

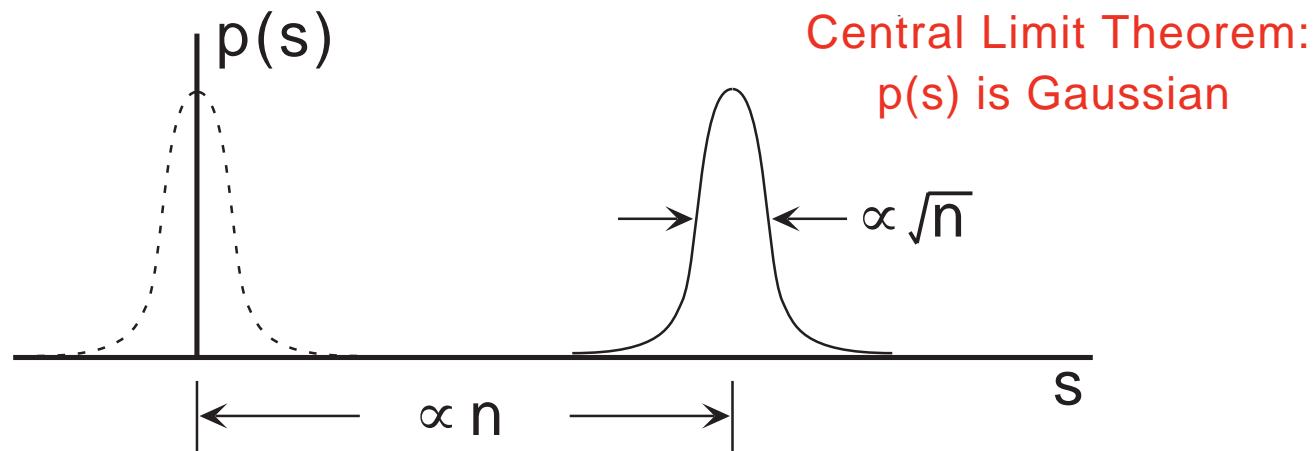
## Facts about sums of RVs

- Exact expressions for  $\langle s \rangle$  and  $\text{Var}(s)$  if S.I.
- $p(s) = p(x) \otimes p(y)$  if S.I.
- $p(s)$  slightly more complicated if not S.I.

- $\otimes$  usually changes functional form
- But not always
- Fourier techniques are very useful

## Very important special case: Central Limit Theorem

- RVs are S.I.
- All have identical densities  $p(x_i)$
- $\text{Var}(x)$  is finite but  $\langle x \rangle$  could be zero
- $n$  is large



If  $x$  is continuous

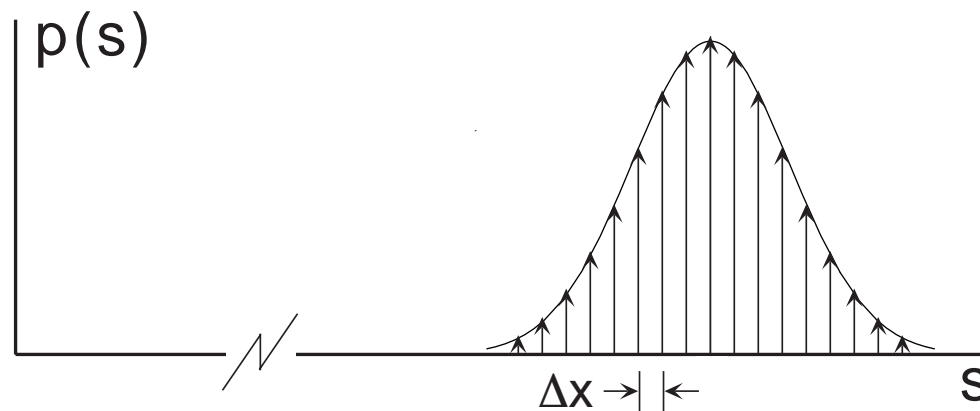
$$p(s) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(s-\langle s \rangle)^2/2\sigma^2}$$

$$\langle s \rangle = n \langle x \rangle$$

$$\sigma^2 = n \sigma_x^2$$

If  $x$  is discrete in equal steps of  $\Delta x$

$$p(s) = \sum_i \underbrace{\frac{\Delta x}{\sqrt{2\pi\sigma^2}} e^{-(s-\langle s \rangle)^2/2\sigma^2}}_{\text{envelope}} \underbrace{\delta(s - i \Delta x)}_{\text{comb}}$$



## Example K.E. of an ideal gas

$$p(v_x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-v_x^2/2\sigma^2}$$

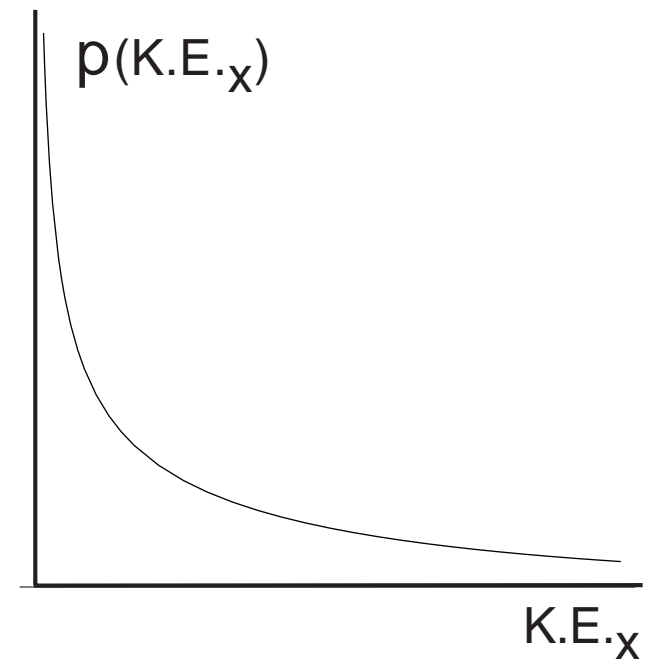
$$\sigma = \sqrt{kT/m}$$

$$\text{K.E.}_x = \frac{1}{2}mv_x^2$$

$$\text{mean} = \frac{1}{2}m \underbrace{\langle v_x^2 \rangle}_{\sigma^2} = \frac{1}{2}kT$$

$$\begin{aligned} \text{mean sq.} &= \left(\frac{1}{2}m\right)^2 \underbrace{\langle v_x^4 \rangle}_{3\sigma^4} \\ &= \frac{3}{4}(kT)^2 \end{aligned}$$

$$\text{variance} = \frac{1}{2}(kT)^2$$



3 directions,  $N$  atoms

$E$  = total K.E.

$$U \equiv \langle E \rangle = 3N \langle K.E.x \rangle = \frac{3}{2} N k T$$

$$\text{Variance}(E) = 3N \text{ Variance}(K.E.x) = \frac{3}{2} N (k T)^2$$

$$p(E) = \frac{1}{\sqrt{2\pi \{(3/2)N(kT)^2\}}} \exp\left[-\frac{(E-(3/2)NkT)^2}{2\{(3/2)N(kT)^2\}}\right]$$

$$\frac{\text{s.d.}}{\text{mean}} = \frac{\sqrt{\frac{3}{2}N} kT}{\frac{3}{2}N kT} = \frac{1}{\sqrt{\frac{3}{2}N}}$$