# 15.053 Tuesday, May 8

- Decision Trees

The example in this lecture is a modification of the example in Bertsimas and Freund: Data, Models, and Decisions.

## Quote of the Day

**Truly successful decision making relies on a balance between deliberate and instinctive thinking.**

**Malcolm Gladwell**
**Blink: The Power of Thinking Without Thinking**

**Every decision has a cost. Do I have to make this decision at all or can I move on to the next thing? What we decided to leave out is almost as important as what we put in.**

**Joshua Schachter**

## Newsvendor Problem

- News vendor Phyllis orders newspapers for sale
  - ordered newspapers cost 20¢ each
  - she sells newspapers for 25¢ each
  - unsold newspapers are worthless
  - Prob(demand = j) = 1/5 for j = 6 to 10.

**$r_{ij}$ is the "net revenue of ordering i papers if the demand is j.**

$r_{ij}$ = 25 i – 20 i = 5 i  if $i \le j$

$r_{ij}$ = 25 j – 20 i  if $i > j$



The newsvendor problem is a classic model in operations management. It models a situation in which one purchases goods in advance of sales, and unsold items are valueless. But it could also be used for times in which unsold items are disposed of for a small revenue.

| | | Papers Demanded | | | | |
|---|---|---|---|---|---|---|
| | | 6 | 7 | 8 | 9 | 10 |
| Papers Ordered | 6 | 30 | 30 | 30 | 30 | 30 |
| | 7 | 10 | 35 | 35 | 35 | 35 |
| | 8 | -10 | 15 | 40 | 40 | 40 |
| | 9 | -30 | -5 | 20 | 45 | 45 |
| | 10 | -50 | -25 | 0 | 25 | 50 |

**$r_{ij}$ is the value of ordering i papers if the demand is j.**



We consider a simple case in which we order 6 to 10 newspapers and we sell 6 to 10 newspapers. Of course, one could order 5 or fewer or 11 or greater, but that would clearly not be optimal.

# Dominated Actions

| Papers Ordered \ Papers Demanded | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 6 | 30 | 30 | 30 | 30 | 30 |
| 7 | 10 | 35 | 35 | 35 | 35 |
| 8 | -10 | 15 | 40 | 40 | 40 |
| 9 | -30 | -5 | 20 | 45 | 45 |
| 10 | -50 | -25 | 0 | 25 | 50 |

| Papers Ordered \ Papers Demanded | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 5 | 25 | 25 | 25 | 25 | 25 |
| 11 | -70 | -45 | -20 | 5 | 30 |

**"Ordering i" *dominates* "ordering k" if $r_{ij} \geq r_{kj}$ for all j.**

**"Ordering 5" is dominated by "ordering 6"**

**"Ordering 11" is dominated by "ordering 10"**



Ordering 5 is not as profitable as ordering 6. So, "ordering 5" is dominated by "ordering 6", regardless of what the demand is. Similarly, ordering "ordering 11" is not as profitable as ordering 10.

# The Maximin Criterion

| | | Papers Demanded | | | | | |
|---|---|---|---|---|---|---|---|
| | | 6 | 7 | 8 | 9 | 10 | min ($r_{ij}$) |
| Papers Ordered | 6 | 30 | 30 | 30 | 30 | 30 | 30 |
| | 7 | 10 | 35 | 35 | 35 | 35 | 10 |
| | 8 | -10 | 15 | 40 | 40 | 40 | -10 |
| | 9 | -30 | -5 | 20 | 45 | 45 | -30 |
| | 10 | -50 | -25 | 0 | 25 | 50 | -50 |

***maximin criterion*: choose the action with the largest guaranteed return**

**maximize $_i$ min $\{r_{ij} : j = 6 \text{ to } 10\}$ = 30.**



We now consider various metrics for measuring the quality of a solution. The maximin criteria is very conservative, and is arguably very pessimistic. For each number that you order, we consider the worst possible profit. This is stored in the column min ($r_{ij}$). We then maximize over all possible orders. In the newsvendor problem, this will always lead to ordering the minimum possible demand for newspapers.

## The Maximax Criterion

| Papers Ordered \ Papers Demanded | 6 | 7 | 8 | 9 | 10 | max ($r_{ij}$) |
|---|---|---|---|---|---|---|
| 6 | 30 | 30 | 30 | 30 | 30 | 30 |
| 7 | 10 | 35 | 35 | 35 | 35 | 35 |
| 8 | -10 | 15 | 40 | 40 | 40 | 40 |
| 9 | -30 | -5 | 20 | 45 | 45 | 45 |
| 10 | -50 | -25 | 0 | 25 | 50 | 50 |

***maximax criterion*****: choose the action with the largest possible return**

**maximize $_i$ max $\{r_{ij} : j = 6 \text{ to } 10\}$ = 50.**



The maximax criteria is really dumb. For each number that you order, we consider the best possible profit. This is stored in the column max ($r_{ij}$). Then you would order the number that maximizes the max. For the newsvendor problem, you would always order the maximum number of newspapers that can be sold.

## The Minimax Regret Criterion

| Papers Ordered \ Papers Demanded | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 6 | 30 | 30 | 30 | 30 | 30 |
| 7 | 10 | 35 | 35 | 35 | 35 |
| 8 | -10 | 15 | 40 | 40 | 40 |
| 9 | -30 | -5 | 20 | 45 | 45 |
| 10 | -50 | -25 | 0 | 25 | 50 |
| $\max_i r_{ij}$ | 30 | 35 | 40 | 45 | 50 |

**regret (i, j) = $\max_k r_{kj} - r_{ij}$**

**regret (6, 6) = 0 regret (6, 7) = 5**



The minimax regret criterion is a somewhat weird criterion. It assumes that you will always look back with regret on what you do. So, if you order 6 papers and the number of customers is 8, you have a regret of $.10 because had you ordered exactly 8 you would have made $.10 more. If you order 10 papers, and the number of customers is 8, then you have a regret of $.40 since had you ordered only 8 newspapers, you would have made $.40 more.

The regret is 0 if you happen to buy the number that is demanded.

