8.324 – Homework 1

Due: Tuesday 20 September 2005

<u>Problem 1</u>. Brown, p. 178. Problem 5. Phase space integral relevant for decay rate.

<u>Problem 2</u> Spatially localized states of a scalar particle (see Brown, p. 183, problem 11). Consider states of the form

$$|\vec{r}, t\rangle \equiv \int d^3\vec{p}\, f(\vec{p})\, |p\rangle\, e^{-i\vec{p}\cdot\vec{r}+iE(\vec{p})t}\,,$$

where $f(\vec{p})$ is a function to be determined and $|p\rangle$ are the familiar one-particle states of momentum $\vec{p}$ and energy $E(\vec{p})$.

(a) A consistency check: Verify that the action of translation operators can be used to take $|\vec{r}, t\rangle$ to $|\vec{r}+\vec{a}, t+\tau\rangle$.

(b) Determine $f(\vec{p})$ by the condition $\langle\vec{r}', t|\vec{r}, t\rangle = \delta^{(3)}(\vec{r}-\vec{r}')$ of perfect localization. Compare your resulting state $|\vec{r}, t\rangle$ with the state $|x\rangle \equiv N\phi(x)|0\rangle$, where N is a normalization constant and $x = (\vec{x}, t)$. Are the states $|x\rangle$ perfectly localized ?

(c) Consider the overlap $C(\vec{r}-\vec{r}', t-t') = \langle\vec{r}', t'|\vec{r}, t\rangle$. Evaluate $C(\vec{r}, t)$ for a spacelike separation: $|\vec{r}| > t$ in terms of derivatives of the Bessel function K_0 [Hint: $C(\vec{r}, t)$ can be written as the time derivative of a Lorentz invariant quantity that is easier to compute]. Suppose we interpret $|\langle\vec{r}, t|\Psi\rangle|^2$ as the probability to find a particle at $\vec{r}$ at time t. Would propagation be causal (*i.e.* confined to the forward light-cone)?

<u>Problem 3</u> Brown, p. 184, problem 12.

<u>Problem 4</u>: Calculating decay rates. For this you should recall that when

$$\langle p_1, p_2, \cdots p_n; +|p; -\rangle = -i\,(2\pi)^4\,\delta^4(p - \sum p_f)\,T(p_1\cdots p_n; p)\,, \tag{1}$$

the decay rate Γ is then given by

$$\Gamma = \Big[\prod_{i=1}^{n}\int \frac{d^3p_i}{(2\pi)^3}\frac{1}{2p_i^0}\Big]\,(2\pi)^4\,\delta^4(p - \sum p_f)\,|T(p_1\cdots p_n; p)|^2\,\frac{1}{2p^0}\,. \tag{2}$$

A factor of $1/k!$ must be added for each group of k identical particles in the final state.

(a) Consider two scalar fields ϕ and χ coupled by

$$\mathcal{L} = -\tfrac{1}{2}(\partial_\mu\phi)^2 - \tfrac{1}{2}M^2\phi^2 - \tfrac{1}{2}(\partial_\mu\chi)^2 - \tfrac{1}{2}m^2\chi^2 + \tfrac{1}{2}\,g\phi\chi^2\,.$$

Assume $M > 2m$ and calculate the decay rate of ϕ-particles to $\mathcal{O}(g)$.

(b) Let A, B, C and D be four real scalar fields with dynamics governed by the Lagrangian

$$\mathcal{L} = -\tfrac{1}{2}\Big((\partial_\mu A)^2 + m^2A^2 + (\partial_\mu B)^2 + (\partial_\mu C)^2 + (\partial_\mu D)^2\Big) - g\,ABCD\,.$$

Calculate the decay rate Γ of the A particle to lowest order in the coupling constant g. How would the decay width change if the interaction $g\,ABCD$ is replaced by gAB^3 ?