Massachusetts Institute of Technology
Department of Physics

Physics 8.07 Fall 2005

Problem Set #9
Due at 4pm Monday, November 21, 2005

1. **Griffiths Problem 8.12 (p. 362)**
You will need the following integral:

$$\int_{-1}^{1} \frac{(1-x^2)\,dx}{(1-2ax+a^2)^{3/2}} = \frac{4}{3}\min(1,a^{-3}) \quad \text{for} \quad a>0 \ .$$

2. **Maxwell Stress Tensor**
Evaluate the time-average Maxwell Stress tensor T_{ij} for the following fields:

   a) A linearly polarized plane electromagnetic wave in vacuum with $\vec{E} = \vec{e}_x\,\mathcal{E}\cos(kz-\omega t)$ (and the appropriate $\vec{B}$).

   b) A circularly polarized plane electromagnetic wave traveling in a linear medium with given (ϵ,μ), with

   $$\vec{E} = \frac{\mathcal{E}}{\sqrt{2}}\left[\vec{e}_x\,\cos(kz-\omega t) + \vec{e}_y\,\sin(kz-\omega t)\right]$$

   (and the appropriate $\vec{B}$).

   c) A static, uniform electric field $\vec{E} = \mathcal{E}\vec{e}_x$ in vacuum (and $\vec{B} = 0$).

   d) A static, uniform magnetic field $\vec{B} = \mathcal{B}\vec{e}_x$ in a linear medium with permeability μ (and $\vec{E} = 0$).

   e) A static, random magnetic field $\vec{B} = \mathcal{B}\vec{n}$ where $\vec{n}$ is a unit vector with random direction. Show that for a randomly-directed field, $\langle T_{ij}\rangle = -p_{\rm mag}\delta_{ij}$ where angle brackets denote an average over the direction of the random vector $\vec{n}$. Find the isotropic magnetic pressure $p_{\rm mag}$ in terms of $\mathcal{B}$ and the permeability μ_0 and compare $p_{\rm mag}$ with the magnetic energy density $u_{\rm mag}$. (This situation describes the turbulent magnetic field of our Galaxy.)

3. **Griffiths Problem 9.5 (p. 373)**

4. **Griffiths Problem 9.16 (p. 392)**

5. **Griffiths Problem 9.19 (p. 396)**