

Lecture 8: Mott's Variable Range Hopping

Mott variable range hopping theory describes the low temperature behavior of the resistivity in strongly disordered systems where states are localized. Consider two states located a distance R apart. The state on the left is at energy E_1 and the state on the right is at energy E_2 . Suppose $W = E_2 - E_1 > 0$. An electron can hop from left to right by the absorption of a phonon with energy ω . The hopping rate is given by

$$\frac{1}{\tau_R} = \omega_0 e^{-2R/\xi} f_1 (1 - f_2) n(W), \quad (1)$$

where ω_0 is the attempt frequency which is given by a typical phonon frequency, ξ is the localization length, $n(W)$ is the Bose factor, and $f_i = \frac{1}{e^{(E_i - \mu_i)/kT} + 1}$ is the Fermi factor for state $i = 1, 2$ with chemical potential μ_1 and μ_2 . The hopping rate for right to left is

$$\frac{1}{\tau_L} = \omega_0 e^{-2R/\xi} f_2 (1 - f_1) (n(W) + 1) \quad (2)$$

by the emission of phonons.

The current is

$$I = eR \left(\frac{1}{\tau_R} - \frac{1}{\tau_L} \right). \quad (3)$$

Expanding to linear order in $\mu_2 - \mu_1$, we find

$$I = \frac{eR\omega_0}{kT} e^{-W/kT} e^{-2R/\xi} (\mu_1 - \mu_2). \quad (4)$$

Setting $\mu_1 - \mu_2 = eV$ we find the expression for the conductance to be

$$G = \frac{e^2 R \omega_0}{kT} \{ e^{-W/kT} e^{-2R/\xi} \}. \quad (5)$$

The key insight of Mott is that G should be determined by optimizing the competition between the overlap term $e^{-2R/\xi}$, which favors short hops, and the energy activation $e^{-W/kT}$, which favors long hops. With longer hops, one has a better chance of reducing the activation energy W . We estimate W by the typical energy level spacing in volume with radius R

$$W = \frac{3}{4\pi R^3 N(0)} \quad (6)$$

where $N(0)$ is the density of states at the Fermi energy. Maximizing the $\{ \}$ in Eq.(5) using Eq.(6), we find

$$\bar{R}^4 = \frac{9\xi}{4\pi N(0)kT} \quad (7)$$

and the conductance

$$G \approx e^{-\left(\frac{T_0}{T}\right)^{1/4}} \quad (8)$$

where

$$kT_0 = \frac{1}{N(0)\xi^3} \quad (9)$$

is the average energy level spacing in a volume ξ^3 . Equation (8) is the famous Mott variable hopping law. It is observed in doped semiconductors at low temperatures.