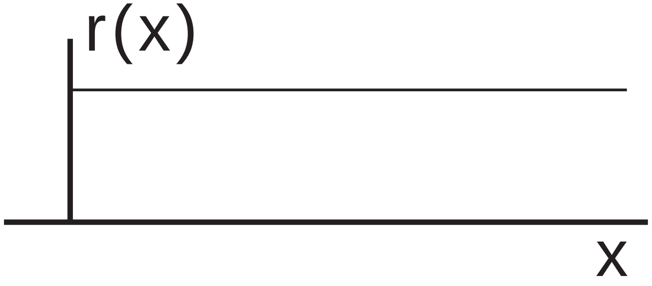
Poisson Random Variable

- S.I. events
- $p(1)$ in $\Delta x \rightarrow r\Delta x$
- r is constant



$$p(n = 0; L) = e^{-rL} \qquad 1^{st} \text{ element of } p(n; L)$$

$$p(n;L+\Delta L) \approx p(n;L)\,\underbrace{p(0;\Delta L)}_{(1-r\Delta L)} + p(n-1;L)\,\underbrace{p(1;\Delta L)}_{(r\Delta L)}$$

$$\frac{p(n;L+\Delta L)-p(n;L)}{\Delta L} = rp(n-1;L)-rp(n;L) \rightarrow \frac{d\,p(n;L)}{dL}$$

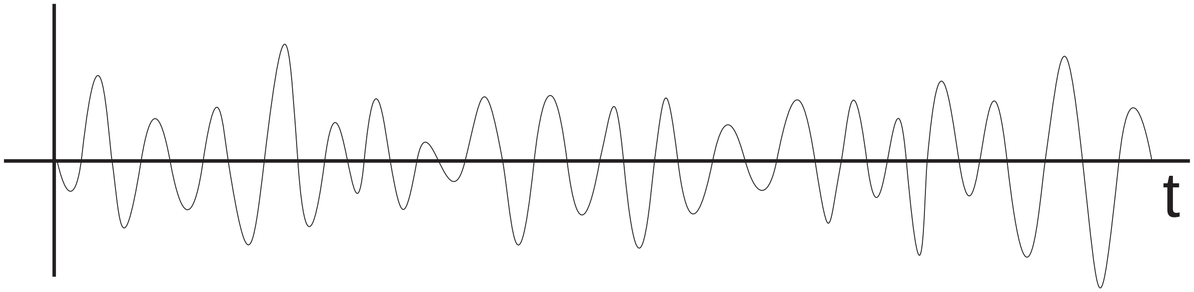
Let $\gamma \equiv rL$ $$\underbrace{\frac{d\,p(n;\gamma)}{d\gamma} + p(n;\gamma)}_{1^{st}\text{ order, linear DE}} = \underbrace{p(n-1;\gamma)}_{\text{already known}}$$

DE: γ is the variable and n is an index

Probability: n is the variable and γ is a parameter

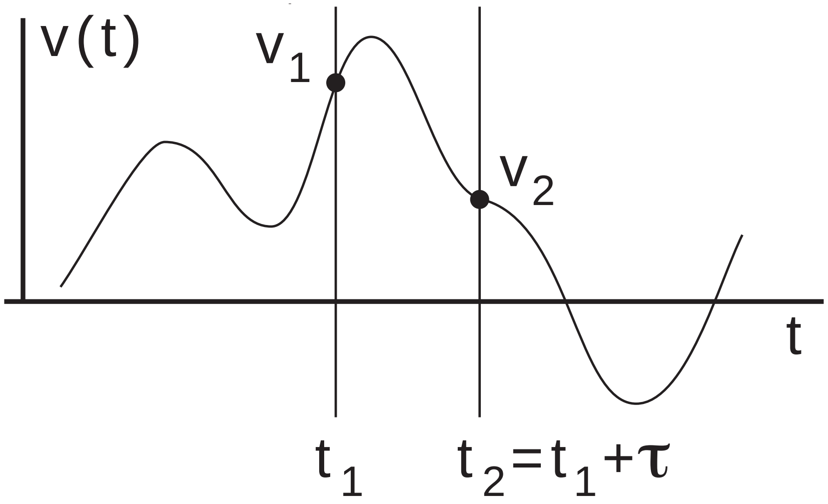
$$p(n;\gamma) = \frac{1}{n!}\,\gamma^n\,e^{-\gamma}$$

