Due Thursday, February 16 at 4pm

Note that the derivation or formula, and not just the end result, is always required.

1. **Photon energy scale I.** (25 points)
   According to quantum mechanics, electromagnetic radiation of frequency ν can be regarded as consisting of photons of energy $h\nu$, where $h = 6.626 \times 10^{-34}$ J·s is Planck's constant.
   a) What is the frequency range of visible photons (400 nm to 700 nm)? What is the energy range of visible photons (both in J and in eV)?
   b) How many photons per second does a low-power (1 mW) He-Ne laser (λ = 633 nm) emit? A cell phone that emits 0.4W of 850 MHz radiation? A microwave oven operating at 2.45 GHz generating a microwave power of 750W? How many photons of the latter frequency have to be absorbed to heat up a glass of water (0.2l, heat capacity 4.18 kJ kg^{-1} K^{-1}) by 10 °C?
   c) At a given power of an electromagnetic wave, do you expect a classical wave description to work better for radiofrequencies, or for X-rays? Why? If you have a He-Ne laser producing a short laser pulse, at what pulse energy do you expect quantum effects to become important?
   d) An adapted human eye (person has spent 30 min in the dark) can see 1 ms long flashes of power 6×10^{-14} W at 510 nm. Assuming that 10% of the incident power reaches the retina, how many photons at the receptors generate the signal that the test person recognizes as a flash of light? (Actually, the cells are sensitive to single photons, but the human brain filters the signal.)

2. **Photon energy scale II.** (20 points)
   a) The thermal energy scale is $k_B T$, where $k_B = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant, and T is the absolute temperature. What energy does room temperature (20 °C) correspond to? What would be the frequency and wavelength of photons of such energy? Is it reasonable that a hot body starts to glow around 1000 °C?
   b) A first-magnitude star emits a light flux of 1.6×10^{-10} W/m^2 as measured on the earth, at an average wavelength of 560 nm. How many photons pass through the pupil of an eye per second?
   c) A photodiode measures light power by converting incident photons into electron-hole pairs, such that the electron current is proportional to the incident light power. The *quantum efficiency* is defined as the probability that an incident photon generates an electron-hole pair. If a typical photodiode has a responsivity of 0.5 A/W for infrared light at 850nm, what is the quantum efficiency of the device? If the quantum efficiency is independent of frequency, what responsivity do you expect for blue light at 400nm?

3. **Diffraction.** (20 points)
   Visible light with a wavelength of 510 nm from a distant source is incident on a 2 μm wide slit. Estimate the width of the pattern observed on a 10m distant screen (e.g., by assuming that the slit acts as a collection of emitters all oscillating in phase).

4. **Radiative collapse of a classical atom.** (35 points)
   The classical atom has a stability problem. We model the hydrogen atom as an electron in a classical circular orbit about a proton. We know that a non-relativistic accelerating electric

charge radiates energy at a rate given by the Larmor formula: $\frac{dE}{dt} = -\frac{2}{3}\frac{q^2}{4\pi\varepsilon_0}\frac{a^2}{c^3}$, where q is the electric charge and a is the magnitude of the acceleration.

(a) Show that the energy lost per revolution is small compared to the electron's kinetic energy. Hence, it is an excellent approximation to regard the orbit as circular at any instant, even though the electron eventually spirals into the proton.

(b) Using the typical size for the atom (1 Å) and the nucleus (10^{-5} Å), calculate how long it would take for the electron to spiral into the proton. (It turns out that this is approximately also the correct quantum mechanical time scale for an excited atom to emit a photon.) Use this time scale to estimate the linewidth (frequency width of the emitted radiation) for an atom at rest.

(c) Compare the velocity of the electron (assuming a radius of 0.5 Å for the orbit) to the speed of light.

(d) Calculate the angular momentum of the electron assuming a radius of 0.5 Å for the orbit. What fundamental quantity has the same units as angular momentum? How does your result compare to that quantity?