## The Minimax Regret Criterion

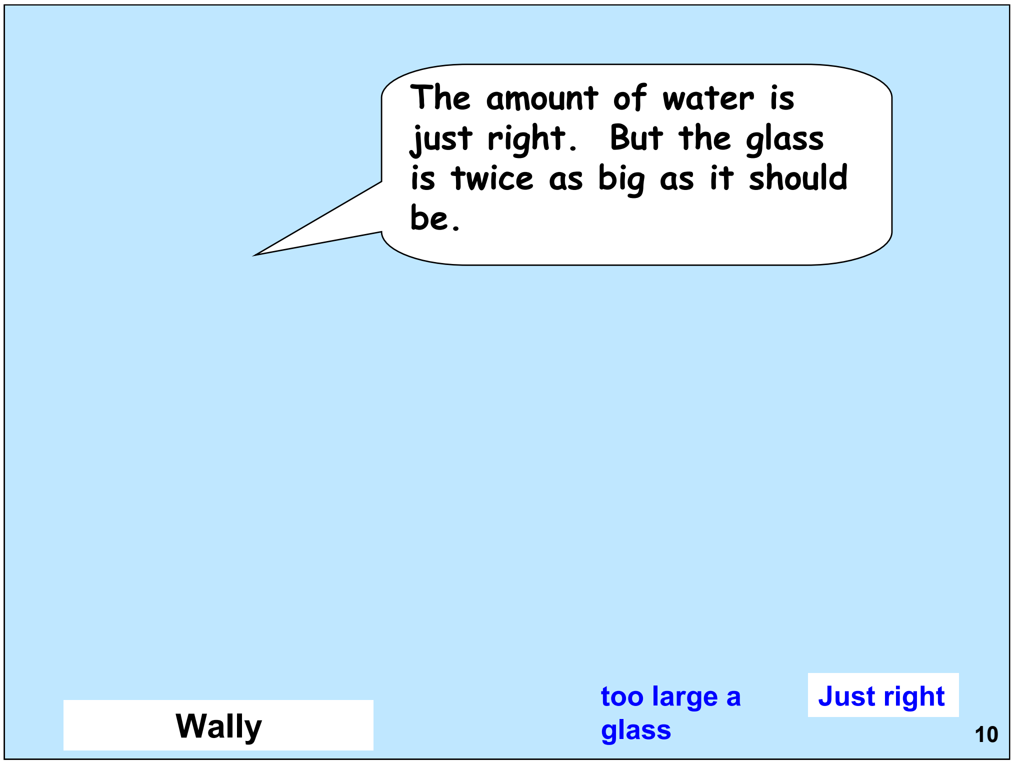
| Papers Ordered | Papers Demanded: 6 | 7 | 8 | 9 | 10 | max regret |
|---|---|---|---|---|---|---|
| 6 | 0 | 5 | 10 | 15 | 20 | 20 |
| 7 | 20 | 0 | 5 | 10 | 15 | 20 |
| 8 | 40 | 20 | 0 | 5 | 10 | 40 |
| 9 | 60 | 40 | 20 | 0 | 5 | 60 |
| 10 | 80 | 60 | 40 | 20 | 0 | 80 |
| $max_i\ r_{ij}$ | 30 | 35 | 40 | 45 | 50 | |

***minimax regret*** **: choose the action that minimizes the maximum possible regret**

***minimax regret*** **= min** $_i$ **max** $_j$ **regret (i, j) = 20**



Here we list all of the regrets. Then we create a column of maximum regrets. The minimax regret criterion is to order the number that minimizes the max regret. In this case, we could order 6 or 7 papers. In both cases, the maximum possible regret is $.20.




Wally doesn't think in terms of the amount of water. As an engineer he focuses on the glass itself, and wonders whether it is the right object for the task at hand.

# Expected Value

| | | Papers Demanded | | | | |
|---|---|---|---|---|---|---|
| | | 6 | 7 | 8 | 9 | 10 |
| Papers Ordered | 6 | 30 | 30 | 30 | 30 | 30 |
| | 7 | 10 | 35 | 35 | 35 | 35 |
| | 8 | -10 | 15 | 40 | 40 | 40 |
| | 9 | -30 | -5 | 20 | 45 | 45 |
| | 10 | -50 | -25 | 0 | 25 | 50 |

**Let p(j) = prob. that exactly j papers are demanded.**

**The expected value of ordering i papers is**

$$E(i) = \Sigma_j \; p(j) \, r_{ij}$$



The expected value ( a horribly named word) is the average value one would obtain if one could repeat this event an arbitrarily large number of times. It is also a standard term used in probability, and is also called the “mean”.

It’s also a very useful way of assessing the return of an investment, and is arguably the most common measure of performance used.

# Expected Value

| Papers Ordered | Papers Demanded: 6 | 7 | 8 | 9 | 10 | Expected Value |
|---|---|---|---|---|---|---|
| 6 | 30 | 30 | 30 | 30 | 30 | 30 |
| 7 | 10 | 35 | 35 | 35 | 35 | 30 |
| 8 | -10 | 15 | 40 | 40 | 40 | 25 |
| 9 | -30 | -5 | 20 | 45 | 45 | 15 |
| 10 | -50 | -25 | 0 | 25 | 50 | 0 |

**Suppose that p(j) = 1/5 for j = 6 to 10.**

**The expected value of ordering i papers is E(i) =** $\sum_j 1/5\ r_{ij}$

**The maximum expected value is 30.**



The expected value of ordering 6 or 7 papers is $.30, which is maximum.

## Another Example of Expected Value

| Papers Ordered | Papers Demanded: 6 | 7 | 8 | 9 | 10 | Expected Value |
|---|---|---|---|---|---|---|
| 6 | 60 | 60 | 60 | 60 | 60 | 60 |
| 7 | 40 | 70 | 70 | 70 | 70 | 64 |
| 8 | 20 | 50 | 80 | 80 | 80 | 62 |
| 9 | 0 | 30 | 60 | 90 | 90 | 54 |
| 10 | -20 | 10 | 40 | 70 | 100 | 40 |

**Suppose that p(j) = 1/5 for j = 6 to 10.**

**Each ordered newspaper is $.20**

**Each sold newspaper is $.30**

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If we changed the selling price from $.25 to $.30, the values $r_{ij}$ would all change as would the expected value of each number ordered.

Note that the expected value increases with the number of papers ordered and then decreases. This always happens with newsvendor problems, and it is the key to the solution approach.

## Back to the newsvendor problem

- **Assumptions:**
  - **D = demand (random variable)**
  - **known probability p(d) that demand is d;**
  - **q is the amount ordered (decision variable);**
  - **c(q, d) = "profit" if q items are ordered and d items are demanded .**
  - **$c_o$ = Overstocking cost = ordering cost.**
    **It is the unit cost of ordering too many.**
    **In previous example this was 20¢.**
  - **$c_u$ = Understocking cost = price - ordering cost**
    **It is the unit "cost" of ordering too few.**
    **In previous example this was 25¢ - 20¢ = 5¢.**
    **It is an opportunity cost.**



The standard assumptions for a newsvendor problem is that you know the demand distribution, you pay a cost for each item ordered, and you receive an amount for each sold.

In order to determine the optimal solution, we focus on two values. The first value is the overstocking cost $c_o$. This is the cost incurred for every newspaper purchased that is not sold. In this case, it is 20¢.

