15.082J/6.855J **Network Optimization**

Homework Set #9
Due Lecture 19

Chapter 13 of AMO

1. Exercise 13.3. Show the partial trees obtained on the way to finding the minimum cost spanning tree.

2. Exercise 13.4. Show the partial trees obtained on the way to finding the minimum cost spanning tree.

3. Read Section 13.8 and do exercise 13.42

4. Prove the following stronger versions of the cut optimality conditions. The proof will not rely on the cut optimality conditions as proved in the book. Rather, the proof of this exercise should be similar to the proof of the cut optimality conditions in the book.

   a. Suppose that (i,j) is a minimum cost arc in the cut (S, N\S). Prove that there is some minimum cost spanning tree that contains (i,j).
   b. Suppose that (i,j) is the unique minimum cost arc in the cut (S, N\S). Prove that every minimum cost spanning tree contains (i,j).

5. Exercise 13.34.

6. Consider the following linear program:

   Maximize $3x_1 + 5x_2 + 7x_3$
   subject to $4x_1 - 6x_2 + 3x_3 \leq 7$
   $-2x_1 + 3x_2 + x_3 \leq 9$
   $x_1 \geq 0 \; x_2 \geq 0 \; x_3 \geq 0$

   a. What is the dual linear program?
   b. What are the complementary slackness conditions?

7. Consider the following version of the assignment problem:

   Maximize $\sum_{(i,j)\in A} c_{ij} x_{ij}$
   subject to $\sum_{j=1 \text{ to } n} x_{ij} \leq 1$ for i = 1 to n
   $\sum_{i=1 \text{ to } n} x_{ij} \leq 1$ forj = 1 to n
   $x_{ij} \geq 0$ for all $(i,j) \in A$

a. What is the dual linear program?
b. What are the complementary slackness conditions?
c. Restate the strong duality theorem in terms of this assignment problem.