8.3. Dipole Forces within the dressed-atom picture

Refs.: [1] J. Dalibard, C. Cohen-Tannoudji, JOSA B 2, 1707 (1985)
[2] C. Cohen-Tannoudji, Les Houches Summer School (1990)
[3] API

- Provides physical insight for
  Dipole traps
  Blue molasses: simplest example for Sisyphus cooling and motion induced changes of population

- Remember [1,3]:
  × Splitting between dressed states
  $$\hbar \Omega(r) = \hbar \sqrt{\Omega_1^2(r) + \delta_L^2}$$
  × Steady state populations of dressed states 1 and 2
  $$\Pi_1^{st} = \frac{\sin^4\theta}{\cos^4\theta + \sin^4\theta} \qquad (\tan 2\theta = -\Omega_1/\delta_L)$$
  $$\Pi_2^{st} = \frac{\cos^4\theta}{\sin^4\theta + \cos^4\theta}$$
  × Relaxation rate for Π_1 and Π_2 to reach steady state:
  $$\Gamma_{pop} = \Gamma\left(\cos^4\theta + \sin^4\theta\right)$$

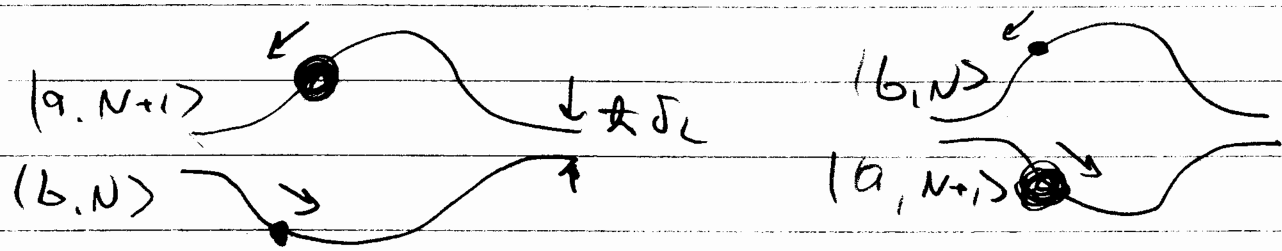
8.3.1.

Mean dipole force, $v = 0$ [1, 2, 3]

$$F_1 = -\frac{\hbar}{2} \nabla \Omega(r)$$

$$F_2 = +\frac{\hbar}{2} \nabla \Omega(r)$$

$$\langle F_{dip} \rangle = F_1 \Pi_1^{St} + F_2 \Pi_2^{St} = -\frac{\hbar \delta_L}{2} \frac{\Omega_1^2}{\frac{\Omega_1^2}{2} + \delta^2} \vec{\alpha}$$

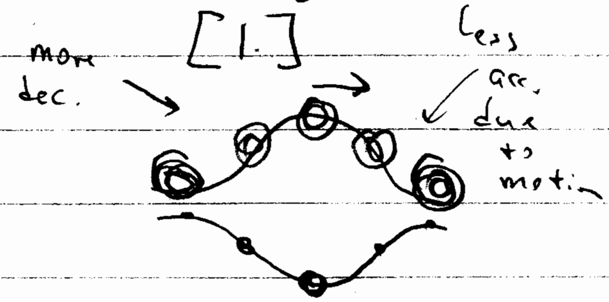


$\delta_L > 0$ $\delta_L < 0$

$$\delta_L = 0 \qquad \Pi_1^{St} = \Pi_2^{St} \Rightarrow \langle F_{dip} \rangle = 0$$

8.3.2

Mean dipole Force for a slowly moving atom [1.]



$$\langle F_{dip} \rangle = F_1 \Pi_1 + F_2 \Pi_2$$

time lag by $\tau_{pop} = 1/\Gamma_{pop}$

$$\Pi_i \simeq \Pi_i^{St}(\vec{r} - \vec{v}\,\tau_{pop})$$

$$\Rightarrow \vec{F}_{dip}(r, v) = \vec{F}_{dip}^{St} - \frac{2\hbar\delta_L}{\Gamma}\left(\frac{\Omega_1^2(r)}{\Omega_1^2(r) + 2\delta_L^2}\right)^3 (\vec{\alpha}\cdot\vec{v})\,\vec{\alpha}$$