We also keep track of an understocking cost, which is not a real cost. It is an opportunity cost. It is how much additional revenue would have been attained by another newspaper in the case that you order fewer papers than is demanded. In this case, it is 25¢ - 20¢ = 5¢.

| Papers Ordered \ Papers Demanded | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|
| 6 | 30 | 30 | 30 | 30 | 30 |
| 7 | 10 | 35 | 35 | 35 | 35 |
| 8 | -10 | 15 | 40 | 40 | 40 |
| 9 | -30 | -5 | 20 | 45 | 45 |
| 10 | -50 | -25 | 0 | 25 | 50 |

**If you order at least as many as the demand, then every extra paper costs you $c_o$ = 20¢.**

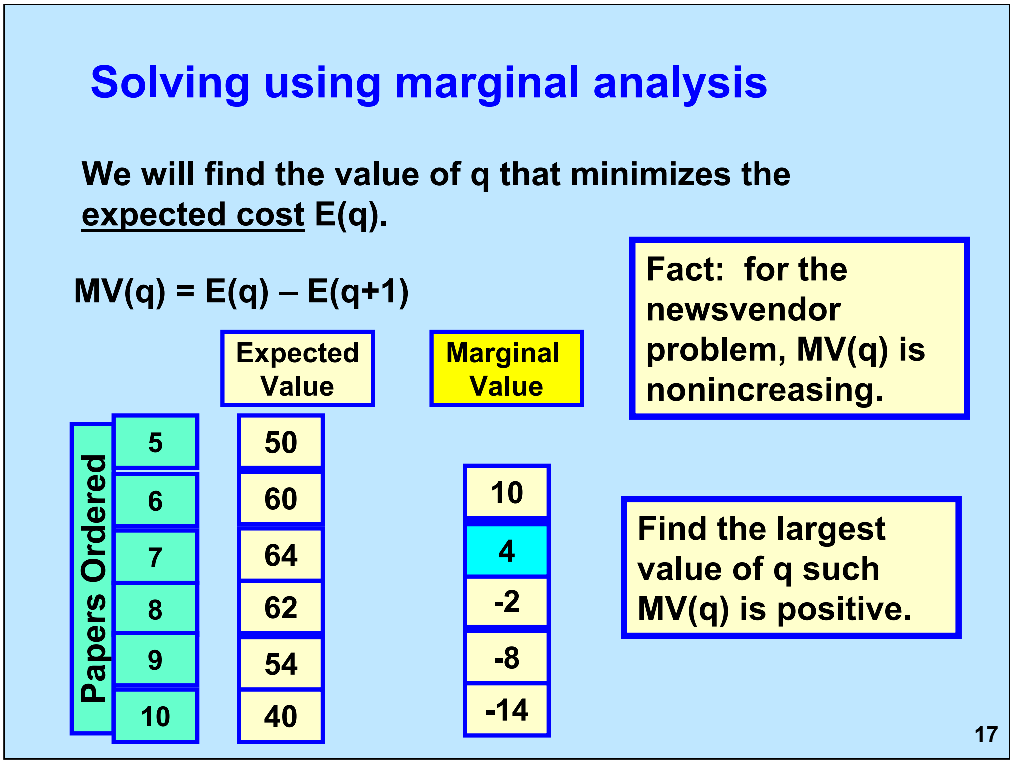
**If you order at most as many as the demand, then every fewer paper ordered costs you $c_u$ = 5¢.**

## Applications of Newsvendor Problem

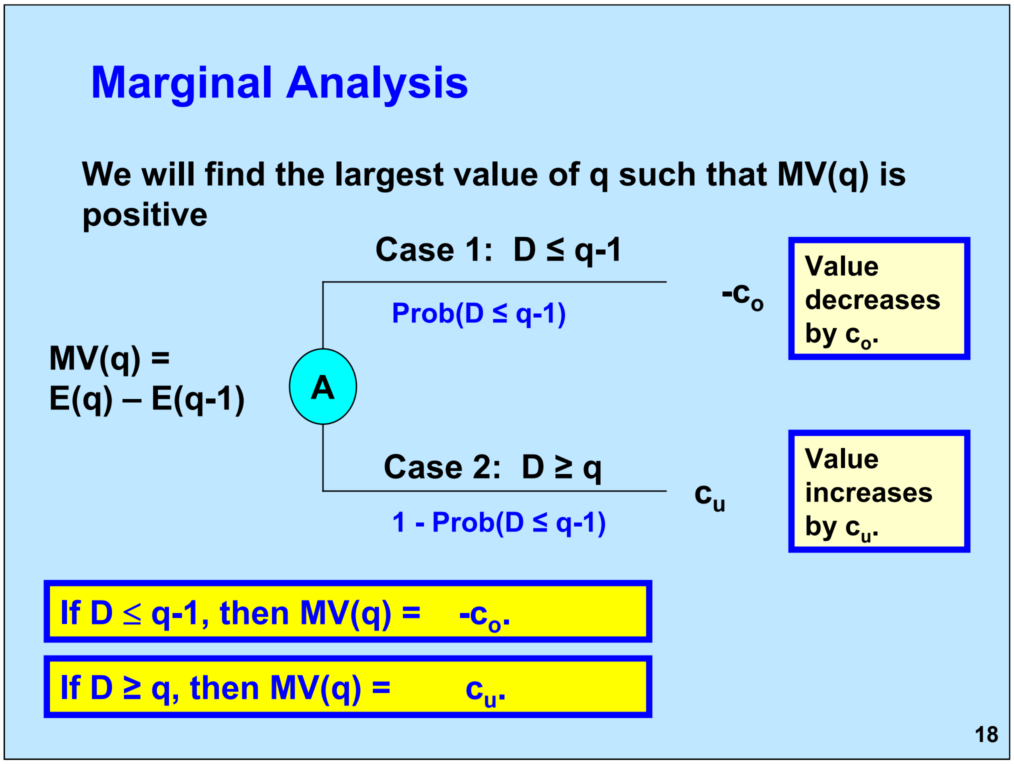
- Relevant to problems with perishable inventories
  - Newspapers, magazines,
  - restaurants with food orders
  - airline overbooking
    - underbooking wastes a seat
    - Overbooking results in having to make an offer to a passenger to give up a seat
  - Flexible reimbursement plans: MIT staff can put in money at the beginning of the year to pay for medical expenses pretax
    - any money left over at the end of the year in the plan is lost.



The newsvendor problem arises in many different applications.




Here are the expected values when the selling price of the newspaper was 30¢. We then list the marginal values. The marginal value for purchasing q newspapers is E(q) – E(q-1), is the value of the q-th newspaper to the expected return. We note that MV(q) is a decreasing function. So, if we choose the largest value q for which MV(q) is positive, we will also be maximizing E(q). In subsequent slides we focus on MV(q).




To compute MV(q), we consider two cases. In the first case, the number demanded is at most q-1, in which case the q-th newspaper is unsold. This leads to MV(q) being $-c_o$. It occurs with probability Prob(D ≤ q-1).

