Massachusetts Institute of Technology

15.023-12.848-ESD.128 Climate Change: Economics, Science and Policy

PROBLEM SET #3 SOLUTION SET SPRING 2004

Problem A1:

As a reminder, our original optimization resulted in the following scenario:

Time Step	Labor (l)	Capital (k)	Energy (e)	Inv (i)	Prod'n (y)	TechChg (a)	Cum. Energy (cume)	Energy Price (pe)	Cons (c)	Log Cons ln(10*c)	Disc't Factor (dfac)	PV Cons ln(c)*dfac	Carbon Emiss (m)	Carbon Stock (stkm)	Atmos Conc (conc)	Energy Growth e(t)/e(t-1)
2000	23	130	600	10	44.56	0.800	0	0.010	29	3.36	1.000	3.36	10.80	808.86	370.87	
2010	25	176	779	11	59.30	0.884	6000	0.010	41	3.70	0.744	2.76	14.01	851.927	390.61	1.298
2020	28	211	892	14	74.69	0.976	13786	0.011	51	3.93	0.554	2.17	16.06	923.679	423.51	1.146
2030	31	267	1001	19	95.19	1.078	22710	0.012	65	4.17	0.412	1.72	18.01	1010.17	463.17	1.121
2040	34	345	1138	25	122.70	1.191	32717	0.013	83	4.42	0.307	1.36	20.48	1109.2	508.57	1.137
2050	37	452	1229	32	157.61	1.316	44094	0.015	107	4.68	0.228	1.07	22.12	1224.95	561.65	1.080
2060	41	593	1272	41	201.41	1.453	56384	0.017	139	4.93	0.170	0.84	22.89	1347.83	617.99	1.035
2070	45	767	1290	51	255.88	1.605	69101	0.020	180	5.19	0.126	0.66	23.21	1468.53	673.33	1.014
2080	50	968	1378	54	325.22	1.773	81998	0.024	238	5.47	0.094	0.51	24.81	1582.79	725.72	1.069
2090	55	1122	1328	29	395.61	1.959	95781	0.032	325	5.78	0.070	0.40	23.90	1703.83	781.22	0.963
2100	61	963	1320	0	435.37	2.164	109059	0.045	376	5.93	0.052	0.31	23.75	1806.05	828.08	0.994
a	0.8						122256					15.1489				

Now, including damages in the production column and re-optimizing, our new result is:

Time Step	Labor (l)	Capital (k)	Energy (e)	Inv (i)	Prod'n (y)	TechChg (a)	Cum. Energy (cume)	Energy Price (pe)	Cons (c)	Log Cons ln(10*c)	Disc't Factor (dfac)	PV Cons ln(c)*dfac	Carbon Emiss (m)	Carbon Stock (stkm)	Atmos Conc (conc)	Energy Growth e(t)/e(t-1)
2000	23	130	600	10	44.56	0.800	0	0.010	29	3.36	1.000	3.36	10.80	808.86	370.87	
2010	25	176	767	11	59.16	0.884	6000	0.010	41	3.70	0.744	2.76	13.81	851.927	390.61	1.279
2020	28	211	879	14	74.43	0.976	13673	0.011	51	3.93	0.554	2.17	15.81	921.652	422.58	1.145
2030	31	266	993	18	94.92	1.078	22459	0.012	65	4.17	0.412	1.72	17.88	1005.81	461.17	1.131
2040	34	344	1091	24	121.51	1.191	32392	0.013	83	4.42	0.307	1.35	19.64	1103.87	506.13	1.098
2050	37	449	1184	32	155.90	1.316	43303	0.014	107	4.67	0.228	1.07	21.31	1211.65	555.55	1.085
2060	41	587	1256	41	199.61	1.453	55144	0.016	138	4.93	0.170	0.84	22.61	1327.51	608.67	1.061
2070	45	760	1296	50	253.90	1.605	67707	0.019	179	5.19	0.126	0.66	23.32	1447.09	663.5	1.031
2080	50	955	1355	54	320.79	1.773	80663	0.024	235	5.46	0.094	0.51	24.39	1564.12	717.16	1.046
2090	55	1109	1355	28	391.84	1.959	94213	0.031	322	5.78	0.070	0.40	24.39	1682.46	771.42	1.000
2100	61	946	1355	0	430.27	2.164	107763	0.043	371	5.92	0.052	0.31	24.39	1791.3	821.32	1.000
a	0.8						121314					15.1427				

Now that there is an incentive to reduce damage from climate change, we see that society chooses to decrease its energy use from 2000 to 2060 in order to reduce atmospheric carbon concentrations slightly throughout the century. This change is relatively small however: given the assumptions of climate sensitivity, damage, and discount rate, it would cost more to reduce emissions significantly than society would gain from reduced climate damage.