Example Jointly Gaussian random variables



Random processes:

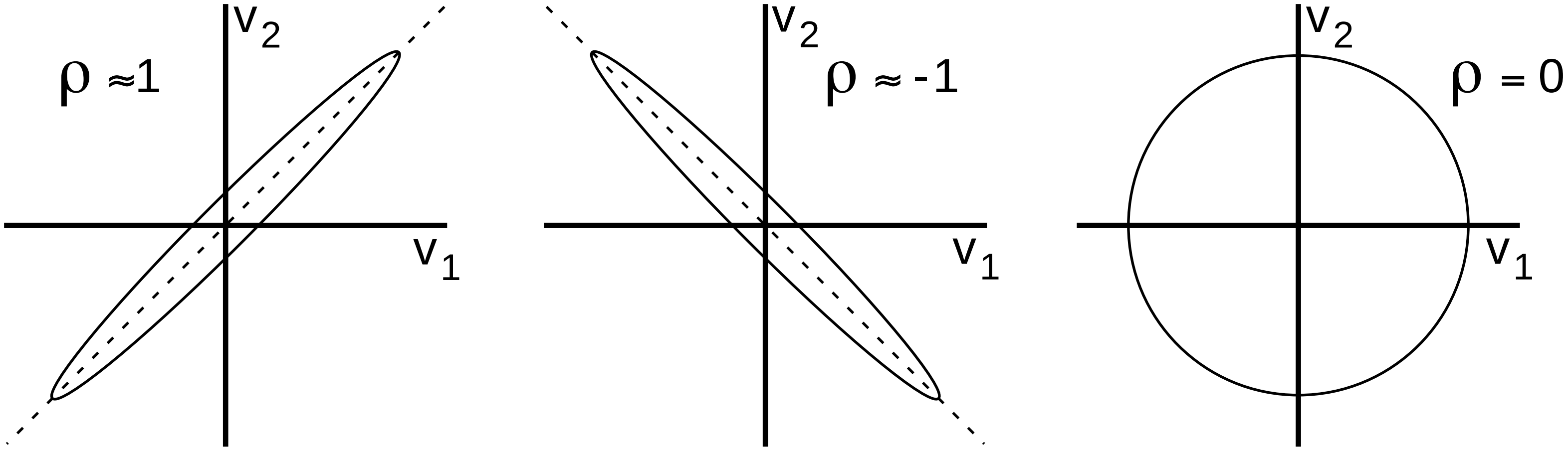
- Noise voltage or current $v(t), i(t)$
- Thermodynamic variable $P(t), T(t), \rho(t)$
- Thermal radiation $E(t), B(t)$
- Seismic background signal $s(t)$



$$p(v_1, v_2) = \frac{1}{2\pi\sigma^2\sqrt{1-\rho^2}} \ \exp\left[-\frac{v_1^2 - 2\rho v_1 v_2 + v_2^2}{2\sigma^2(1-\rho^2)}\right]$$

σ is constant $\qquad \rho = \rho(\tau) \ \rightarrow \ 1$ as $\tau \rightarrow 0$

$\rightarrow \ 0$ as $\tau \rightarrow \infty$



v_2 and v_2
are S.I.

$$p(v_1) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{v_1^2}{2\sigma^2}\right]$$

