

15.060 Data, Models, and Decisions
Homework 1
Due: MONDAY, September 17, 2007

1. Bertsimas and Freund, Exercise 2.12
 2. Bertsimas and Freund, Exercise 2.14
 3. Bertsimas and Freund, Exercise 2.32
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4. United Eastern Airlines (UEA) routinely holds (from early sale) 20 seats per flight for regular business customers who buy their tickets about 1 week before their flight. Historically the revenue per flight from these customers is \$20,000 with probability 0.65 and \$30,000 with probability 0.35. This month however, the UEA expects many “last-minute” customers who are typically willing to pay even more for their tickets. These purchases occur within the last two days prior to the flight. The projected revenue per flight from these last-minute customers is estimated to be $\$32,000 \times F$, where F is the fraction of seats that would be taken away from the aforementioned business customers. UEA estimates that the demand from these last-minute customers will be 8 seats with probability 0.30 and 10 seats with probability 0.70, so the company considers reserving 8, 9 or 10 seats for these customers. If, within several hours of flight time, there are any remaining unsold seats, then these unsold seats can typically be sold to regular business customers, but at a 50% discount. Use a decision tree to determine if the company should modify their usual plan and hold some seats for the very lucrative last-minute customers, and if so, determine how many seats should be held. Perform the sensitivity analysis with respect to the probability that the revenue from business customers is \$30,000 (which currently is 0.35). At what value (if any) would your recommended strategy change?

5. A game of roulette consists of a random spin of a wheel where a ball will land either on 0, 00, or 1, 2, 3, etc., through 36 (for a total of 38 numbers). One way to bet on roulette is to bet on a color. Odd numbers between 1 and 27 have the color red as well as the numbers 30, 32, 34, and 36. The remaining numbers between 1 and 36 are black. A bet on a color of \$1 will pay off \$2 if the ball lands on that color, and will pay off nothing otherwise. However, the numbers 0 and 00 are not associated with any color. What is the probability that a bet on RED will pay off \$2? What is the expected value of a bet on red?

A gambler considers betting a certain amount on a black color. He is planning to play once only and intends to follow the strategy of waiting until there are 4 spins in a row on which the ball lands on red, and then on the fifth spin he will bet on black. He is planning to wait as long as it takes to see 4 spins in a row where the ball lands on red. His thinking is that it is unlikely that there would be 5 red spins in a row and the chances of seeing 0 or 00 are too meager. His friend claims, however, that this strategy is a waste of time and he might as well bet immediately. Is the friend’s advice sound? What is the probability that his bet will win? What is the probability of winning the fifth spin if the first four were reds? Justify your answer.
