

15.070

Homework Assignment 1

Given: September 14, 2005

Due: September 21, 2005

Problem 1 *Exercise 1 from Lecture 1.*

Problem 2 *Exercise 4 from Lecture 1. Prove rigorously the identity $A(c) = \bigcap_{r \geq 1} C_r$.*

Problem 3 *Consider the following process. At the end of each day a cashier empties the cash register and records the amount in dollars. The amount is expressed in whole dollar amounts. This happens every day starting from day one for the rest of the eternity.*

- A. Write down the sample space Ω corresponding to the process of recording the revenue each day.*
 - B. Consider the following event: one of the days not a single item was sold and, as a result, there was no revenue. Express this event mathematically.*
 - C. Prove that this event is measurable, that is, it belongs to the product σ -field \mathcal{F}_∞ .*
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Problem 4 *Prove part (c) of Proposition 3 in Lecture 1.*

Problem 5 *Consider a random variable $X : \Omega \rightarrow \mathbb{R}$ and the corresponding distribution function $F(x) = \mathbb{P}(\omega : X(\omega) \leq x)$. Establish the following properties of F :*

- A. $F(x)$ is non-decreasing.*
- B. $\lim_{x \rightarrow -\infty} F(x) = 0, \lim_{x \rightarrow \infty} F(x) = 1$.*
- C. $F(x)$ is right-continuous. That is $\lim_{x \downarrow x_0} F(x) = F(x_0)$ for every real value x_0 .*
- D. Give example of a random variable such that F is not continuous.*