In the second case, the number demanded is at least D. In this case, the q-th newspaper is sold, and the revenue increases by $c_u$. This occurs with probability Prob(D ≥ q) = 1 - Prob(D ≤ q-1).

## Solving using marginal analysis

**If $D \le q-1$, then $MV(q) = -c_o$.**

**If $D \ge q$, then $MV(q) = c_u$.**

$$MV(q) = - c_0 \, Prob(D \le q - 1) + c_u (1 - Prob(D \le q - 1))$$

$$MV(q) > 0 \text{ iff } - c_0 \, Prob(D \le q - 1) + c_u (1 - Prob(D \le q - 1)) > 0$$

$$\text{iff } Prob(D \le q - 1)(c_0 + c_u) - c_u < 0$$

$$\text{iff } Prob(D \le q - 1) < c_u / (c_0 + c_u)$$



We can now compute the expected value of MV(q) using the probabilities.

MV(q) has positive expected value when $Prob(D \le q - 1) < c_u / (c_0 + c_u)$.

So, to maximize E(q), we find the largest value of q such that MV(q) is positive, which is also the largest value of q such that $Prob(D \le q - 1) < c_u / (c_0 + c_u)$. This is the solution to the newsvendor problem.

## Solution for Newsvendor Problem

**The optimum value q of newspapers to order is the largest value of q such that**

$$\text{Prob}(D \le q - 1) < c_u / (c_0 + c_u)$$

**Example 1.**

- **ordered cost: 20¢ each**
- **selling cost 25¢ each**
- **Prob(demand = j) = 1/5 for j = 6 to 10.**

$c_u = 5¢$

$c_o = 20¢$

$c_u / (c_0 + c_u) = 1/5$

$\text{Prob}(D \le 6 - 1) = 0$

$\text{Prob}(D \le 7 - 1) = 1/5$



In our original instance, 6 is the largest value of q such that $\text{Prob}(D \le q - 1) < c_u / (c_0 + c_u) = 1/5$. Note that $\text{Prob}(D \le 6 - 1) = 0$, and $\text{Prob}(D \le 7 - 1) = 1/5$. (It turns out that 7 is an alternative optimum solution.)

# Example 2 for Newsvendor Problem

**Suppose that p(j) = 1/5 for j = 6 to 10.**

**Each ordered newspaper is $.20**

**Each sold newspaper is $.30**

$c_u$ = 10¢ $c_o$ = 20¢

$c_u / (c_0 + c_u) = 1/3$

Prob(D ≤ 7 - 1) = 1/5

Prob(D ≤ 8 - 1) = 2/5

**Optimum order is 7 newspapers.**

| Papers Ordered | Expected Value |
|---|---|
| 5 | 50 |
| 6 | 60 |
| 7 | 64 |
| 8 | 62 |
| 9 | 54 |
| 10 | 40 |



If each newspaper is sold for $.30, 7 is the largest value of q such that Prob(D ≤ q - 1) < $c_u / (c_0 + c_u)$ = 10/30=1/3. Note that Prob(D ≤ 7 - 1) = 1/5, and Prob(D ≤ 8 - 1) = 2/5.

## Examples of Optimal Ordering

**Choose the greatest value of q such that**
**$\text{Prob}(D \le q - 1) < c_u / (c_0 + c_u)$.**

**Suppose that D takes on values from 1 to 100, and that $\text{Prob}(D \le q-1) = (q-1)/100$ for each q.**

**Suppose the cost of selling a newspaper is 25¢. What is the optimal order amount if**

1. **Cost of ordering = 20¢.**
2. **Cost of ordering = 10¢.**
3. **Cost of ordering = 5¢.**



Case 1. $c_u / (c_0 + c_u) = 5/25 = 1/5$. $q = 20$
$\text{Prob}(D \le 20 - 1) = 19/100$

Case 2. $c_u / (c_0 + c_u) = 15/25 = 3/5$. $q = 60$
$\text{Prob}(D \le 60 - 1) = 59/100$

Case 3. $c_u / (c_0 + c_u) = 20/25 = 4/5$. $q = 80$
$\text{Prob}(D \le 80 - 1) = 79/100$

# Mental Break

## Juan Lee's Decision Problem

It is January 10th, and Juan Lee is currently a fourth year undergraduate in Management Science at Sloan. He has decided to seek out a job as a consultant. He already has received an offer from ABC consulting for $72,000 per year. He has until February 1st to decide whether to accept the offer. An old classmate of his, Mary Kumar, has told him that she has recommended him highly to her consulting firm, and feels that there is an excellent chance that they would give him an offer for $80,000. However, they are not prepared to make any decision until February 15th. If they made him an offer, he would need to decide by March 1. He also has the option of taking part in the consulting job fair in the middle of March. He is fairly certain that he could get a consulting job at that time, but is uncertain as to what he would be paid.



This is an example used for Decision trees.

## Juan's Goal

- Assume here that Juan views all of these consulting jobs as excellent opportunities, and that the only differentiating factor is money.

- Juan has decided to maximize his expected salary.

## Juan's probabilities

- Juan does not know how to evaluate his "excellent chances" at Mary's firm, nor does he know what to expect from the consulting fair.

- Juan's best guesses
  - Probability of getting Job at Mary's firm: 60%
  - Different possibilities for consulting fair offers.
    - $90,000: 10%
    - $70,000: 50%
    - $60,000: 40%

## A comment on probabilities

- **Juan's best guesses are not probabilities in the sense of frequencies.**
- **They are called *Bayesian* or *subjective probabilities*.**
- **Most decision problems rely on "best guesses" both in terms of the outcomes (events) and the probabilities for those events.**
  - **Could be based on detailed analysis**
  - **Could be a first guess for a number that is not known at all.**

