Quantum Physics II (8.05) Fall 2004
Outline

1. **General structure of quantum mechanics.** 8.04 was based primarily on wave mechanics. We review that foundation with the intent to build a more formal basis for quantum mechanics, emphasizing vector (Hilbert) space and Dirac notation. This will allow us to study quantum mechanics in many contexts more general than the "particle in a potential" problems familiar from 8.04. We introduce the two-state system (using the Stern-Gerlach experiments on spin-1/2 particles as an example) giving us a simple example of a Hilbert space which is in many ways quantum mechanics boiled down to its essentials.

   References: Griffiths, Ch. 3, Appendix; Ohanian, Ch. 4; Shankar, Ch. 1; Cohen-Tannoudji, parts of Ch. II, III, IV, V. Sakurai, Ch. 1; R. L. Jaffe, "Supplementary notes on the foundations of quantum mechanics, Dirac Notation, ..." .

   - Brief review of 8.04 treatment of a particle in a potential. State of a system, wavefunctions, probability interpretation, normalization. Operators, eigenvalues, eigenfunctions, completeness, orthonormality, measurement. Time development, the Schrödinger wave equation.
   - Stern-Gerlach experiment and spin-1/2 particles as an example of a two-state system. The operators $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$ as matrices. (These become our canonical example of operators acting in a finite dimensional Hilbert space.) (Sakurai 1.1, Cohen-Tannoudji IV)
   - Quantum states, the space of states, inner products. Hilbert space, bases. Dirac notation: kets, the dual space, bras. Wavefunctions in Dirac notation, inner product reconsidered.
   - Operators in Dirac notation, Hermitian operators and measurement of observables, completeness, expansion in eigenstates.
   - Unitary operators and change of basis.
   - Postulates of quantum mechanics.
   - Application to the operators $\hat{x}$ and $\hat{p}$ – constructing the operators, representations of states and operators in the basis of eigenstates of $\hat{x}$ and $\hat{p}$. Infinite dimensional Hilbert space. Nondenumerable bases.
   - Commuting operators. Compatible observables. Complete sets of commuting observables. Noncommuting operators. Incompatible observables. Uncertainty relations.

- Application of operator methods to the harmonic oscillator. Hamiltonian and raising and lowering operators, operator algebra. The ground state, the spectrum. Matrix representation of operators. (Griffiths 2.3; Ohanian 6.1; Cohen-Tannoudji V)

2. **Quantum Dynamics**. The time development of quantum systems from an operator point of view. Several different inquiries into the relation between classical and quantum dynamics.

   References: Ohanian Ch 5; Cohen-Tannoudji, parts of III, IV, V; Sakurai 2.1-2.4.

   - Paths in the space of states, unitary time evolution, the Hamiltonian as the generator of time evolution.
   - The Schrödinger equation and the time dependence of states in the Schrödinger picture.
   - Time dependence of operators, the Heisenberg picture, Ehrenfest's equation, and the correspondence to classical physics.
   - The evolution of p and x in the harmonic oscillator.
   - Coherent states of the harmonic oscillator and the classical limit. Construction of, properties of, and time evolution of coherent states. Expansion of coherent states in energy eigenstates. (Supplementary notes on coherent states; Cohen-Tannoudji V, Complement G_V)

3. **Two-state systems.** As we have seen already, the simplest quantum systems are those where only two states are important. They illustrate many aspects of quantum dynamics, and have many interesting modern applications.

   References: Feynman, Ch. 9. Supplementary notes on "Neutrino Oscillations and Kaon Physics" by Jaffe and Rajagopal; Cohen-Tannoudji, Ch. 4; Sakurai, 2.10; Ohanian, 9.2.

   - The ammonia molecule. An example of dynamics in a two-state system with a time-independent Hamiltonian. The ammonia maser: dynamics in a two-state system with a time-dependent Hamiltonian.
   - Spin precession and NMR. Spin-1/2 particle in a static magnetic field. Eigenstates of $\hat{S}_{\vec{n}}$ revisited. Unitary time evolution as precession among these states. Realization that this is the most general two state system with a time independent Hamiltonian. Nuclear magnetic resonance: a time-dependent term in the Hamiltonian. Rotating frame. Resonance condition.

