From:
Lecture notes of D.E. Pritchard

4.6.5.2

Squeezing by FM @ 2 ω_o

Parametric modulation of the frequency of a classical harmonic oscillator causes progressive squeezing of the two quadrature components of its motion.

Consider a mass m which moves in a potential

$$U(t) = \tfrac{1}{2} k x^2 + \frac{\varepsilon}{2} k x^2 \sin 2\omega_o t \;\text{ with } \omega_o = \sqrt{k/m}. \tag{1}$$

The corresponding equation of motion, which represents a harmonic oscillator potential which is modulated in frequency by a small ($\varepsilon \ll 1$) amount at twice its natural frequency, is

$$\ddot{x} + \omega_o^2 x = -\omega_o^2 \varepsilon \sin(2\omega_o t)\, x \tag{2}$$

Since ε is small, substitute

$$x(t) = B(t)\cos\omega_o t + C(t)\sin\omega_o t, \tag{3}$$

the usual solution for an undamped harmonic oscillator *except* that B and C are presumed to be slowly varying functions of time. Therefore

$$\ddot{x}(t) = -\omega_o^2 x(t) - \omega_o \dot{B}\sin\omega_o t + \omega_o \dot{C}\cos\omega_o t$$

and substitution in the equation of motion (Eq. 2) gives

$$-\omega_o \dot{B}\sin\omega_o t + \omega_o \dot{C}\cos\omega_o t = \omega_o^2 \varepsilon [B\cos\omega_o t + C\sin\omega_o t]\sin 2\omega_o t.$$

using trig identities [eg. sin x cos y = ½ (sin (x+y) + sin (x-y)] and averaging away the terms rapidly oscillating at 3 ω_o gives

$$-\dot{B}\sin\omega_o t + \dot{C}\cos\omega_o t = \frac{-\varepsilon\omega_o}{2}[B\sin\omega_o t + C\cos\omega_o t].$$

The coefficients of sin $\omega_o t$ and cos $\omega_o t$ must be separately equal, hence $\dot{B} = +\varepsilon\omega_o/2B$, $\dot{C} = -\varepsilon\omega_o/2\,C$ and hence the coefficients B (t) and C (t) are

$$B(t) = B_o e^{+\varepsilon\omega_o t/2} \quad C(t) = C_o e^{-\varepsilon\omega_o t/2} \tag{4}$$

Thus squeezing parameter, r, is $r = \varepsilon\omega_o t/2$. Thus one quadrature component of the motion increases with time which the other one decreases. This represents a progressive squeezing of the motion -- after a long time almost any initial conditions will lead to motion predominately in the cos $\omega_o t$ component. The total energy of the oscillator will then be further amplified by the parametric drive term.