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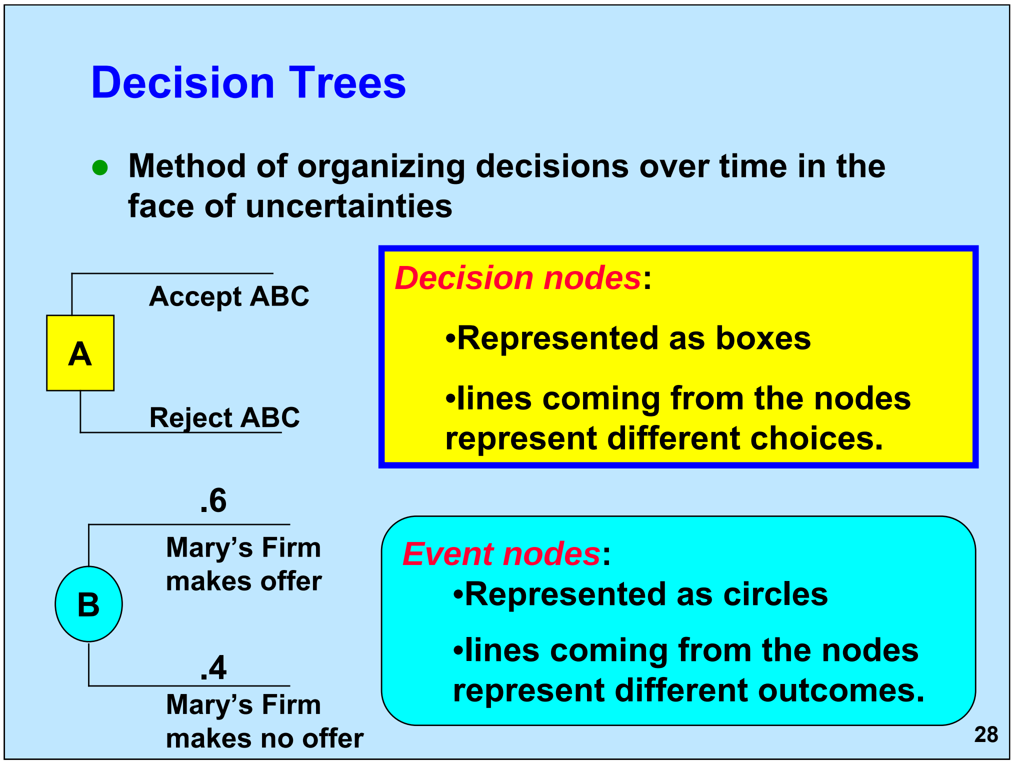
One of the first realizations in using probability in decision making is that it is rarely about repeatable events. It is much more an internal assessment about how likely an event is. Juan believes that there is a 60% chance of getting a job at Mary's firm. He knows what a 60% probability is. It is the probability of getting one of the numbers 1 to 6 on a fair 10-sided die. In his mind, he views the probability of getting a job from Mary as equally likely.

Juan's use of probability is called "subjective probability" and it is often referred to as Bayesian probability, although Bayesian has additional meanings.

A natural question is: can one use subjective probabilities in making calculations on probabilities? What this question means is "will the answers be useful and meaningful?"

In our class we will use the subjective probabilities as though they are the probabilities we are used to. In practice, using subjective probabilities is far better than ignoring the uncertainties altogether. And the value of using the subjective probabilities depends a lot on whether Juan is reasonably good at expressing his lack of certainty, and whether he is reasonably consistent in assigning probabilities to events.

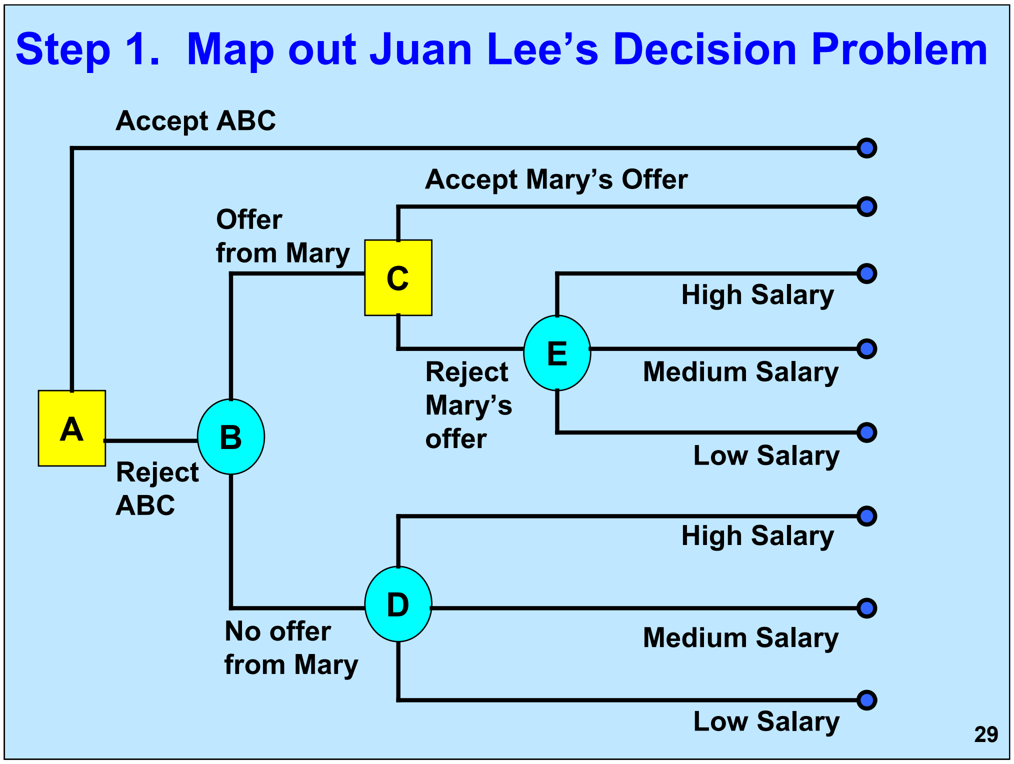



Decision trees map out a decision problem over time.

There are two types of nodes of the decision tree: decision nodes and event nodes. Decision nodes are represented as rectangles, and event nodes are represented as ovals.

At a decision node, the decision maker needs to make a decision.

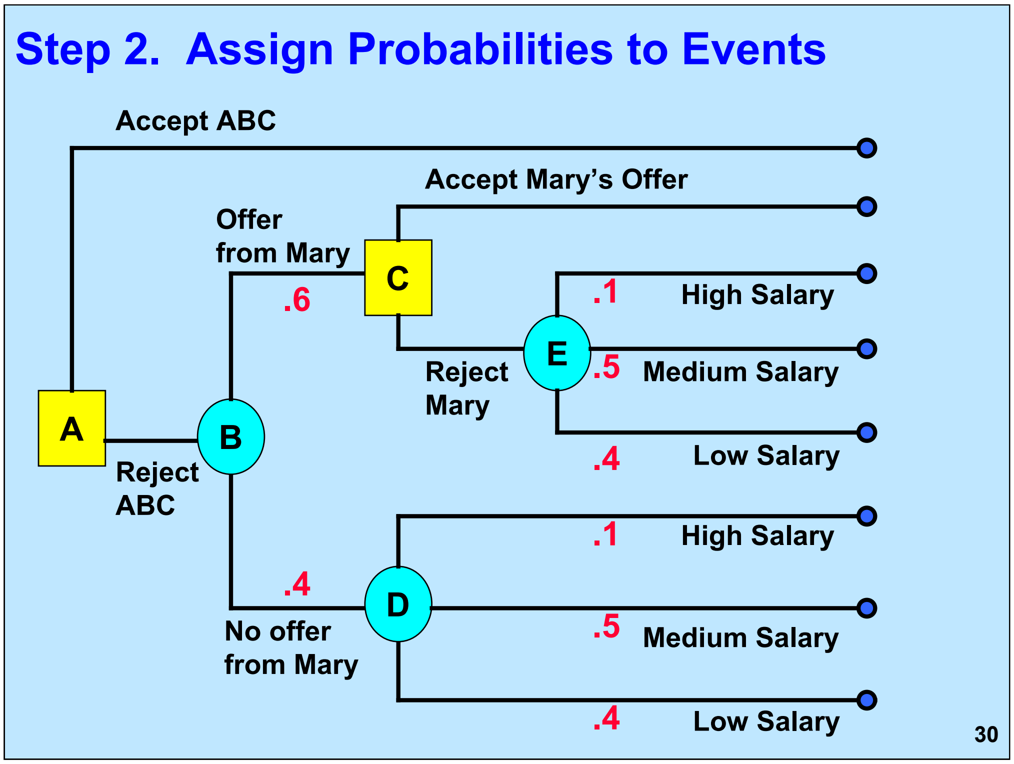
At an event node, the decision maker finds out the outcome of an event. This type of node is different than we are used to in probability. Usually, we think of an event as having a probability (and it does). But in a decision tree, the event node refers to the decision maker finding out the outcome of an event, and then being able to use this information in subsequent decisions.