A2:

When the climate sensitivity is only 1.5 degrees C, as one would expect there are even fewer reductions of emissions (concentration is 825 ppm in 2100) (temperature change is only 1.7 degrees instead of 2.6).

And, when the climate sensitivity is 4 degrees C the temperature reaches 3.9 degrees C,

Time Step	Labor (l)	Capital (k)	Energy (e)	Inv (i)	Prod'n (y)	TechChg (a)	Cum. Energy (cume)	Energy Price (pe)	Cons (c)	Log Cons ln(10*c)	Disc't Factor (dfac)	PV Cons ln(c)*dfac	Carbon Emiss (m)	Carbon Stock (stkm)	Atmos Conc (conc)	Energy Growth e(t)/e(t-1)
2000	23	130	600	10	44.56	0.800	0	0.010	29	3.36	1.000	3.36	10.80	808.86	370.87	
2010	25	176	753	10	58.97	0.884	6000	0.010	41	3.70	0.744	2.76	13.55	851.927	390.61	1.255
2020	28	210	855	14	74.02	0.976	13527	0.011	51	3.93	0.554	2.17	15.40	919.026	421.38	1.136
2030	31	264	959	18	94.17	1.078	22080	0.012	64	4.17	0.412	1.72	17.27	999.206	458.14	1.122
2040	34	341	1060	24	120.49	1.191	31675	0.013	83	4.42	0.307	1.35	19.08	1091.69	500.55	1.105
2050	37	445	1145	31	154.23	1.316	42273	0.014	106	4.67	0.228	1.06	20.60	1194.82	547.83	1.080
2060	41	581	1221	40	197.25	1.453	53719	0.016	137	4.92	0.170	0.84	21.98	1304.93	598.32	1.067
2070	45	752	1282	49	251.10	1.605	65929	0.019	177	5.18	0.126	0.65	23.08	1419.95	651.06	1.050
2080	50	945	1377	53	317.86	1.773	78753	0.023	233	5.45	0.094	0.51	24.79	1536.8	704.63	1.074
2090	55	1097	1377	28	387.24	1.959	92523	0.029	319	5.77	0.070	0.40	24.79	1661.29	761.71	1.000
2100	61	933	1377	0	423.85	2.164	106293	0.042	367	5.90	0.052	0.31	24.79	1775.78	814.21	1.000
a	0.8						120063					15.1350				

Time Step	Δ CO2 Forcing (W/m^2)	Δ T (° C)	damage (% prod)	damage (trillion disc'ted $)
2000	0	0	0	0
2010	0.300608	0.243255	8.74E-05	0.003837
2020	0.743256	0.601451	0.000535	0.021909
2030	1.236437	1.000539	0.001479	0.057392
2040	1.764162	1.42758	0.003012	0.111239
2050	2.308695	1.868222	0.005158	0.181459
2060	2.847161	2.303955	0.007844	0.262624
2070	3.369583	2.726705	0.010987	0.348432
2080	3.864607	3.127284	0.014453	0.431713
2090	4.358097	3.526622	0.018379	0.497693
2100	4.785153	3.872201	0.022158	0.488673
				2.404971

CS	4
K_v	9.2
Const	0.2
Tau	0.60663
Damage	0.0133

we see slightly more reductions. Remember the impact that our NPV calculation has: not only are future damages significantly discounted, but because we take the log of consumption, as consumption grows (which it is predicted to do) any given amount of damage is also worth less. So even though damages are significant in 2100, optimization only reduces concentrations slightly.

A3:

First result to note: When the damage constant is high (0.5), there is a significant incentive to reduce energy use in order to minimize climate damage on production. (If this high damage constant was coupled with a high climate sensitivity, there would be even more incentive).

Concentration in 2100	Discount Rate = 0.2	Discount Rate = 0.5
Damage = 0.0133	827	813
Damage = 0.5	607	632

On the discount rate: When the damage constant is high, an increase in discount rate leads to an increase in concentrations in 2100. This is because society discounts the future more, therefore damage in the future isn't as important, therefore more energy use in the present is acceptable. Here the climate damage is the overwhelming variable... However, when the damage constant is 0.0133, it is effectively zero. So we are really examining the impact of the discount rate on the economy in the absence of damage: ie, a high discount rate leads to high initial consumption but low investment. The result of the low investment is less capital which means (given the form of the production equation) less reason to use more energy.