Standing wave $\Omega_1(x) = 2\Omega_1 \cos kx$

λ-average: $\langle F_{dip} \rangle = -\alpha v$

For $\delta_L \gg \Omega_1$ (note that $\vec{\alpha} \approx \vec{k}$) $\quad \alpha \approx \frac{\hbar\delta_L}{\Gamma} \frac{\Omega_1^6}{\delta_L^6} k^2$

We will late obtain this result in a different way

8.3.3

Energy balance in a small displacement [1]

$$dW = -\langle \bar{F}_{dip} \rangle \, \delta \vec{r}$$ Work

$$= \underbrace{dU_A}_{\text{Atoms}} + \underbrace{dU_F}_{\text{radiation Field}}$$

$$U_A = \sum \Pi_i E_i$$

internal energy of dressed state

$$dU_F = \underbrace{\left(\Gamma_{2\leftarrow 1} \Pi_1 \hbar\Omega - \Gamma_{1\leftarrow 2} \Pi_2 \hbar\Omega\right) dt}_{=0 \text{ For symmetric sidebands}}$$

When atom moves, Π_i's change $\Rightarrow$ asymmetric sidebands.

8.3.4

Atomic Momentum Diffusion [1]

Forces change signs during the radiative cascade

Remember: $2D_p = \frac{d}{dt}\left(\langle p^2 \rangle - \langle p \rangle^2\right)$ Definition of momentum diffusion coefficient

$\langle \dot{E}_{heat} \rangle = D/M$

$$D_p = \int_0^\infty d\tau \left[\langle F(0) F(\tau) \rangle - \langle F(0) \rangle^2\right]$$

Momentum diffusion is caused by fluctuations of the force

$$\frac{d}{dt}\left(\langle \vec{p} \cdot \vec{p} \rangle - \langle \vec{p} \rangle \langle \vec{p} \rangle\right) =$$

$$2 \langle \vec{p} \cdot \vec{F}(0) \rangle - \langle \vec{p} \rangle \langle \vec{F}(0) \rangle =$$ $\vec{p}$ is time integral of the force

$$2 \int_{-\infty}^{0} dt \left[\langle \vec{F}(t) \vec{F}(0) \rangle - \langle \vec{F}(t) \rangle \langle \vec{F}(0) \rangle\right]$$

