Massachusetts Institute of Technology
Department of Physics

Physics 8.07 Fall 2004

Midterm Exam

1. **Polarized medium (40 points)**
A point charge q_0 is placed at $\vec{x} = 0$ in a polarizable medium.

a) Give the displacement field $\vec{D}(\vec{x}\,)$ (magnitude and direction) for $\vec{x} \neq 0$.

b) Measurements show that the electrostatic potential is spherically symmetric and is given by

$$V(r) = \begin{cases} q_0/(4\pi\epsilon_0 r) \ , & r < r_0 \\ q_0[r_0^2 + (r - r_0)^2]/(4\pi\epsilon_0 r^3) \ , & r > r_0 \ . \end{cases}$$

Determine the electric field $\vec{E}(\vec{x}\,)$ (magnitude and direction) for $\vec{x} \neq 0$.

c) What is the bound surface charge σ_b at $r = r_0$?

d) What is the bound charge density ρ_b for $r > r_0$?

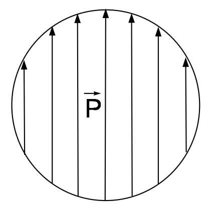
2. **Current flowing in a wire (40 points)**
A uniform current density $\vec{J} = J\vec{e}_z$ flows down a long cylindrical wire of radius a. The current density is uniform for $0 \leq s \leq a$.

a) Find the magnetic field $\vec{B}(\vec{x}\,)$ (magnitude and direction) everywhere. (Use the appropriate cylindrical orthonormal basis vectors $\vec{e}_s$, $\vec{e}_\phi$, and/or $\vec{e}_z$.)

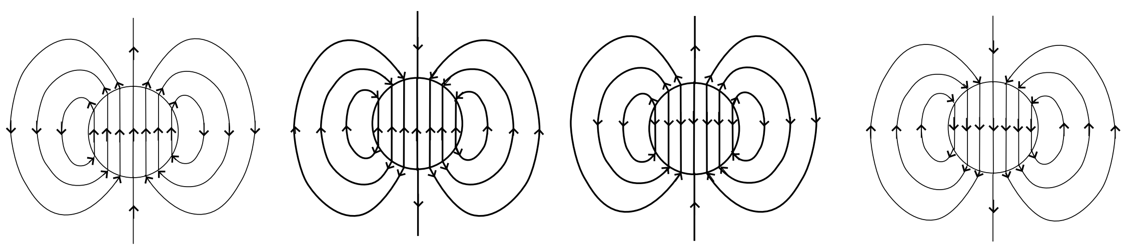
b) What is force per unit volume (magnitude and direction, as a function of position) on the current in the wire?

c) By whatever means you can think of, find the vector potential $\vec{A}(\vec{x}\,)$ (magnitude and direction) everywhere, with boundary condition $\vec{A} = 0$ for $s = 0$.

Multiple Choice Questions (10 points each)



3. A sphere is uniformly polarized as shown in the diagram above. Circle the case below that shows the correct electric field lines.



(a) (b) (c) (d)

4. A circular disk of radius R has uniform surface charge density σ and rotates like a wheel about its central axis with angular velocity $\vec{\omega}$. The magnetic field for $r \gg R$ is given by which of the following expressions?

(a)

$$\frac{\mu_0 \sigma \omega R^2}{4\pi r^2}(\cos\theta\, \vec{e}_r - \sin\theta\, \vec{e}_\theta) \ ,$$

(b)

$$\frac{\mu_0 \sigma \omega R^4}{4\pi r^2}(\cos\theta\, \vec{e}_r - \sin\theta\, \vec{e}_\theta)$$

(c)

$$\frac{\mu_0 \sigma \omega R^2}{16 r^3}(2\cos\theta\, \vec{e}_r + \sin\theta\, \vec{e}_\theta) \ ,$$

(d)

$$\frac{\mu_0 \sigma \omega R^4}{16 r^3}(2\cos\theta\, \vec{e}_r + \sin\theta\, \vec{e}_\theta) \ .$$