Concentration in 2050	Discount Rate = 0.2	Discount Rate = 0.5
Damage = 0.0133	544	563
Damage = 0.5	441	477

(Alternatively, concentration in 2100 for Discount = 0.3 and Damage = 0.5 was 617 ppm)

Problem B1:

Here are the emissions from the BAU case ("B0") and the B1 case.

GHG emissions (MT Carbon Equivalent) B0

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	1730.152	363.8006	1104.717	446.5392	319.0358	960.734	1028.462	1228.455	545.0962	264.3623	213.259	409.1604
2000	1932.072	380.6484	1224.121	500.0863	329.7603	943.5026	1126.69	1224.914	595.8764	287.7136	225.1127	433.5899
2005	2124.765	417.5514	1326.701	546.7711	344.2847	1015.376	1224.956	1414.966	677.7747	326.2322	242.7322	468.5675
2010	2298.654	448.9409	1415.332	591.4173	374.9428	1157.659	1330.289	1645.114	768.2006	368.7502	267.7939	508.9099
2015	2444.27	472.0941	1481.212	624.4854	402.5309	1260.241	1409.797	1886.895	862.2511	409.3975	292.6529	549.9113
2020	2573.885	490.8494	1531.202	648.7892	433.0504	1340.621	1473.627	2181.443	969.2192	451.8616	314.9185	587

GHG emissions (MT Carbon Equivalent) B1

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	1730.152	363.8006	1104.717	446.5392	319.0358	960.734	1028.462	1228.455	545.0962	264.3623	213.259	409.1604
2000	1932.072	380.6484	1224.121	500.0863	329.7603	943.5026	1126.69	1224.914	595.8764	287.7136	225.1127	433.5899
2005	2124.765	417.5514	1326.701	546.7711	344.2847	1015.376	1224.956	1414.966	677.7747	326.2322	242.7322	468.5675
2010	2316.818	363.8	1104.7	446.4999	318.9999	960.6928	1342.682	1651.988	770.2305	371.9339	270.4001	511.1396
2015	2470.714	363.8	1104.7	446.5566	319	960.7	1427.083	1896.901	865.4127	414.3751	296.7445	553.2478
2020	2607.67	363.8	1104.7	446.5566	319	960.7	1494.939	2193.888	973.3458	458.7134	320.5253	591.2591

Note that carbon prices vary from region to region because there is no trading:

shadow prices of carbon (1995 US dollars / ton Carbon Equivalent) B1

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0	0	0
2010	0	119.5102	85.4393	64.4695	14.5893	15.6077	0	0	0	0	0	0
2015	0	153.5779	111.4298	89.3862	34.6173	35.3162	0	0	0	0	0	0
2020	0	174.403	128.9365	108.0293	59.4249	52.433	0	0	0	0	0	0

Problem B2:

change in welfare from BAU (%) B1

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0	0	0
2010	0.0336	-0.2046	-0.5674	-0.4634	-0.0003	-0.1735	-0.5828	0.0358	0.1067	0.0666	0.0895	0.0052
2015	0.0433	-0.2462	-0.679	-0.6315	-0.1176	-0.2583	-0.8756	0.0607	0.1639	0.1085	0.1373	0.0071
2020	0.0591	-0.244	-0.6905	-0.7523	-0.276	-0.4291	-1.1813	0.103	0.2357	0.172	0.1812	0.0181

change in welfare from BAU (%) (b2 case)

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0	0	0
2010	0.0273	-0.0626	-0.285	-0.3525	0.0877	0.6833	-0.4306	0.028	0.0842	0.0556	0.0686	0.0147
2015	0.038	-0.0956	-0.4136	-0.5399	0.0357	0.8428	-0.6991	0.0522	0.1417	0.0982	0.1132	0.0199
2020	0.054	-0.1089	-0.4803	-0.6941	-0.0737	0.8229	-0.9941	0.0921	0.2137	0.1585	0.1569	0.0321
	-0.0063	0.142	**0.2824**	0.1109	0.088	**0.8568**	**0.1522**	-0.0078	-0.0225	-0.011	-0.0209	0.0095
	-0.0053	0.1506	**0.2654**	0.0916	**0.1533**	**1.1011**	**0.1765**	-0.0085	-0.0222	-0.0103	-0.0241	0.0128
	-0.0051	0.1351	**0.2102**	0.0582	**0.2023**	**1.252**	**0.1872**	-0.0109	-0.022	-0.0135	-0.0243	0.014

