

Fitting of Data

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Chapters 4-6

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Reminder

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Fitting

- Fitting: Find a functional form that describes data within errors
- Why fit data?
 - extract physical parameters from data
 - test validity of data or model
 - interpolate/extrapolate data

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How to fit data?

- Vary parameters of function until you find a global maximum of goodness-of-fit criterion
- Goodness-of-fit
 - chi-square ← **most common**
 - likelihood
 - (Kolmogorov-Smirnov test)

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Chi-square fit

Chi-square → $\chi^2 = \sum_{i=1}^N \frac{(x_i - e_i)^2}{\sigma_i^2}$

Data points → x_i Model expectation → e_i

Estimated uncertainty → σ_i^2

Best Fit: Global minimum of Chi-square

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Goodness of fit

- You EXPECT data to fluctuate by \sim error
- For Gaussian errors, only 68% of your data points should agree with model/fitted function by better than 1σ
- Chi-square per Degree of Freedom (DoF) should be ~ 1
 - DoF: Number of data points - number of fitted parameters
- Chi-square allows you to test if model/fitted function is compatible with data

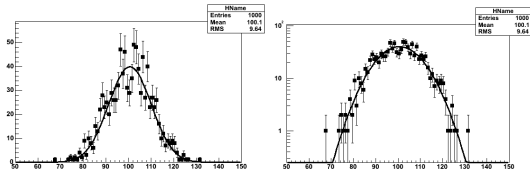
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Example

- Suppose you have 10^4 atoms
- Probability of decay is 10^{-2} /sec
- Count # of decays/second
- Repeat experiment many times (always starting with 10^4 atoms)
- What distribution do you expect for the number of observed counts?

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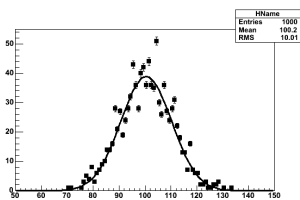
Example



Chi-square/DoF = 60/53

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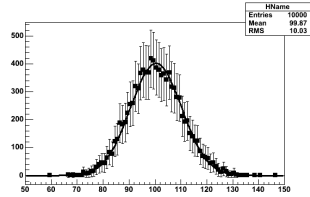
Example



Chi-square/DoF = 1200/55

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Example



Chi-square/DoF = 4.5/71

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Chi-square distribution

- Even if your error estimate is correct and the model is correct
 - Chi-square will fluctuate from $\chi^2/\text{DoF} < 1$ to $\chi^2/\text{DoF} > 1$
 - The shape of the χ^2 -distribution depends only on the number of degrees of freedom
 - allows calculation of probability that data and model agree
- Chi-square probability
 - Percentage of all measurements that you expect to have a worse chi-square than what you see

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Fitting

- Linear relationship: Analytical
- In general: Numerical minimization of chi-square
 - Many methods
 - simple grid-search
 - Non-linear Newton
 - Levenberg-Marquardt
 - Simulated annealing
 - Criteria
 - Dimension of parameter space
 - Data statistics
 - Challenge
 - Speed vs finding true global minimum

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