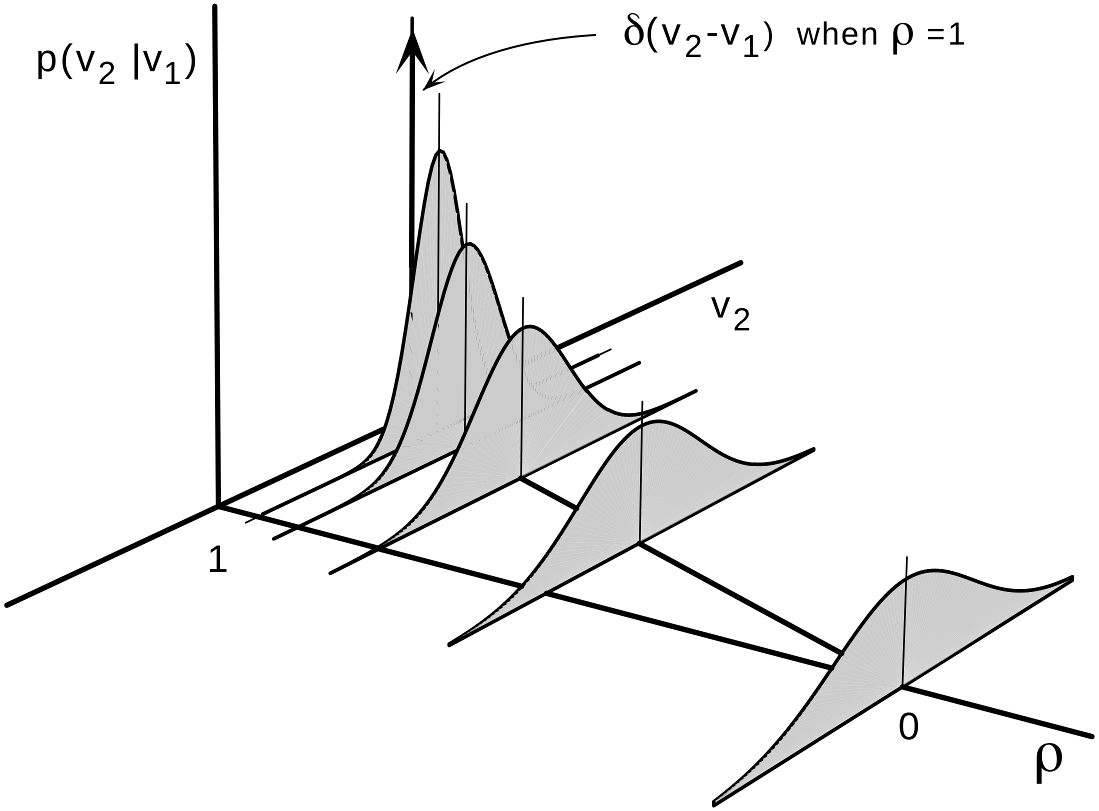
a zero mean gaussian with variance σ^2

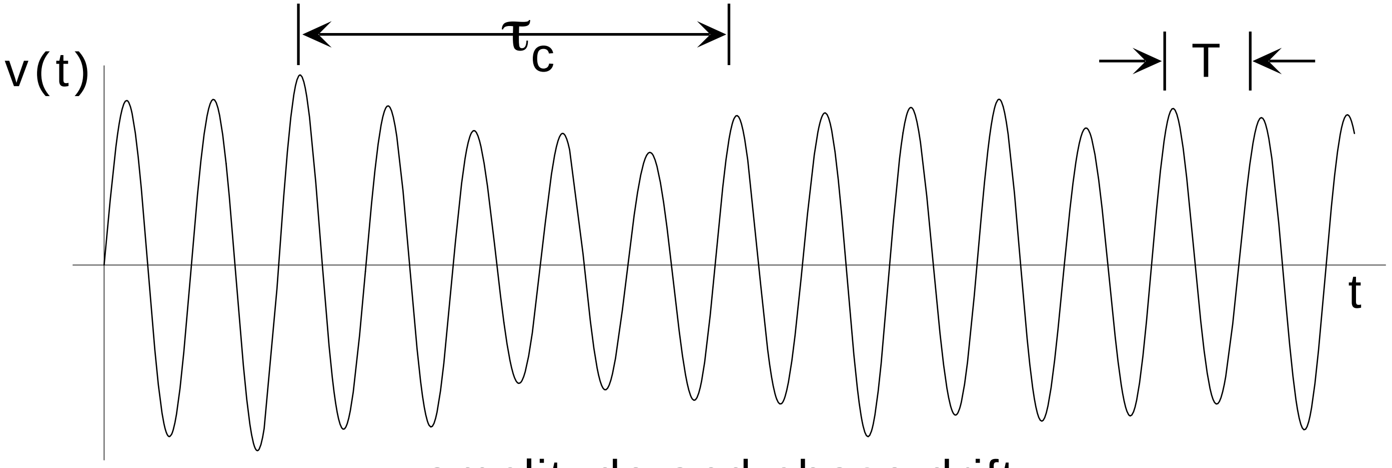
$$p(v_2|v_1) = \frac{1}{\sqrt{2\pi\sigma^2(1-\rho^2)}} \exp\left[-\frac{(v_2-\rho v_1)^2}{2\sigma^2(1-\rho^2)}\right]$$

