Langevin Model for inelastic collisions

$$V(r) = -C\, r^{-n} \qquad\qquad E b^2 = \frac{L^2}{2m}$$

$$V_{eff}(r) = V(r) + E\, b^2/r^2$$

Maximum of centrifugal barrier at r_b

$$\frac{\partial V_{eff}}{\partial r} = 0 = n C r^{-n-1} - 2 E b^2 r^{-3}$$

$$\Rightarrow \quad r_b = \left(\frac{2 E b^2}{n C}\right)^{-\frac{1}{n-2}} \qquad n > 2$$

small E | large r_b

Height of barrier $h = V_{eff}(r_b)$

$$h = -C\left(\frac{2 E b^2}{n C}\right)^{\frac{n}{n-2}} + E b^2 \left(\frac{2 E b^2}{n C}\right)^{\frac{2}{n-2}}$$

$$= \left(\frac{n}{2}\right)^{-\frac{2}{n-2}} \left[1 - \frac{2}{n}\right] E^{\frac{n}{n-2}}\, b^{\frac{2n}{n-2}}\, C^{-\frac{2}{n-2}}$$

$$= h(b, E) \qquad\qquad \uparrow \text{ with } b$$

<u>Model</u>: no reaction if $E \le h(b, E)$

$$E = h(b_{bar}, E)$$

$$b_{bar} = \left(\frac{n}{2}\frac{C}{E}\right)^{1/n} \left(1 - \frac{2}{n}\right)^{-\frac{n-2}{2n}}$$

Cross section

$$Q_{bar} = \pi\, b_{bar}^2 \;\propto\; E^{-2/n}$$

$$n = 6 \qquad Q_{bar} \propto E^{-1/3}$$

for $n=4$

$Q \propto \frac{1}{v}$

rate $K \propto Qv$ is independent of E

Langevin cross section

[Interaction between ion and neutral atom is r^{-4};
calculation of diffusion and mobility coefficients
for gas mixtures]

partial waves ℓ : replace $\frac{E b^2}{r^2}$ by $\frac{\hbar^2 \ell(\ell+1)}{2m r^2}$

$$h = \left(\frac{n}{2}\right)^{-\frac{2}{n-2}} \left[1 - \frac{2}{n}\right] \left[\frac{\hbar^2 \ell(\ell+1)}{2m}\right]^{\frac{n}{n-2}} C^{-\frac{2}{n-2}}$$

highest partial wave contributing at a
temperature T

$h = k T$, solve for ℓ_{max}

$$\ell_{max}(\ell_{max}+1) = (\quad)\,(k_B T)^{\frac{n-2}{n}}$$

$\ell_{max} < 1 \Rightarrow$ only s waves

Na 1mK

Cs ~~4~~ 3μK

elastic collision

$V(r) = -C_6/r^6$

- classical: $\sigma_{tot} \to \infty$
  long range

  $\frac{d\sigma}{d\Omega}$ diverges at small deflection angles Θ

Closest approach r_0
Collision time $\tau = r_0/v$
Energy uncertainty: $\hbar\tau^{-1} = \frac{\hbar v}{r_0} = C_6/r_0^6$

Condition for no collision q.m.

$\Rightarrow \quad r_0 = \left(\frac{C_6}{\hbar v}\right)^{1/5}$

$\sigma \propto r_0^2 \propto T^{-1/5}$

- s wave limit: $r_0 \simeq \lambda_{DB}$
  breakdown of semiclassical trajectories
  $\Leftrightarrow \quad r_0 = \frac{\hbar}{mv}$

  $\hbar\tau^{-1} = \frac{\hbar^2}{m r_0^2} = \frac{C_6}{r_0^6} \Rightarrow r_0 = \left[\frac{m C_6}{\hbar^2}\right]^{1/4}$

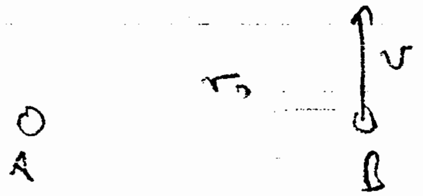
  r_0^2 estimate of s-wave cross section

  but: depends on scattering phase

$$kT_{\text{S-wave}} \propto mv^2 = \frac{\hbar^2}{m r_0^2}$$

inserting r_0: $$(kT)_{\text{S-WAVE}} = \frac{\hbar^3}{m^{3/2} C_6^{1/2}}$$

$$\approx 1 \text{ mK} \quad \text{for Na}$$



$$L = m v r_0$$

$$= \hbar \frac{r_0}{\lambda_{DB}}$$

$\lambda_{DB} \gg r_0$ $\quad \ell = 0 \quad$ s wave