

15.070

Homework Assignment 5

Given: November 23, 2005

Due: December 5, 2005

Problem 1 Let $dX(t) = V(t)dB(t)$ for $V \in H^2$.

A. Give an example when X^2 is not a martingale.

B. Show that $X^2(t) - \int_0^t V^2(s)ds$ is a martingale.

Problem 2 The following theorem holds. If $U \in H^2$ and Novikov's condition

$$\mathbb{E}[\exp(\frac{1}{2} \int_0^t U^2(s)ds)] < \infty$$

is satisfied, then $\mathbb{E}[\exp(\int_0^t U(s)dB(s))] < \infty$.

Establish this result in a special case when U is a simple function consisting of a single piece. Namely $U(s, \omega) = U(\omega)$ for all s . NOTE: remember that U is adapted to the same filtration as $B(t)$.

Problem 3 Problem 1 from Lecture 18.

Problem 4 Problem 2 from Lecture 18.

Problem 5 Problem 3 from Lecture 18.