For problem B2, I have reported the change in welfare from BAU, and in addition, the change in welfare from the B1 case so we can see the difference resulting from the addition of trading. Here we see that all five trading partners gain, but especially the former Soviet Union (who are selling large numbers of permits). This makes sense: under most circumstances, trading leads to improvements in welfare for both parties as the nations with cheap reductions (FSU) sell permits for more than the reductions cost them, and nations with expensive reductions (EEC)

shadow prices of carbon (1995 US dollars / ton Carbon Equivalent) B2

	USA	JPN	EEC	OOE	EET	FSU
1995	0	0	0	0	0	0
2000	0	0	0	0	0	0
2005	0	0	0	0	0	0
2010	0	40.9774	40.9774	40.9774	40.9774	40.9774
2015	0	64.5568	64.5568	64.5568	64.5568	64.5568
2020	0	86.3096	86.3096	86.3096	86.3096	86.3096

buy permits for less than their welfare improvement from increasing emissions. In fact, we can see this from the carbon price table in 2020: $86 is more than the $52 that it cost the FSU to reduce its last ton of carbon, and is less than the $129 that it cost the EEC in the no-trading case. (Note that the FSU sales are not technically “hot air” – their constraint is below their BAU emissions, it is just inexpensive for them to meet that constraint. “Hot air” refers to the 1990 Kyoto constraint, which *would* be above FSU’s BAU emissions)

Also, the Energy Exporting nations gain as well. There isn’t enough detail in the Toy to be able to determine what the EEX impact comes from: carbon emissions are approximately the same between both runs, so it is possibly a result of shifts between different energy sectors (eg, if the permit buyers were oil importers and the permit sellers were coal users, then that would explain a better oil market).

B3:

change in welfare from BAU (%) (b3 case)

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0	0	0
2010	-0.1572	-0.0994	-0.3737	-0.5552	0.4521	1.3567	-1.0031	0.0528	0.2116	0.1093	0.1643	0.0735
2015	-0.2407	-0.1423	-0.5123	-0.8053	0.5792	1.6983	-1.5469	0.1093	0.3418	0.1828	0.262	0.1085
2020	-0.2768	-0.1471	-0.5504	-1.0137	0.6149	1.721	-2.1261	0.197	0.496	0.2829	0.3553	0.152
	-0.1845	-0.0368	-0.0887	-0.2027	0.3644	0.6734	-0.5725	0.0248	0.1274	0.0537	0.0957	0.0588
	-0.2787	-0.0467	-0.0987	-0.2654	0.5435	0.8555	-0.8478	0.0571	0.2001	0.0846	0.1488	0.0886
	-0.3308	-0.0382	-0.0701	-0.3196	0.6886	0.8981	-1.132	0.1049	0.2823	0.1244	0.1984	0.1199

shadow prices of carbon (1995 US dollars / ton Carbon Equivalent) B3

	USA	JPN	EEC	OOE	EET	FSU
1995	0	0	0	0	0	0
2000	0	0	0	0	0	0
2005	0	0	0	0	0	0
2010	65.3025	65.3025	65.3025	65.3025	65.3025	65.3025
2015	98.8078	98.8078	98.8078	98.8078	98.8078	98.8078
2020	127.0861	127.0861	127.0861	127.0861	127.0861	127.0861

Of course the US will lose from entering the agreement, as it takes on a constraint that it did not otherwise have. However, as it enters the trading regime it is going to buy permits, causing the price to rise. This will negatively impact other permit buyers (JPN, EEC, OOE), and benefit sellers (EET, FSU). The EEX lose again as their market decreases, and the remainder of the world wins as they see lower energy prices and probably a more competitive market for energy intensive goods. One interesting comparison is the EEC: its carbon price in this case is similar to that in B1, but its welfare loss is not as large. This is possibly because there was some compensation from reduced competition from the US.