- Neutrino oscillations. Two different bases related by a unitary transformation: weak interactions produce either $|\nu_e\rangle$ or $|\nu_\mu\rangle$; eigenstates of the Hamiltonian are $|\nu_1\rangle$ or $|\nu_2\rangle$. Computation of the probability that an electron neutrino will be found in an initially purely $|\nu_\mu\rangle$ beam, as a function of the distance traveled. Experiments that use neutrinos from accelerators, the sun, and cosmic rays.
- Kaon physics. Production of neutral kaons and absorption of neutral kaons in matter (one basis); decay of neutral kaons (another basis). Regeneration. ϕ decays and EPR correlations.

4. **Angular Momentum and Spin** Quantum mechanics in three dimensions involves new kinematics: the theory of angular momentum, and new dynamics: forces that depend on the distance r between particles. In this section we develop an operator-based description of angular momentum and then revisit spin.

   References: Griffiths, Ch. 4; Ohanian, Ch. 7; Cohen-Tannoudji, VI, VII, IX.

   - Schrödinger equation in three dimensions with central forces. Reduced mass. Separation of variables.
   - Angular momentum operators, commutators, raising and lowering operators. Matrix representation.
   - Eigenvalues and eigenstates of angular momentum, wavefunctions, properties of spherical harmonics.
   - Absence of half-integer orbital angular momentum.
   - We revisit spin, now after having learned about angular momentum in more generality. Operator algebra of spin-1/2, Pauli matrices, rotation of spinors. Combining spin and spatial states.

5. **The Radial Equation and operator methods.** The dynamics in problems with central forces occurs in the radial reduction of the Schrödinger equation. An operator technique known as the supersymmetric method (or factorization method) provides a powerful tool to find solutions in important cases.

   References: Ohanian, Ch. 6, 8. Supplementary notes on "The Supersymmetric Method in Quantum Mechanics".

   - Review (from 8.04) of the radial wave equation, its solution, and the spectrum of atomic hydrogen and other one-electron atoms. Atomic orbitals and their spatial shape. Spectroscopic notation.

- Particle in a general central potential. Bound states. Scattering states. Phase shift. Qualitative features of radial wavefunctions.
- $V = 0$ as a central potential. Operator method giving spherical Bessel functions.
- The supersymmetric method. Constructing Hamiltonians with related spectrums. General properties.
- Using operator methods to solve radial wave equations. The supersymmetric method applied to hydrogen and other examples.

6. **Addition of angular momentum.**

   References: Griffiths, Ch. 4; Ohanian, 9.3; Cohen-Tannoudji, Ch. X.

   - Semiclassical derivation of spin-orbit coupling; motivates need to consider $\vec{J} = \vec{L} + \vec{S}$. Magnitude of energy splittings in hydrogen due to $\vec{L} \cdot \vec{S}$ coupling.
   - Alternative complete sets of commuting operators. Coupled and uncoupled bases. Addition of angular momentum as change of basis. Determination of what j's are allowed for a given ℓ and s. Determination of Clebsch-Gordan (C-G) coefficients and their properties.
   - C-G coefficients for $L \times (S = 1/2)$.
   - C-G coefficients for $(S = 1/2) \times (S = 1/2)$
   - Spin and orbital angular momentum, angular momentum of several particles.
   - Hyperfine interaction in the ground state of the hydrogen atom.

7. **Introduction to the Quantum Mechanics of Identical Particles.**

   References: Griffiths, Ch. 5.1, 5.2, 6.3, 6.4; Cohen-Tannoudji, Ch. XIV.

   - N-particle systems. Identical particles are indistinguishable.
   - Exchange operator, symmetrization and antisymmetrization.
   - Exchange symmetry postulate. Bosons and fermions.
   - Pauli exclusion principle. Slater determinants.
   - Non-interacting fermions in a common potential well.
   - Exchange "force" and a first look at hydrogen molecules and helium atoms.
   - Rotation states of the hydrogen molecule and the effects of the spin of the protons: ortho- and para-hydrogen.