15.070

Homework Assignment 6

Given: December 5, 2005 **Due:** December 12, 2005

Problem 1 *Suppose* $u_n, n \geq 1$ *and* $v_n, n \geq 1$ *are sequences of not necessarily independent random variables. Let* $S_n = \sum_{1 \leq k \leq n} v_k$ *and* $N(t) = \max\{n : \sum_{1 \leq k \leq n} u_k \leq t\}$*. Suppose both* S_n *and* $N(t)$ *satisfy FSLLN with constants* μ^{-1} *and* λ*. Namely*

$$\lim_{n\to\infty} \frac{S_{\lfloor nt \rfloor}}{n} \to \mu^{-1} t$$
$$\lim_{n\to\infty} \frac{N(nt)}{n} \to \lambda t$$

almost surely u.o.c. Establish that $\frac{N(S_{\lfloor nt \rfloor})}{n} \to \frac{\lambda}{\mu} t$ *and* $\frac{S_{N(nt)}}{n} \to \frac{\lambda}{\mu} t$*. almost surely u.o.c. Recall that* $f_n \in D[0,\infty)$ *converges to* $f \in D[0,\infty)$ *uniformly on compact sets (u.o.c.) if for every* $T > 0$ $\|f_n - f\|_T \to 0$*.*

Problem 2 *Given* $x \in D$ *with no downward jumps let* $y = \Psi(x), z = \Phi(x)$*. Fix* $t_0 > 0$ *and consider a modified process* $\hat{x}(t) = z(t_0) + x(t_0 + t) - x(t_0), t \geq 0$*. Establish that* $\Phi(\hat{x})(t) = z(t_0 + t), \Psi(\hat{x})(t) = y(t_0 + t) - y(t_0), t \geq 0$*.*

Problem 3 *We have established in Theorem 21.8* Ψ, Φ *are Lipschitz continuous with constants* 1 *and* 2 *respectively. Show that these constants are tight.*