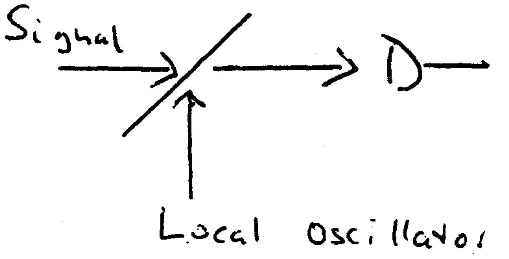
4.6.5.3.

Homodyne detection

(B.L. Schumaker, Opt. Lett. 9, 189 (1984))

$$E = i\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,\hat{\varepsilon}\left[a\, e^{i(kr-\omega t)} - a^{+} e^{-i(kr-\omega t)}\right]$$ Heisenberg rep.

One-port homodyning $= -2\sqrt{\frac{\hbar\omega}{2\varepsilon_0 V}}\,\hat{\varepsilon}\left[a_1 \sin(kr-\omega t) + a_2 \cos(kr-\omega t)\right]$



quadrature operator

$$a_1 = \frac{1}{2}(a + a^{+})$$

$$a_2 = \frac{1}{2i}(a - a^{+})$$

$$\left.\begin{array}{lll} \text{Transmisson} & T & = t^2 \\ \text{Reflection} & 1-T & = r^2 \end{array}\right\} \text{of beam splitter}$$

$$\text{Signal} \quad a = \underbrace{\overbrace{A_1 + \Delta a_1}^{a_1}}_{\cos\omega t} + i\,\underbrace{\overbrace{(A_2 + \Delta a_2)}^{a_2}}_{\sin\omega t}$$

$a_1, a_2, \Delta a_1, \Delta a_2$ Operators

A_1, A_2 Numbers

$A_1 = \langle a_1 \rangle$

Local oscillator

$$b = B_1 + \Delta b_1 + i\,\Delta b_2 \qquad (\cos\omega t + \text{noise})$$

Photocurrent I is proportional to

$$I \propto (ta + rb)^2 = r^2 B_1^2 + 2rt A_1 B_1 + 2rB_1(r\Delta b_1 + t\Delta a_1)$$

Strong LO limit $r^2B^2 \gg t^2 A_1^2$

[phase sensitive detection of only the $\cos\omega t$ quadrature component]

Signal current: $2rt\,A_1 B_1$

noise current $I^2 - \langle I \rangle^2 = [2rB_1]^2(r^2\Delta b_1^2 + t^2 \Delta a_1^2)$

$$SNR = \frac{t\,A_1}{\left[t^2\langle \Delta a_1^2\rangle + (1-t^2)\langle \Delta b_1^2\rangle\right]^{1/2}}$$

maximum (and independent of LO noise) if $t \simeq 1$ (with $(1-t^2)B^2$ finite).

It seems that attenuated high power LO has less noise than low power LO.

Indeed, attenuation by a factor r attenuates both B_1 and Δb_1. For a minimum uncertainty state (coherent state) $\langle \Delta b_1^2\rangle = 1/4$ [in units which give B_1^2 = # of photons N; $(B_1 + \Delta b_1)^2 \simeq N + 2B_1\Delta b_1$].

$$\langle [(B_1+\Delta b_1)^4\rangle \simeq N^2 + \underbrace{4N\langle \Delta b_1^2\rangle}_{N} = N^2 + N$$

Shot noise

In a certain sense, attenuation even attenuates the minimum uncertainty Δb_1!

However, for vanishing signal $A_1 \to 0$, the photocurrent is $r^2 B_1^2$ with a noise of $2rB_1\left(r^2\langle \Delta b_1^2\rangle + (1-r^2)\langle \Delta a_1^2\rangle\right)^{1/2}$.

With $N_r = r^2 B_1^2$ (number of photons after attenuation):

$$\text{noise} = 2\sqrt{N_r}\left(r^2\langle \Delta b_1^2\rangle + (1-r^2)\langle \Delta a_1^2\rangle\right)^{1/2}$$

With $\Delta b_1^2 \doteq 1/4$ this would be below shot noise, if one could have $\Delta a_1^2 = 0$. However, even the vacuum fluctuations have $\Delta a_1^2 = 1/4$, thus $\left[r^2\langle \Delta b_1^2\rangle + (1-r^2)\langle \Delta a_1^2\rangle\right]^{1/2} \geq 1/2$ and noise $\geq \sqrt{N_r}$

However, if the vacuum is squeezed in the quadrature component 1, the combined beam has reduced fluctuations in this quadrature component.

Thus it is correct to say that attenuation ($\hat{=}$ beam splitter) attenuates the noise even below the minimum uncertainty; but the open port of the beam splitter couples in the vacuum fluctuations thus reestablishing the minimum uncertainty. A squeezed vacuum generates therefore a squeezed transmitted beam.

This was recently used to do spectroscopy with squeezed light (Kimble et al., Phys. Rev. Lett. 68, 3020 (1992)).