a Gaussian with mean ρv_1 and variance $\sigma^2(1-\rho^2)$

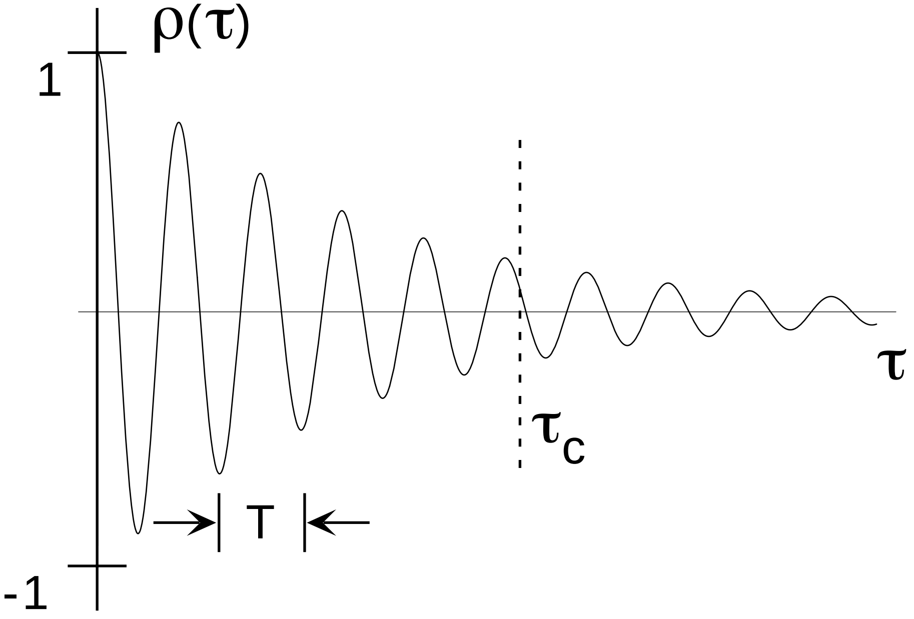
As ρ goes from $1 \rightarrow 0$, the mean goes from $v_1 \rightarrow 0$

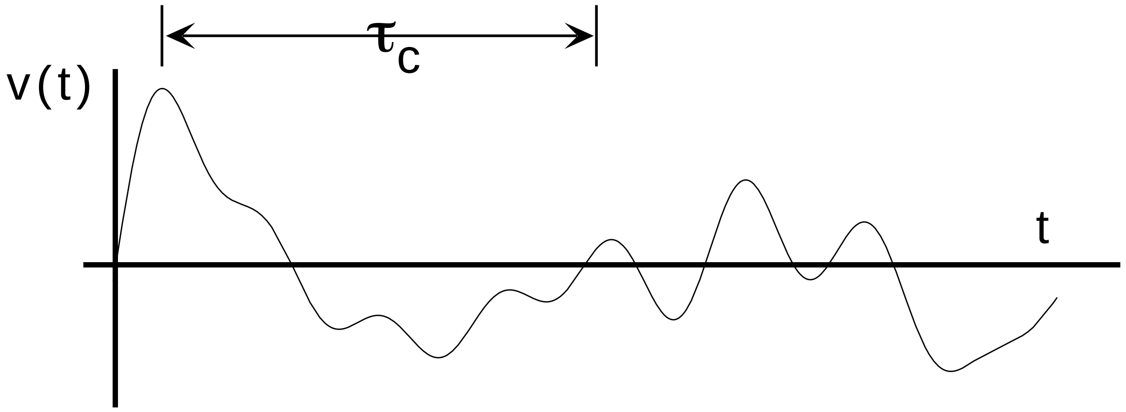
and the variance goes from $0 \rightarrow \sigma^2$.

