

Problem Set #4
Due at 4pm Friday, October 7, 2005

1. Conducting Sphere in a Gradient Field (twice the credit of one problem)

This problem is a modification of Griffiths Example 3.8. A solid conducting sphere of radius R is placed in an external electric field

$$E_i(\vec{x}) = C_i + D_{ij}x_j$$

where C_i and D_{ij} are constants and $D_{ii} = 0$. The sphere is centered at $\vec{x} = 0$ and has zero net charge. You may find it helpful to write $x_i = r\hat{r}_i$. Note that this problem does not involve Legendre polynomials.

- a) Suppose that the conducting sphere develops an induced dipole moment p_i and quadrupole moment Q_{ij} . What potential $V_{\text{ind}}(\vec{x})$ do they create for $r > R$?
- b) Derive expressions for p_i and Q_{ij} in terms of R , C_i , and D_{ij} .
- c) What is the surface charge distribution on the sphere, $\sigma(\hat{r})$? Compare the dipole part with Griffiths eq. (3.77).
- d) Suppose that the external field is changed so that $D_{ii} \neq 0$. What charge distribution creates such a field?
- e) Consider 4 infinite line charges placed parallel to the z -axis, offset from $(x, y) = (0, 0)$ as follows:

line charge λ at	$(x, y) = (0, d)$
line charge λ at	$(x, y) = (0, -d)$
line charge $-\lambda$ at	$(x, y) = (d, 0)$
line charge $-\lambda$ at	$(x, y) = (-d, 0)$

Evaluate $E_i(\vec{x})$ to first order in \vec{x}/d . Find D_{ij} at $\vec{x} = 0$. Make a rough sketch of the electric field lines showing the line charge with no conducting sphere.

2. Molecular Dipoles

- a) In lecture we derived the following expression for the mean dipole moment of a polar molecule in a gas or liquid placed in an external electric field $E\vec{e}_z$:

$$\langle p_z \rangle = p \left(-\frac{1}{a} + \coth a \right) ,$$

where p is the molecular dipole moment and $a = pE/kT$. Consider a water molecule in the strong electric field 10^6 V/m at room temperature. Evaluate a to two significant digits. By what factor is the average dipole reduced due to random orientations? (See Griffiths Section 4.1 for the dipole moment of a water molecule.)

- b) Lightning occurs when a strong atmospheric electric field causes molecules to become ionized, liberating free charge which then flows so as to decrease the electric field. Nitrogen molecules are non-polar. Treating the nitrogen molecule as a conducting sphere of radius equal to the bond length, estimate the potential energy of a nitrogen molecule in an electric field 10^6 V/m (more than 10 times greater than the field in a real thunderstorm). Is the energy large enough to ionize nitrogen? If not, can you guess how electrons get liberated to make a lightning bolt?

3. Electric polarization

- a) Griffiths Problems 4.10 (p. 169)
- b) Griffiths Problems 4.11 (p. 170). In the case $L \ll a$, find the electric field inside the electret.
- c) Explain why Griffiths eq. (4.18) does not give the interior electric field for either part a) or part b) above.

4. Displacement Field

Using $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, (i) compute \vec{D} everywhere, (ii) verify that it obeys the boundary conditions of Griffiths eqs. (4.26) and (4.27) at the interface of the polarized materials, and (iii) make a sketch of the \vec{D} field lines, for

- a) The uniformly polarized sphere of Griffiths Example 4.2;
- b) The radially polarized sphere of Problem 3(a) above; and
- c) The bar electret of Problem 3(b) above, in the limit $L \ll a$. (You do not have to compute \vec{D} exactly for the fringing field at the edge of the cylinder; however, do make a qualitative sketch of the \vec{D} field lines.)

d) In which of these cases can \vec{D} be written as the gradient of a potential everywhere?

5. **Griffths** Problem 4.18 (p. 184)
Neglect the fringing fields.