Massachusetts Institute of Technology
Department of Physics

Physics 8.07 | Fall 2005

Problem Set #3
Due at 4pm Friday, September 30, 2005

1. **Cylindrical Capacitor**

   a) **Griffiths** Problem 2.39 (p. 106)

   b) How much electric energy per unit length is stored in the capacitor when its charge per unit length is λ?

2. **Mean Value Theorem**

   a) **Griffiths** Problem 3.1 (p. 115)

   b) **Griffiths** Problem 3.2 (p. 115)

3. **Method of Images**

A point charge q is situated a distance a from the center of a thin grounded ($V = 0$) conducting spherical shell of radius $R > a$. Assume $V \to 0$ as $r \to \infty$.

   a) Find the potential everywhere ($r < R$ and $R > r$).

   b) Find the induced surface charge density $\sigma(\theta)$ on the inner surface of the sphere, as a function of θ.

   c) What is the total induced surface charge $Q_{\text{ind}} = \int \sigma\, da$, and what is the force on the charge q? As the charge is brought to a very small distance $R - a \ll R$ from the sphere, how does the force on the charge compare with the force on a charge the same distance from a grounded conducting plane?

   d) Suppose that instead of being grounded, the conductor has zero total charge (summing over both its inner and outer surfaces). Find the potential everywhere and find the force on the charge q.

4. **Separation of Variables (twice the credit of one problem)**
A rectangular box has side lengths a, b, and c, with $0 \le x \le a$, $0 \le y \le b$, and $0 \le z \le c$. There are no charges inside the box. The potential is zero on four of the sides of the box: $x = 0$, $x = a$, $y = 0$, $y = b$. The other two sides ($z = 0$ and $z = c$) have $E_z = E_0 =$ constant.

a) Using separation of variables, find two sets of basis functions $V_{mn}^{(1)}(x, y, z)$ and $V_{mn}^{(2)}(x, y, z)$ satisfying Laplace's equation and the boundary conditions along the four sides with $V = 0$. Be sure to indicate the values taken by the indices (m, n).

b) Write the potential inside the box $V(x, y, z)$ as a sum over the basis functions. Impose boundary conditions along the remaining two sides to arrive at an expression for the expansion coefficients.

c) Assuming $a = b = c$, evaluate $E_z(a/2, a/2, a/2)/E_0$ to at least three significant figures, by summing enough terms of the series. **Note: this part requires the use of a computer or programmable calculator. You may use Matlab or similar package on** the MIT server**, or write a simple program.**

5. **Legendre series**
A sphere of radius R has a certain charge density distribution $\sigma(\theta)$. There are no other charges. The potential on the sphere at radius R is

$$V = V_0 + V_1 \cos\theta + V_2 \cos 2\theta$$

where V_0, V_1, and V_2 are constants.

a) Find $V(r, \theta)$ for $r < R$ and $r > R$.

b) Find the electric field $\vec{E} = E_r \vec{e}_r + E_\theta \vec{e}_\theta$ for $r < R$ and $r > R$. Then verify the tangential jump condition for $\vec{E}$ across the sphere.

c) Using the normal jump condition for $\vec{E}$ find $\sigma(\theta)$.