This diagram reflects a mapping out of Juan Lee's decision.

First Juan has to make a decision on whether or not to accept ABC. If he accepts ABC, the decision process ends for Juan. Otherwise, he next finds out whether there is an offer or not from Mary. If there is an offer, he can accept it or not.

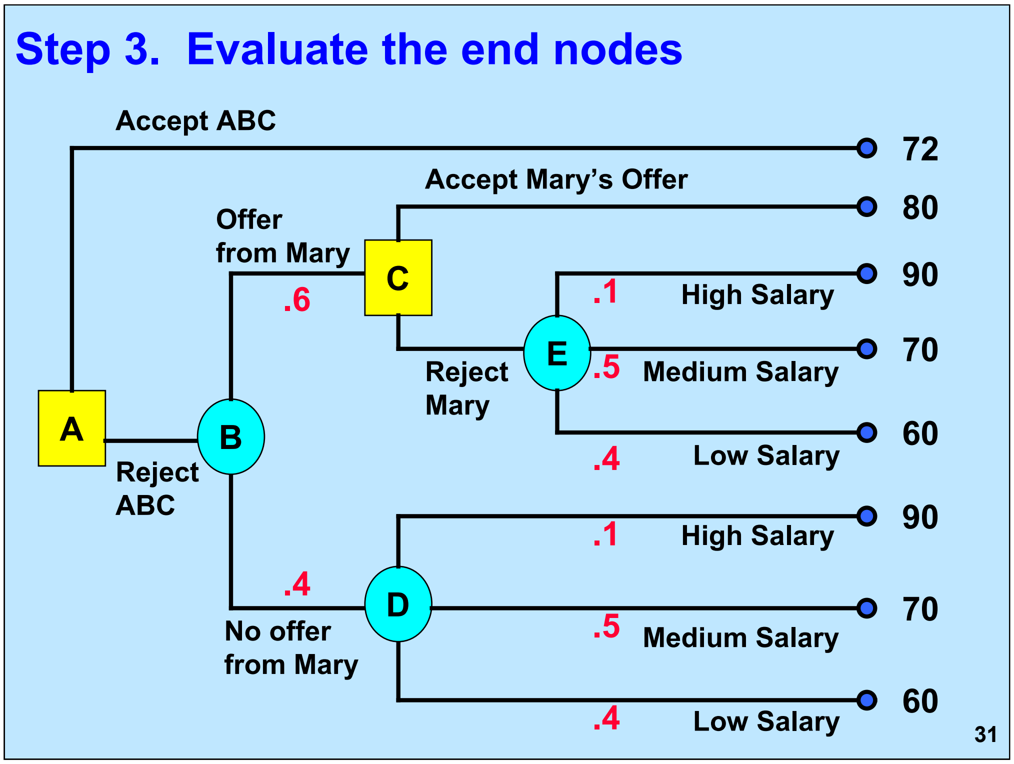
If Juan does not receive an offer from Mary or if receives one and rejects it, he then moves to the job fair and learns which of three salaries he will get.




We now assign a probability to events.

The probability that he gets an offer from Mary is .6.

The probability of getting the high, medium and low salaries are taken from a previous slide.




The essence of calculating the values of nodes is to start at the end, and work backwards to the beginning. This is because each end node reflects a possible decision path for Juan, including outcomes of events. We refer to all of the nodes on the far right as "endnodes."

For example, the second endnode from the top on the right has a value of 80. This node reflects the following history:

1. Juan rejects ABC's offer
2. Juan gets an offer from Mary
3. Juan accepts Mary's offer.

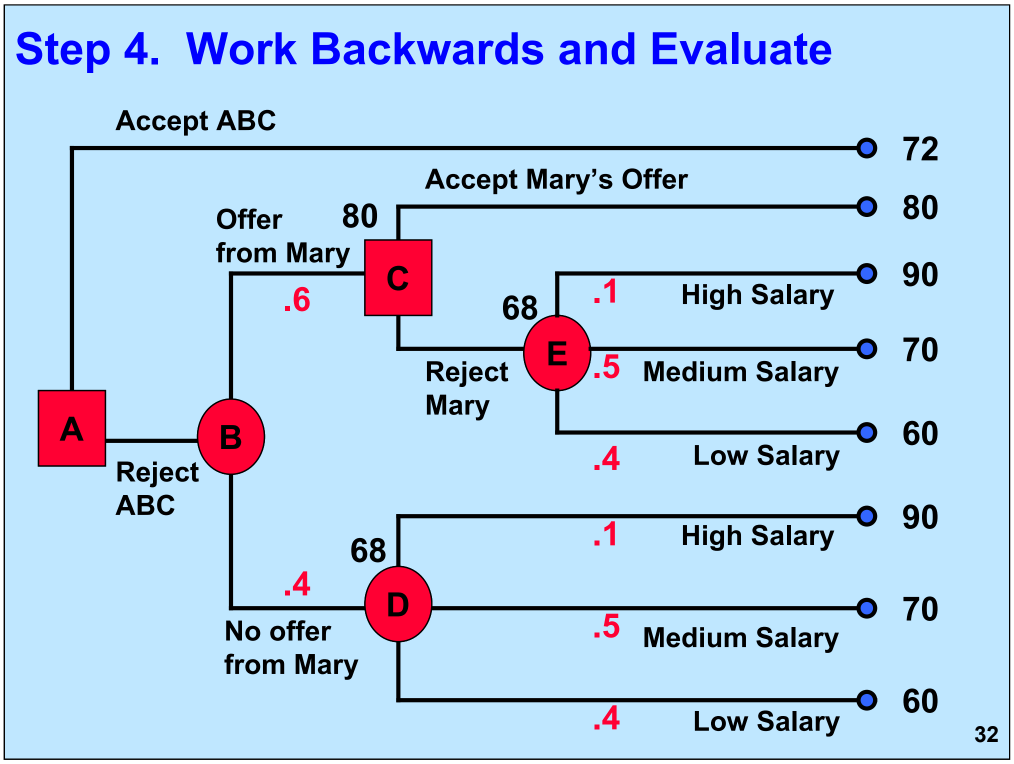
As such, this is worth $80,000 to Juan, which is the value of the offer.

