MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044 Statistical Physics I — Spring Term 2004

Problem Set #7
Due by 1:10 PM: Monday, April 5

Problem 1: Energy of a Film

A surface film has a surface tension $\mathcal{S}$ given by $\mathcal{S}(T, A) = -NkT/(A - b)$ where b is a constant with the units of area.

a) Show that the constant area heat capacity C_A is independent of area at fixed temperature.

b) Show that the internal energy of the film is a function of temperature alone and find an expression for the energy in terms of the heat capacity.

Problem 2: Bose-Einstein Gas

Later in the course we will show that a gas of atoms obeying Bose-Einstein statistics undergoes a phase transition at a very low temperature to another gas phase with quite different properties. [This transition was first observed in the laboratory in 1995 and received a great deal of publicity. Wolfgang Ketterle's group here at MIT are world leaders in this field. He shared the 2001 Nobel Prize in Physics for his work on Bose-Einstein Condensation.] The equation of state and the heat capacity of the gas below the transition temperature are given by the expressions

$$P(T, V) = aT^{5/2} + bT^3 + cV^{-2}$$

$$C_V(T, V) = dT^{3/2}V + eT^2V + fT^{1/2}$$

where a through f are constants which are independent of T and V.

a) Find the differential of the internal energy $dE(T, V)$ in terms of dT and dV.

b) Find the relationships between a through f due to the fact that $E(T, V)$ is a state function.

c) Find $E(T, V)$ as a function of T and V.

d) Find the entropy, $S(T, V)$, as a function of T and V.

Problem 3: Paramagnet

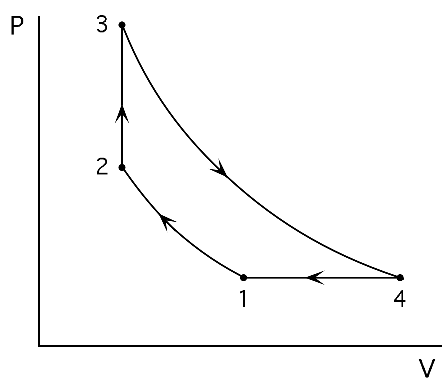
Consider a paramagnetic material where the equation of state relating the magnetization, M, to the applied magnetic field, H, is $M = AH/(T - T_0)$. T_0 and A are constants and the expression is valid only for $T > T_0$.

a) Show that the heat capacity at constant magnetization, C_M, is independent of M at constant T.

b) Find an expression for the internal energy, $E(T, M)$, in terms of A, T_0 and $C_M(T)$.

c) Find the entropy, $S(T, M)$.

Problem 4: Sargent Cycle



The diagram above is an approximation to a Sargent cycle run on an ideal gas. A constant pressure path and a constant volume path are connected by two adiabatic paths. Assume all processes are quasi-static and that the heat capacities, C_P and C_V, are constant.

a) Which of the four states (1,2,3 and 4) has the highest temperature and which has the lowest?

b) T_2 could be either hotter of colder than T_4 depending on the specific values of P and V at the four corners of the cycle. Demonstrate graphically one version of the cycle where T_4 is clearly less than T_2. Demonstrate another extreme where T_4 would necessarily be greater than T_2.

c) Prove that the efficiency of this cycle running as an engine is $\eta = 1 - \gamma(T_4 - T_1)/(T_3 - T_2)$ where $\gamma \equiv C_P/C_V$.

d) Find an expression for the total work done, W, in one cycle. Express your results in terms of N, k, γ, and the T's.

e) Show that the first law, $|Q_H| - |Q_C| = |W|$, applied to this cycle (together with the assumption that the heat capacities are constants) leads to the requirement that $C_P - C_V = Nk$.

Problem 5: Entropy Change

A mass M of liquid at a temperature T_1 is mixed with an equal mass of the same liquid at a temperature T_2. The system is thermally insulated but the liquids are maintained at some constant (atmospheric perhaps) pressure. Show that the entropy change of the universe is

$$2MC_P \ln \frac{(T_1 + T_2)}{2\sqrt{T_1 T_2}},$$

and prove that it is necessarily positive.