Constraint (MT Carbon Equivalent)

	USA	JPN	EEC	OOE	EET	FSU
2010	1730	364	1105	447	319	961
2015	1730	364	1105	447	319	961
2020	1730	364	1105	447	319	961

GHG emissions (MT Carbon Equivalent) B3

	USA	JPN	EEC	OOE	EET	FSU
2010	1892.14	385.912	1148.214	449.2245	276.22	773.1831
2015	1905.125	386.5033	1139.1	444.057	274.8799	775.2914
2020	1921.649	385.3101	1127.942	439.0745	275.7092	775.2717
	NEW SALES					
	161.9885	22.1114	43.4971	2.6853	-42.8158	-187.5509
	174.9733	22.7027	34.3828	-2.4822	-44.1559	-185.4426
	191.4971	21.5095	23.2252	-7.4647	-43.3266	-185.4623
	OLD SALES					
		32.101	83.8481	25.0166	-27.7873	-113.1778
		35.2685	83.6331	23.6778	-26.0464	-116.5312
		35.1961	75.7097	19.9356	-21.7361	-109.1116

We can examine who sells and who buys permits by comparing the B3 case to the constraint (which for all except the US is equal to the B1 case). For example, we can see that in 2020

JPN is buying 21.5 MT carbon equivalent permits because its emissions of 385.3 are 21.5 higher than its constraint of 363.8 MtCeq. Here is the proof that FSU and EET are selling permits, and JPN and EEC are buying permits. We can see the impact of the US entry by comparing the sales in the B2 case (“Old Sales”) to the B3 sales (“New Sales”): Permits buyers are buying fewer permits, and permit sellers are selling more. As would be expected from the entry of a new buyer. The OOE were buying permits in B2, and in the B3 case they are now selling permits as the price of carbon rises.

B4:

change in welfare from BAU (%) (b4 case)

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN	IND	DAE	BRA	ROW
1995	0	0	0	0	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0	0	0	0	0
2010	-0.1484	-0.0824	-0.3537	-0.5543	0.4691	1.2917	-1.0805	-0.4126	0.2137	0.0974	0.1753	0.0715
2015	-0.2353	-0.1231	-0.5066	-0.8403	0.6879	1.78	-1.754	-0.922	0.3661	0.1762	0.2954	0.1095
2020	-0.2737	-0.1298	-0.571	-1.1004	0.884	2.0894	-2.5289	-1.594	0.5639	0.2889	0.4255	0.1728
	0.0088	0.017	0.02	0.0009	0.017	-0.065	-0.0774	-0.4654	0.0021	-0.0119	0.011	-0.002
	0.0054	0.0192	0.0057	-0.035	0.1087	0.0817	-0.2071	-1.0313	0.0243	-0.0066	0.0334	0.001
	0.0031	0.0173	-0.0206	-0.0867	0.2691	0.3684	-0.4028	-1.791	0.0679	0.006	0.0702	0.0208

As we would expect, the EET and the FSU are the big winners from China’s entry, and CHN and the EEX are the big losers. Interestingly, the other trading members do not lose very much: it turns out that China’s entry does not impact carbon prices that much (implying that China happens to have internal costs of reduction very similar to the external price – and in fact, in 2010 China actually sells a few permits) (see following charts:), and the reduction in competition may compensate for some of it.

shadow prices of carbon (1995 US dollars / ton Carbon Equivalent)

	USA	JPN	EEC	OOE	EET	FSU	EEX	CHN
1995	0	0	0	0	0	0	0	0
2000	0	0	0	0	0	0	0	0
2005	0	0	0	0	0	0	0	0
2010	64.6313	64.6313	64.6313	64.6313	64.6313	64.6313	0	64.6313
2015	104.0346	104.0346	104.0346	104.0346	104.0346	104.0346	0	104.0346
2020	144.51	144.51	144.51	144.51	144.51	144.51	0	144.51

Constraint (MT C equivalent)

	USA	JPN	EEC	OOE	EET	FSU	CHN
2010	1730	364	1105	447	319	961	1228.5
2015	1730	364	1105	447	319	961	1228.5
2020	1730	364	1105	447	319	961	1228.5

GHG emissions (MT Carbon Equivalent) B4

	USA	JPN	EEC	OOE	EET	FSU	CHN
2010	1896.313	386.8394	1150.9	450.6035	276.8374	776.3665	1215.534
2015	1897.228	385.903	1134.974	441.6666	272.8611	768.5675	1252.247
2020	1892.732	381.8147	1112.023	430.4975	269.9238	753.1944	1313.27
Total Sales							
	166.1615	23.0388	46.1828	4.0643	-42.1984	-184.3675	-12.966
	167.0763	22.1024	30.2577	-4.8726	-46.1747	-192.1665	23.7471
	162.5809	18.0141	7.3065	-16.0417	-49.112	-207.5396	84.7701
Change from B3:							
	-4.173	-0.9274	-2.6857	-1.379	-0.6174	-3.1834	
	7.897	0.6003	4.1251	2.3904	2.0188	6.7239	
	28.9162	3.4954	15.9187	8.577	5.7854	22.0773	