Note: $a<0$, $v>0$, $k>0$, $a_{max}>0$

Chirped slowing

$$F = \underbrace{-\hbar k \frac{\Gamma}{2}}_{M a_{max}} \frac{I/I_0}{1 + I/I_0 + \left[\frac{2(\delta + kv)}{\Gamma}\right]^2} \qquad (1)$$

① Select deceleration a (<0)

② Determine nominal detuning δ' from

$$a = \frac{-I/I_0}{1 + I/I_0 + \left(\frac{2\delta'}{\Gamma}\right)^2} a_{max} \qquad (2)$$

[has solution if $a < \frac{I/I_0}{1+I/I_0} a_{max}$]

③ Select initial velocity V_0 (cancels out)

$V(t) := V_0 + at$

$\delta(t) := \delta' - k V(t)$

$v' := v - V(t)$

just definitions

$\delta(t)$ is the laser detuning (chirp)

④ Substitute into (1): $\delta + kv = \delta' + kv'$

$$F = M a_{max} \frac{-I/I_0}{1 + I/I_0 + \left(\frac{2(\delta' + kv')}{\Gamma}\right)^2}$$

⑤ transform to decelerating frame (add fictitious force $-Ma$ from (2)

$$F(v') = M a_{max} \left[\frac{-I/I_0}{1 + I/I_0 + \left[\frac{2(\delta' + kv')}{\Gamma}\right]^2} + \frac{I/I_0}{1 + I/I_0 + \left(\frac{2\delta'}{\Gamma}\right)^2}\right]$$

This is exact (!) for arbitrary v'