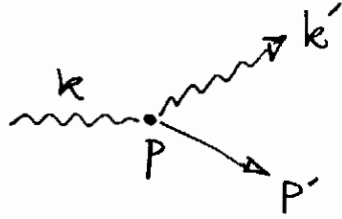


PROBLEM 2



$$k + p = k' + p' \quad \text{four vectors}$$

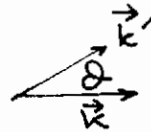
$$k - k' + p = p'$$

$$(p')^2 = m^2 c^2 = k^2 + k'^2 + p^2 + 2p \cdot (k - k') - 2k \cdot k' \quad (1)$$

$$k^2 = 0, k'^2 = 0, p^2 = m^2 c^2$$

$$2p \cdot (k - k') = 2mc (E - E')/c$$

$$k \cdot k' = \frac{EE'}{c^2} (1 - \cos\theta)$$



$$(1) \rightarrow m^2 c^2 = m^2 c^2 + 2mc (E - E')/c - \frac{2EE'}{c^2} (1 - \cos\theta)$$

$$EE' (1 - \cos\theta) = mc^2 (E - E')$$

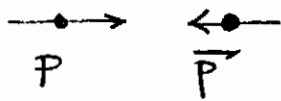
$$E = h\nu = hc/\lambda$$

$$1 - \cos\theta = mc^2 \left(\frac{\lambda'}{hc} - \frac{\lambda}{hc} \right) \Rightarrow \lambda' - \lambda = \frac{h}{mc} (1 - \cos\theta)$$

PROBLEM 3

a) $P + \bar{P} \rightarrow t + \bar{t}$

Minimum energy $\Rightarrow t \neq \bar{t}$ produced at rest



$$2E = 2m_t c^2 = 300 (939 \text{ MeV})$$

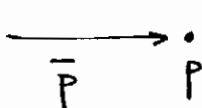
$$2E = 281.70 \text{ GeV}$$

Each proton starts w/ 939 MeV so

$$2E_{\text{BEAM}} = 281.70 - 2(939) = 279.82 \text{ GeV}$$

Cost is $\# 2.7982 \times 10^8$

b) In rest frame of proton



$$(P + \bar{P})^2 = 4m_t^2 c^2$$

$$2M_p^2 c^2 + 2P \cdot \bar{P} = 4m_t^2 c^2$$

$$\left. \begin{aligned} P &= (m_p c, \vec{0}) \\ \bar{P} &= (\bar{E}/c, 0) \end{aligned} \right\} P \cdot \bar{P} = m_p \bar{E}$$

$$M_p^2 c^2 + M_p \bar{E} = 2m_t^2 c^2 \quad \rightarrow \quad \bar{E} = \frac{2m_t^2 - M_p^2}{M_p} c^2$$

$$\begin{aligned} E_{\text{BEAM}} &= \bar{E} - M_p c^2 = 2 \frac{m_t^2 - M_p^2}{M_p} c^2 = 2 [(1150)^2 - 1] M_p c^2 \\ &= 11249.5 M_p c^2 = 10563.28 \text{ GeV} \end{aligned}$$

Cost is $\# 10.563 \times 10^9$

Recommendation: build the two rings!

PROBLEM 4

a) $P_{K\mu} + P_{P\mu} = P_{\pi\mu} + P_{\Lambda\mu}$

In each case $P_{\mu} = (E/c, \vec{P})$

b) P and Λ are at rest, isolate P_{π} :

$$P_K + P_P - P_{\Lambda} = P_{\pi}$$

Square $m_{\pi}^2 c^2 = (m_P - m_{\Lambda})^2 c^2 + m_K^2 c^2 + 2(m_P - m_{\Lambda}) E_{Kc}$

$$E_K \equiv E^* = \frac{m_K^2 + (m_P - m_{\Lambda})^2 - m_{\pi}^2}{2(m_{\Lambda} - m_P)} c^2$$

c)

$$E^* = 722.50 \text{ MeV}$$

$$\text{K.E.} = E^* - m_K c^2 = 228.5 \text{ MeV}$$

d) Process no longer works if $E^* < m_K c^2$. Limit occurs at $E^* = m_K c^2$ or

$$2(m_{\Lambda} - m_P) m_K = m_K^2 + (m_P - m_{\Lambda})^2 - m_{\pi}^2$$

$$m_{\pi}^2 = (m_K + m_P - m_{\Lambda})^2$$

$$m_{\pi} = m_K + m_P - m_{\Lambda}$$

$$\text{so } m_{\pi} c^2 = 317 \text{ MeV}$$

Process occurs provided $m_{\pi} c^2 < 317 \text{ MeV}/c$.

PROBLEM 5

- a) Neutrinos with $E = 5 \text{ MeV}$ have some speed β .
(depends on their mass)

Let $t_\gamma = \text{light travel time}$

$t_\nu = \text{neutrino travel time}$

$$t_\nu - t_\gamma = \frac{d}{\beta c} - \frac{d}{c} \quad d = \text{distance to LMC}$$

$$\Delta t = \frac{d}{c} \left(\frac{1-\beta}{\beta} \right) \approx \frac{d}{c} (1-\beta)$$

$$1-\beta = \frac{c \Delta t}{d} = \frac{\Delta t}{(d/c)} \leq \frac{1 \text{ hr}}{30000 \text{ yrs}}$$

$$1-\beta < \frac{3600 \text{ seconds}}{5 \times 10^4 \times \pi \times 10^7 \text{ seconds}} = \frac{3.6 \times 10^3}{15.7 \times 10^{11}} = .23 \times 10^{-8}$$

$$\left. \begin{array}{l} \beta \geq 1 - 2.3 \times 10^{-9} \\ \beta^2 \geq 1 - 4.6 \times 10^{-9} \end{array} \right\} \Rightarrow \gamma \geq \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{4.6 \times 10^{-9}}}$$

$$\gamma \geq \frac{10^5}{6.78} \geq 1.5 \times 10^4$$

$$m_\nu c^2 \gamma = E_\nu$$

$$m_\nu c^2 < \frac{5 \text{ MeV}}{1.5 \times 10^4} = 3.33 \times 10^{-4} \text{ MeV} = \underline{\underline{33.3 \text{ KeV}}}$$

b) If neutrinos have a mass m_ν and energy E_ν , then they travel with

$$\gamma = \frac{E_\nu}{m_\nu c^2}$$

The half-life to an earth observer is greater than 50,000 yrs.

$$\tau_{1/2}^{\text{earth}} \geq 50,000$$

$$\tau_{1/2}^{\text{proper}} = \frac{\tau_{1/2}^{\text{earth}}}{\gamma} \geq \left(\frac{m_\nu c^2}{E_\nu} \right) 50,000 \text{ yrs.}$$

If we take the lower limit on γ from part a)

$$\tau_{1/2}^{\text{proper}} = \frac{50,000}{15,000} = 3\frac{1}{3} \text{ yrs.}$$