Portfolio Optimization and the Endurance Investors' Case

15.060 Data, Models and Decisions

November, 2007

Development of Portfolio Model Data

$P_{A,t}$ = stock price of asset A at end of month t

$D_{A,t}$ = dividends paid during month t

$R_{A,t}$ = monthly rate of return of asset A in month t :

$$R_{A,t} = (P_{A,t} + D_{A,t} - P_{A,(t-1)}) / P_{A,(t-1)}$$

R_A = *future* monthly rate of return of asset A

R_A is a random variable

Estimate of mean and standard deviation of R_A :

$$\text{MEANR}_A = \frac{1}{T} \sum_{t=1}^{T} (R_{A,t})$$

$$\text{VAR}_A = \frac{1}{T-1} \sum_{t=1}^{T} (R_{A,t} - \text{MEAN } R_A)^2$$

$$\text{SD}_A = \sqrt{\text{VAR}_A}$$

Estimate of covariance and correlation of R_A and R_B :

$$\text{COV}(R_A , R_B) = \frac{1}{T-1} \sum_{t=1}^{T} (R_{A,t} - \text{MEANR}_A) (R_{B,t} - \text{MEANR}_B)$$

$$\text{CORR}(R_A , R_B) = \frac{\text{COV}(R_A , R_B)}{\text{SD}_A \ \text{SD}_B}$$

Critique of Methodology

- Uses past performance to estimate future performance
- Some months might be very unusual
- Might instead use "exponential smoothing"
- How many months should we use? 12, 24, 60, 120, ...
- This method (based on past data) is a good estimator of future covariance and correlation, but it is not a very good estimator of future expected returns
- There are much better ways to estimate $MEANR_A$ based on the Capital Asset Pricing Model (CAPM):

$$MEANR_A = R_f + \beta \cdot (MEANR_{MARKET} - R_f)$$

R_f is the risk-free rate of return

MARKET is the entire stock market/economy

$$\beta = \frac{COV(R_A, R_{MARKET})}{VAR_{MARKET}}$$

Converting monthly estimates to annual estimates? Rates of return are multiplicative, not additive. Handled using logarithms.

More Advanced Modeling Methods for Portfolio Optimization

Set-Up and Basic Data Requirements

$i = 1, \ldots, n$ is the group of assets under consideration

We currently own H_i shares of asset i

$PCURR_i$ = yesterday's closing price of asset i

$PFUT_i$ = future price of asset i (say, one quarter from now)

$PFUT_i$ is a random variable

We have estimates of the following data:

$EPFUT_i$ (expected future price of asset i)

$COV(PFUT_i , PFUT_j)$ (covariance of future price of assets i and j)

Decision Variables:

Y_i = the amount we will purchase or sell of asset i

$Y_i \geq 0$ if we are buying

$Y_i \leq 0$ if we are selling

Expected Value and Risk of the Portfolio

EXPECTED VALUE =

$$\sum_i EPFUT_i \cdot (H_i + Y_i)$$

STANDARD DEVIATION =

$$\sqrt{\sum_i \sum_j COV(PFUT_i, PFUT_j)(H_i + Y_i)(H_j + Y_j)}$$

Transaction Fees

Transaction fee of A_i, measured in \$ per share traded

Typically A_i = \$0.03 (or less) for large investors, depending on the asset

Total transaction costs are:

$$\sum_i A_i | Y_i |$$

Price Impact

Suppose average daily volume of Boeing is 200,000 shares

Suppose a large investment firm wants to purchase 100,000 shares

The closing price of Boeing yesterday was $60.00

What will happen?

For example, it might turn out that the firm can purchase 20,000 shares at $61.00
40,000 shares at $62.00
40,000 shares at $63.00

If the transaction fees are $0.03/share, then:

Transaction fees: $3,000 = 0.03 • 100,000

Impact on price: $220,000 = $20,000 • 1.00
+ 40,000 • 2.00
+ 40,000 • 3.00

Cost of price impact is *significantly* greater than the transaction fees

We model: $PNEW_i = PCURR_i + C_i \cdot Y_i$

(C_i is typically determined by regression on past data)

Therefore:

$$PRICE\ IMPACT = (PCURR_i + C_i \cdot Y_i) \cdot Y_i - (PCURR_i) \cdot Y_i$$

$$= C_i \cdot (Y_i)^2$$

Liquidity

If a firm owns 50% of all outstanding stock in asset i , it will be more difficult to sell the stock.

