equiv N < \epsilon > = -HM \neq 0$!!

Energy = energy to create H field ①

+ energy to assemble M ②

+ energy to move M into H ③

① does not appear when using $đW = HdM$. It is included in U when using $đW = -MdH$.

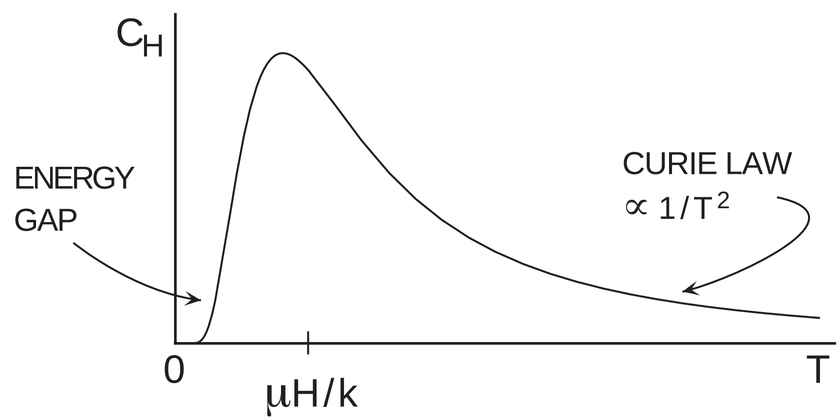
② We did not create the $\vec{\mu}$. They do not interact. $\Rightarrow U = 0$ in the current example.

③ $-\vec{H} \cdot \vec{M}$, energy of macroscopic moment in $\vec{H}$ equals $N < \epsilon >$ in the current example.

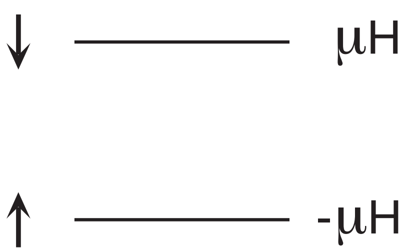
$$C_M \equiv \left(\frac{đQ}{dT}\right)_M = T\left(\frac{\partial S}{\partial T}\right)_M = 0 \text{ since } S = S(M)$$

$$C_H \equiv \left(\frac{đQ}{dT}\right)_H = \frac{1}{dT}(\underbrace{dU}_{0} - H\,dM)_H = -H\left(\frac{\partial M}{\partial T}\right)_H$$

$$= Nk\left(\frac{2\mu H}{kT}\right)^2 \frac{e^{2\mu H/kT}}{(e^{2\mu H/kT}+1)^2}$$



Spin 1/2 Paramagnet, Canonical Ensemble



$$Z_1 = e^{+\mu H/kT} + e^{-\mu H/kT}$$

$$p_\uparrow = \frac{e^{+\mu H/kT}}{Z_1} \qquad p_\downarrow = \frac{e^{-\mu H/kT}}{Z_1}$$

$$M = N\mu(p_\uparrow - p_\downarrow)$$

$$= \mu N \tanh(\mu H/kT) \qquad \surd$$