Outlook: $k_B T = D/\alpha$

Radiative cascades

Force switches between F_1 and $F_2 = -F_1$

$$|F_1| = |F_2| = \frac{\hbar}{2} \nabla \Omega_1$$

On resonance; Correlation time $\tau = 2/\Gamma$

$$\Rightarrow D_{dip} \sim \frac{\hbar^2 (\nabla \Omega_1)^2}{2\Gamma}$$

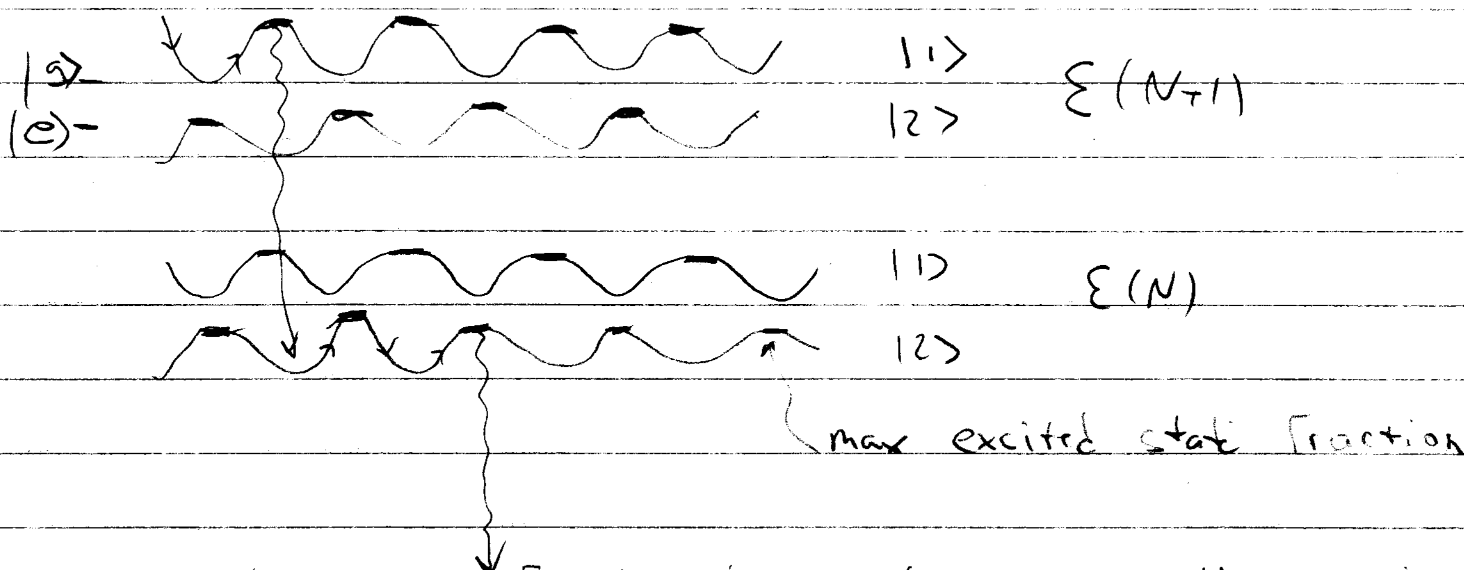
result for arbitrary δ [1]

$$D_{dip} = \frac{\hbar^2}{2\Gamma}\left(\frac{\Omega_1^2}{\Omega_1^2 + 2\delta^2}\right)^3 (\nabla \Omega_1)^2$$

8.3.5

Atoms moving in a standing wave [1,2]

$\Gamma \leq kv$; atom moves several wavelengths in one lifetime



excited state fraction is largest at the maxima of the potential energy both in |1> and |2>. $\Rightarrow$ Preferential emission at the potential hills = Sisyphus cooling

Cooling rate $\propto U_0 \Gamma_{1\to 2} \Pi_1$

8.3.6

Cooling in a standing wave [1]

- Summary of concepts for the simplest limit

$\delta_L \gg \Omega_1 \gg \Gamma$, neglecting factors on the order of unity.