Let T_i = total number of outstanding shares of stock in asset i

Suppose we do not want to own more than 4% of the total outstanding shares in asset i

$$H_i + Y_i \leq 0.04 \cdot T_i$$

Turnover

We can impose a constraint on the volume traded of asset i :

$$| Y_i | \leq LIMIT_i$$

and we can impose a bound on the total volume of all trades:

$$\sum_i PNEW_i \cdot | Y_i | \leq 0.\mathbf{05} * \left(\sum_i PCURR_i \cdot H_i \right)$$

Cash Balance

We keep a small amount of the fund in cash to use for transactions, otherwise we must balance out the cash used and created:

$$\left| \sum_i PNEW_i \cdot Y_i \right| \leq LIMIT$$

<u>No Short Selling</u>

$H_i + Y_i \geq 0$

<u>Closeness to Benchmark</u>

Investment firms often restrict the weights of stocks in their portfolios to be close to some benchmark. Here is one way that this is done if the benchmark is the S&P500:

T_i is the number of outstanding shares of asset i in the S&P500

$$MCAP_i = PCURR_i \cdot T_i$$

$$WBENCH_i = MCAP_i / \sum_{k=1}^{500} MCAP_k$$

$$TOTALINVEST = \sum_i PCURR_i \cdot (H_i + Y_i)$$

We then use the following constraint:

$$\sum_i \left| \frac{PCURR_i \cdot (H_i + Y_i)}{TOTALINVEST} - WBENCH_i \right| \leq LIMIT$$

The Overall Model

Maximize $\sum_i \text{EPFUT}_i \cdot (H_i + Y_i) - A_i \,|\, Y_i \,| - C_i \cdot (Y_i)^2$

Subject to:

$$\sqrt{\sum_i \sum_j \text{COV}(\text{PFUT}_i, \text{PFUT}_j)(H_i + Y_i)(H_j + Y_j)} \leq \text{TARGET Std. Dev.}$$

$H_i + Y_i \leq 0.04 \cdot T_i$ (Liquidity)

$|\, Y_i \,| \leq \text{LIMIT}_i$ (Turnover)

$\sum_i \text{PNEW}_i \cdot |\, Y_i \,| \leq 0.05 * (\sum_i \text{PCURR}_i \cdot H_i)$ (Turnover)

$|\sum_i \text{PNEW}_i \cdot Y_i \,| \leq \text{LIMIT}$ (Cash Balance)

$H_i + Y_i \geq 0$ (No short selling)

$\sum_i \left| \frac{\text{PCURR}_i \cdot (H_i + Y_i)}{\text{TOTALINVEST}} - \text{WBENCH}_i \right| \leq \text{LIMIT}$ (Benchmark)

$\text{PNEW}_i = \text{PCURR}_i + C_i \cdot Y_i$ (Price Impact)

Multi-Period Portfolio Models

Capture the dynamics that decisions can be delayed in order to take advantage of economies of scale in trading as well as to lessen the price impact of trades for large institutional investors

Based on “dynamic programming” or “dynamic optimization”

Take 15.071 Decision Methodologies for Managers (“The Edge”)
Spring semester, Professor Bertsimas

Additional Reading

Basic level:

- *Mean-Variance Analysis in Portfolio Choice and Capital Markets,* Basil Blackwell, New York, (1987), by H. Markowitz.

- *Active Portfolio Management: Quantitative Theory and Applications,* Probus Pub Co. (1995), by Richard C. Grinold, Ronald N. Kahn

- *Investments*, Irwin Series in Finance, 5th edition (2002) by Zvi Bodie, Alex Kane and Alan J. Marcus

Advanced level:

- *Foundations for Financial Economics,* Elsevier Science Publishers, North-Holland, (1988), by Chi-fu Huang and Robert H. Litzenberger.

Final Note

The Nobel Memorial Prize in Economic Science was awarded in 1990 to Merton Miller, William Sharpe, and Harry Markowitz for their work on portfolio theory and portfolio models (and the implications for asset pricing).