As another example, consider the fourth endnode with a value of 70. This reflects the following history:

1. Juan rejects ABC's offer
2. Juan gets an offer from Mary
3. Juan rejects Mary's offer
4. Juan goes to the job fair and is given a medium salary.

As such, this is worth $70, which is the value of the medium salary.

For some problems, one needs to look carefully at the full history to figure what the endnode is worth. For this problem, it is more straightforward to compute the value of an endnode.




Once the endnodes are given values, we can work backwards (from right to left) and compute the value of all other nodes.

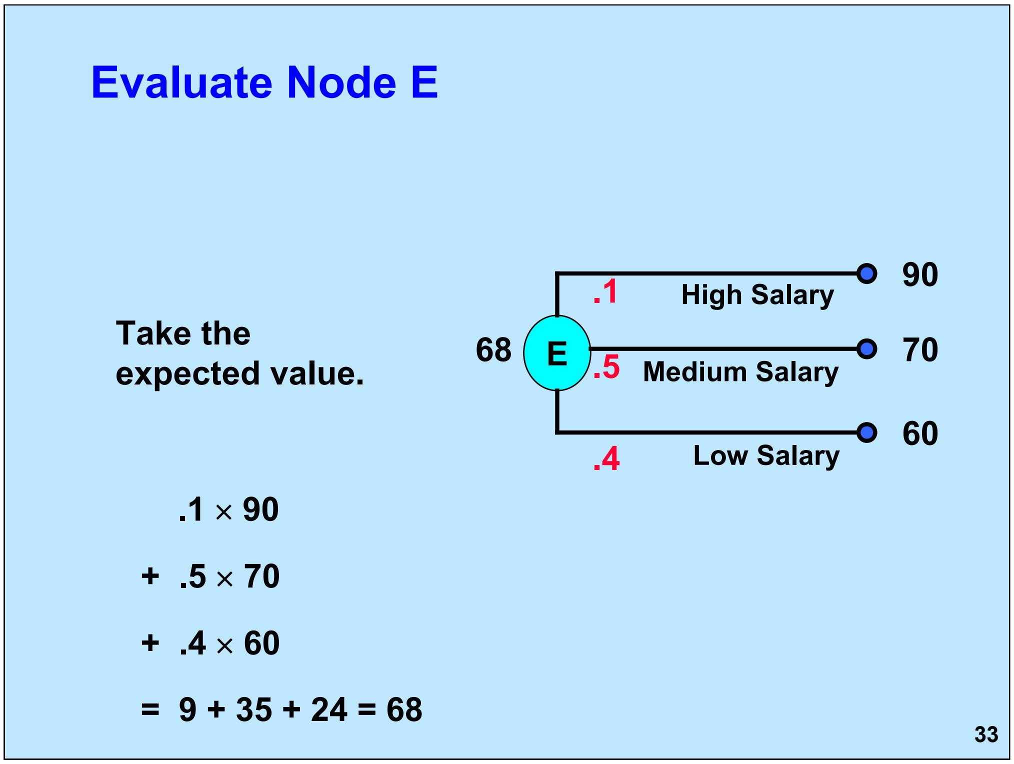
There are two types of calculations, one for event nodes and one for decision nodes.

You can click on the nodes during the slide show and see the calculations in more detail. Then click on the return button (curly arrow button on the slide) to return to this slide. Or if you forget, just type in 32 and press enter.

The value of an event node is an expected value calculation. For example, the value of node E is .1 x 90 + .5 x 70 + .4 x 60, which is the expected value at node E. This value is $68,000 or 68 on the decision tree. What this means is the following: If Juan rejects the ABC offer, obtains an offer from Mary, and then rejects the offer from Mary, he has an expected annual salary of $68,000. (Recall once again that "expected value" is a technical term from probability. Juan has no chance at all of getting exactly $68,000 in this example.)

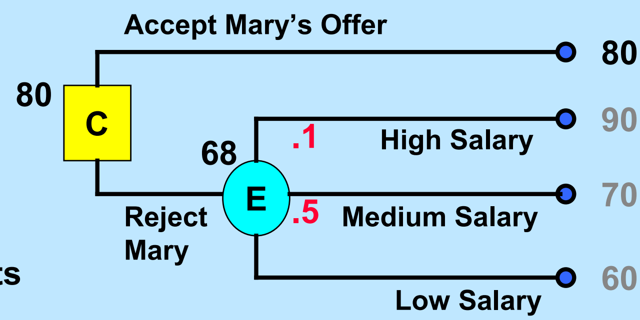
A natural question is whether we need to calculate the value of an event node by calculating its expected value. The answer is that this is the most useful way of approaching decision trees, but it can run into difficulties if the decision maker is either risk preferring (very unusual) or risk averse (very common). We will show how one can deal with risk aversion in the next lecture. For now, we will work with expected values.

The other type of calculation is for a decision node. For example, consider node C, which is a decision node. At a decision node, Juan will choose the best decision, that is, the one with the highest expected return. For node C, he has a choice of accepting Mary's offer for $80,000, or going to the job fair and getting an expected income of $68,000. In this case, Juan will take the $80,000. So, at a decision node, the value is the max of the values of the different decisions.




The next few slides treat the evaluations of different nodes. The button with the curly arrow is the return button.

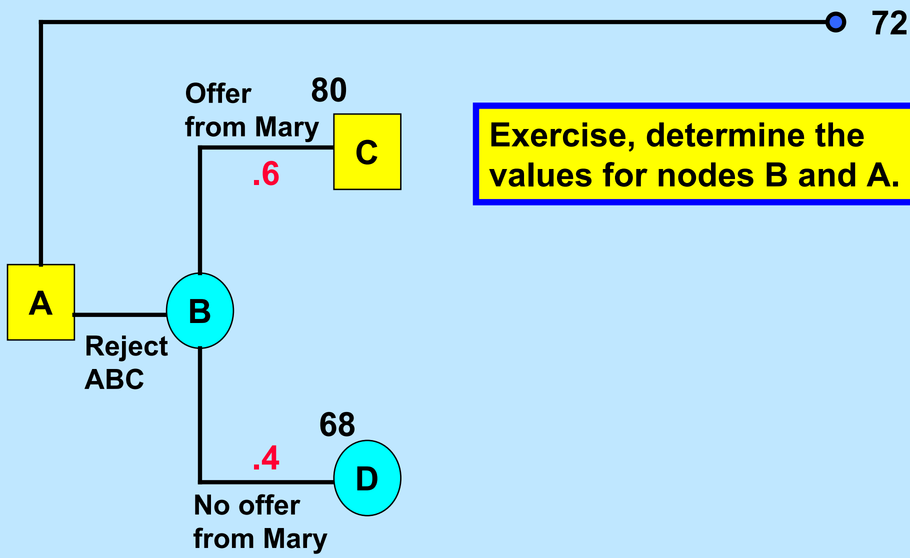
# Evaluate Node C




**For a decision node, take the best outcome leading from its branches.**

**Max (80, 68) = 80**

# Evaluate Nodes A and B




Exercise, determine the values for nodes B and A.

