Massachusetts Institute of Technology
Department of Physics

Physics 8.07 Fall 2005

Midterm Exam

1. **Electrostatics (40 points)**

An unknown charge distribution produces the electrostatic potential

$$V(\vec{x}\,) = \frac{V_0}{2R^2}(2z^2 - x^2 - y^2) \quad \text{for} \quad r^2 \equiv x^2 + y^2 + z^2 < R^2 \ .$$

a) What is the charge density distribution $\rho(\vec{x}\,)$ for $r < R$?

b) Suppose that the charges responsible for creating $V(\vec{x}\,)$ reside on the (non-conducting) sphere $r = R$ and suppose that there are no charges anywhere else. In addition, the net charge integrated over the sphere is zero. Find $V(r,\theta)$ for $r > R$ where $\cos\theta = z/r$.

c) Find the surface charge density $\sigma(\theta)$ at $r = R$.

d) In terms of V_0, R, and ϵ_0, how much potential energy is stored in the system?

The following formulae may be useful in this problem:

$$(2l+1)\int_{-1}^{1} P_l(x)P_{l'}(x)\,dx = 2\delta_{ll'} \ ,$$

$$P_0(x) = 1 \ , \quad P_1(x) = x \ , \quad P_2(x) = \frac{1}{2}(3x^2 - 1) \ .$$

2. **Currents in wires and space (40 points)**
The magnetic geometry of a magnetic reconnection experiment (the VTF at PSFC, MIT) can be approximated by that of four infinite wires parallel to the z-axis carrying currents with the same magnitude, I, but in alternating directions. The signs and locations of the currents are

$$\begin{array}{rccl} \text{current} & I & \text{at} & (x,y)=(0,d) \\ \text{current} & I & \text{at} & (x,y)=(0,-d) \\ \text{current} & -I & \text{at} & (x,y)=(d,0) \\ \text{current} & -I & \text{at} & (x,y)=(-d,0) \end{array}$$

It follows that in the central region where $x^2+y^2 \ll d^2$ the magnetic vector potential is approximated by $\vec{A} = A_z\,\vec{e_z}$, where $A_z = b_0(y^2-x^2)$ and $b_0 = \mu_0 I/(\pi d^2)$.

a) Make a sketch of the magnetic field lines (with directions indicated) in the x-y plane including all four conductors and the fields around them.

b) Find expressions for the components of $\vec{B}$ for $x^2+y^2 \ll d^2$.

Due to (plasma) currents in the volume of the experiment the expression for $\vec{A}$ is altered into $\vec{A} = A_z\,\vec{e_z}$ where $A_z = b_0[(1/\alpha)y^2 - \alpha x^2]$, valid for $x^2+y^2 \ll d^2$.

c) With this new expression for $\vec{A}$ show that $\vec{\nabla}\cdot\vec{B} = 0$ (as required by the Maxwell equations).

d) Calculate the current density $\vec{J}$ for $x^2+y^2 \ll d^2$, in terms of b_0, α, and μ_0.

Multiple Choice Questions (10 points each, no partial credit)

3. The electric field inside a large piece of dielectric is $\vec{E}_0$ and the polarization is $\vec{P}$. Now a thin wafer-shaped cavity perpendicular to $\vec{P}$ is hollowed out of the material. The electric field in the center of the cavity is approximately

(a) $\vec{E}_0 + \frac{1}{\epsilon_0}\vec{P}$

(b) $\vec{E}_0 - \frac{1}{\epsilon_0}\vec{P}$

(c) $\vec{E}_0 + \frac{1}{3\epsilon_0}\vec{P}$

(d) $\vec{E}_0 - \frac{1}{3\epsilon_0}\vec{P}$

4. Two infinite, parallel conducting sheets have identical surface current $\vec{K}$. The force per unit area on one sheet is

(a) $\mu_0 K^2$ towards the other plate

(b) $\mu_0 K^2$ away from the other plate

(c) $\frac{1}{2}\mu_0 K^2$ towards the other plate

(d) $\frac{1}{2}\mu_0 K^2$ away from the other plate