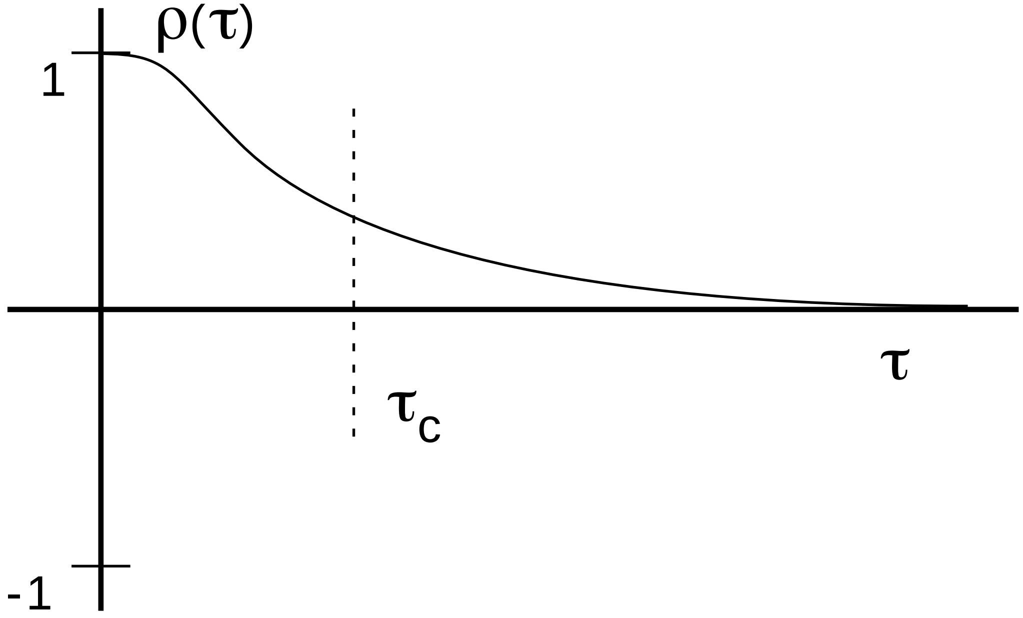

$\delta(v_2 - v_1)$ when $\rho = 1$
$p(v_2 | v_1)$
v_2
1
0
ρ



amplitude and phase drift




τ_c
v(t)
t


ρ(τ)
1
-1
τ
τ_c

Averages

$$< v_1 v_2 > \;=\; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_1 v_2\, p(v_1, v_2)\, dv_1\, dv_2$$

$$=\; \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} v_1 v_2\, p(v_2|v_1)\; p(v_1)\, dv_1\, dv_2$$

$$=\; \int_{-\infty}^{\infty} v_1 p(v_1) \qquad \underbrace{\int_{-\infty}^{\infty} v_2 p(v_2|v_1)\, dv_2}_{\text{conditional mean } = \rho(\tau) v_1} \qquad dv_1$$

$$=\; \rho(\tau) \underbrace{\int_{-\infty}^{\infty} v_1^2 p(v_1)\, dv_1}_{<v^2>=\sigma^2} \qquad \Rightarrow \rho =< v_1 v_2 > /\sigma^2$$