## Key Aspects of a Decision Tree

**Time flows from left to right**

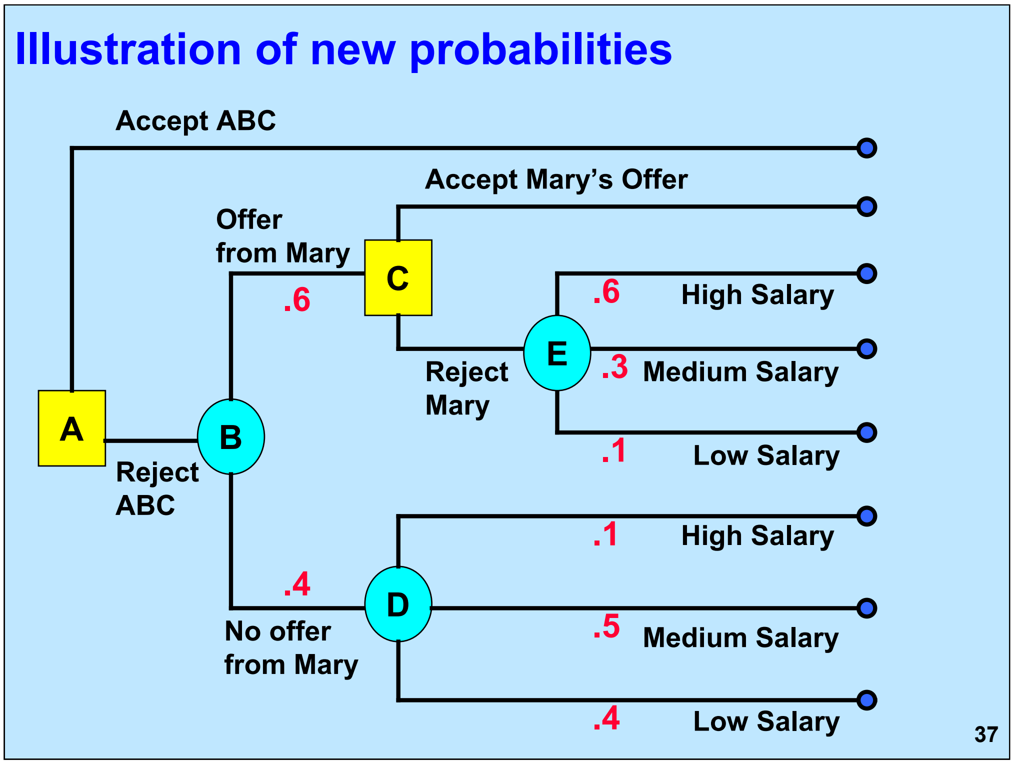
**Branches from a decision node represent decisions and take into account all decisions or events leading to that node**

- **Example. Juan expects to get Mary's job, but does not know how he is viewed in the job market. If he does get Mary's offer, he believes that he must be viewed highly. Accordingly, he would adjust his probabilities of getting salaries in the job fair as follows:**
  - **High: 60%**
  - **Medium: 30%**
  - **Low: 10%**



This slide reflects an interesting use of subjective probability. People will often change their probability assessments when they get new information. Here we suppose that Juan will change his probability assessments if he gets the offer from Mary's firm. (If he is rejected by Mary's firm, for some reason he does not change his probability assessments. It may Juan's ego, or it may reflect reality.)

So, we assume here that the probability of a high offer increases to 60%, and the probability of medium and low offers also adjust.




Note that the probability of .6 for a high salary occurs at event node E. The event node E reflects the following history:

1. Juan has rejected the ABC offer
2. Juan received an offer from Mary
3. Juan rejected the offer from Mary

Under these circumstances, Juan believes the probability of a high salary is .6.

If you look at node D, you will note that the probability there of a high salary is .1, which is Juan's probability if he gets no offer from Mary.

In this case, the value of node E is

.6 x 90 + .3 x 70 + .1 x 60 = 54 + 21 + 6 = 81.

So, at node C, the optimal decision is to reject the offer from Mary's firm. It's ironic that without the offer, Juan's expected salary would be $68,000 and the offer from Mary's firm would be very attractive. But after getting the offer, it now is optimal to reject it.

## Key Aspects of a Decision Tree

- Branches from an event node represent a set of mutually exclusive and collectively exhaustive

- Final nodes in the tree have an associated value

- Values of other nodes are computed working backwards. The value of an event node is the expected value of its endpoints. The value of a decision node is the highest value of its endpoints.

## But the tree is only the beginning

- **Typically in decision trees, there is a great deal of uncertainty surrounding the numbers.**
  - **Decision Trees work well in such conditions**
- **This is an ideal time for sensitivity analysis the "old fashioned way."**
  - **One varies numbers and sees the effect**
  - **One can also look for changes in the data that lead to changes in the decisions.**



One of the great things about Decision trees is that it is easy to change the numbers and get new results. This type of sensitivity analysis can do a variety of things:

1. It can give additional confidence in decisions
2. It can reveal how sensitive the decisions are to the probabilities or other numbers in the tree. Usually, the probabilities are best guesses. If the decision is highly sensitive to one of these guesses, it may be better to gather more information. Similarly, if the decision is not changed when the probabilities change, then you don't need to worry about the accuracy of the guess, and there is no need to gather more information.
3. It can give you a sense for whether to consider new options. If all decisions in the decision tree appear to be poor, then there may be other options to consider.

## The Airfare Problem

You are trying to get the cheapest airfare that you can. You just called up and found that the ticket home will cost \$400, and it cannot be refunded or exchanged. You can also buy a ticket for \$450, which can be refunded for \$430 (and thus costs you \$20). The price of tickets will change in one week, and you will have one more chance to buy a ticket. There is a 50% chance that the ticket would cost \$300, and a 50% chance that it would cost \$600. What should you do to minimize the expected expense?

**Exercise. Write the Airfare problem as a decision tree, and solve it.**



We leave this as an exercise.

# The Air Fare Problem

## Summary Conclusions

- Decision trees are a very useful technique for mapping out sequential decisions under uncertainty.

- They can be useful for mapping out other logic as in the newsvendor problem.