$\hbar = 1$

$$\Omega_1(x) = 2\Omega_1 \cos kx$$

$$|1\rangle = |a\rangle + \left(\frac{\Omega_1(x)}{\delta_L}\right)|b\rangle$$

$$|2\rangle = |b\rangle - \left(\frac{\Omega_1(x)}{\delta_L}\right)|a\rangle$$

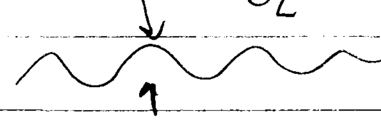
First order perturbation theory

$$\Gamma_{1\to2} = \Gamma\left(\frac{\Omega_1(x)}{\delta_L}\right)^4$$

$$\Gamma_{2\to1} = \Gamma$$

$U_0 = \frac{\Omega_1^2}{\delta_L}$ AC Stark effect

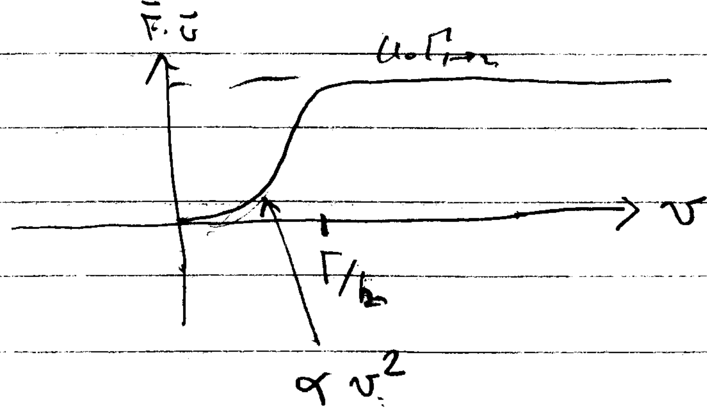
Dressed state potential



$v \gtrsim \Gamma/k$: Sisyphus cooling

Cooling rate $\vec{F}\cdot\vec{v} = U_0 \Gamma_{1\to2}$

v small $|F| = |\alpha v|$



estimate α

$$\alpha = \lim_{v\to0} \frac{F(v)}{v}$$

$$\approx \frac{U_0\Gamma_{1\to2}}{(\Gamma/k)^2}$$

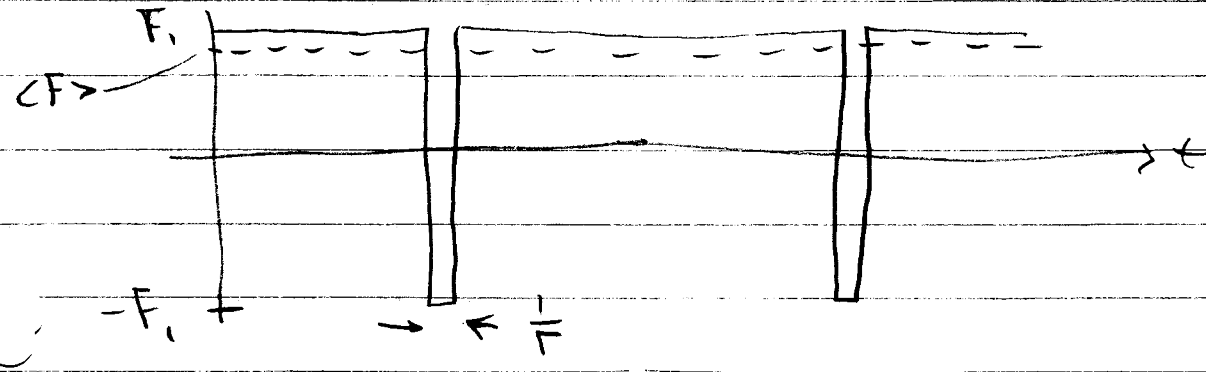
$$= \frac{\Omega_1^2}{\delta_L}\frac{\Gamma\Omega_1^4}{\delta_L^4}\frac{k^2}{\Gamma^2}$$

$$= \frac{\Omega_1^6 k^2}{\delta_L^5\Gamma}$$ ← as obtained before

Diffusion

$$D = \int \langle \Delta F(0)\, \Delta F(t) \rangle \, dt$$

atom mainly in state $|1\rangle$, Force $F_1 = -\nabla U_1(r) \approx k U_0$
For a duration $1/\Gamma_1$ at a rate $\Gamma_{1\to 2}$, the force jumps to $-\nabla U_2(r) = +\nabla U_1(r)$



Dominant contribution to D from the spikes

$$D = (k U_0)^2 / \Gamma \; \underbrace{\frac{\left(\frac{1}{\Gamma}\right)}{\left(\frac{1}{\Gamma_{1\to 2}}\right)}}$$

probability that $F(0) = -F_1$ in the ensemble average

$$= k^2 U_0^2 \, \Gamma_{1\to 2} / \Gamma^2$$

Ultimate temperature $\quad k_B T = D/\alpha = U_0$

- Two-level atoms cannot be localized in the nodes of a standing wave
- We have neglected the Doppler effect and heating due to photon recoil. They dominate for $U_0 < \Gamma$ and prevent cooling below the Doppler limit