15.082 Practice Midterm

Practice for a closed book midterm.

Students are permitted two sheets of notebook paper with

writing on one side only

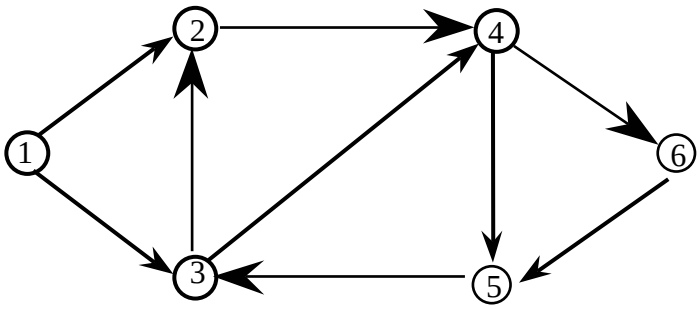
This practice midterm is designed to be longer than the real midterm by about 20 minutes.

Note: the emphasis on the following:

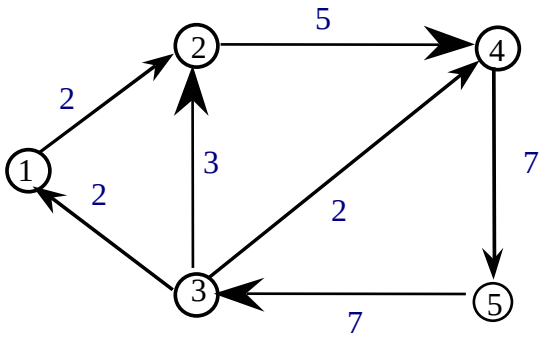
1. **Understanding the algorithms and being able to replicate key steps**
2. **Understanding the essential aspects of the run time analysis**
3. **Understanding theoretical aspects of the algorithms and being able to use these theoretical aspects with comfort**
4. **Being able to model problems as a shortest path or max flow or min cut problem.**

The scores add up to 90 points only. I removed a 10 point question on preflow push because that will not be covered on the exam.

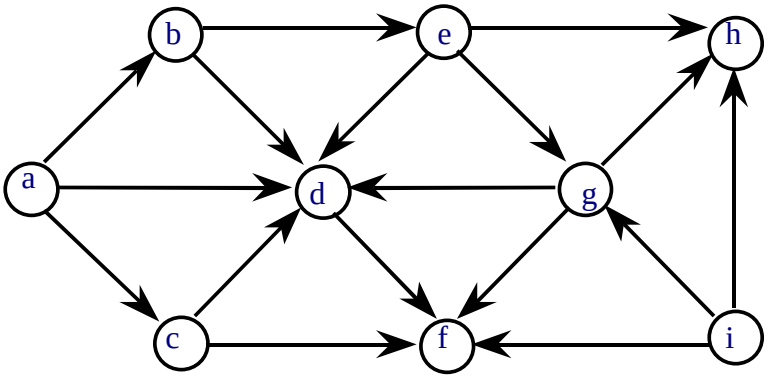
1. (10 points) Please provide a breadth first search tree and a depth first search tree for the following network. (The search algorithm keeps track of predecessors, and the predecessors form the tree.)



2. (10 points) Express a flow decomposition of the flow described below. (As usual, any path in the flow decomposition should be directed from a supply node to an demand node.)

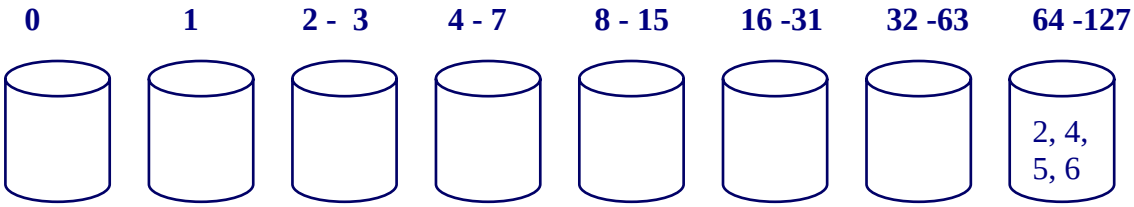


3. (10 points) Establish that the following graph is acyclic by topologically ordering the node labels, or else determine a directed cycle. (You will need to copy the graph onto your blue book.)



4. (10 points) Here are the buckets for some iteration of the R-heap algorithm. The algorithm is called again as part of the find-min operation". What is the state of the buckets after the find-min computation is completed, that is after node 2 is deleted from the buckets and made permanent. (What is the range of each bucket, and what nodes are in which buckets?)

node	1	2	3	4	5	6
d(node)	0	69	1	79	73	122



5. (10 points, 2.5 points per part). In each of the examples below, you will be provided with a function f and a function g. You are to state whether f(n) = O(g(n)) or not.

   a. f(n) = 10n; g(n) = $n^2 - n$.
   b. f(n) = n if n is odd, and 0 if n is even; g(n) = n if n is even, and 0 if n is odd.
   c. f(n) = $50n + 2^{1000}$; g(n) = n-1;
   d. f(n) = $(\log n)^{100}$; g(n) = $n^{.01}$;

6. (10 points) Let $f(n,m,k) = n^3/k + knm$. Let $g(n,m) = \min \{ f(n,m,k) : k > 0 \}$.

   Show that $g(n,m) = \Theta(n^2m^{.5})$, that is $g(n,m) = O(n^2m^{.5})$, and $n^2m^{.5} = O(g(n,m))$.

   HINT: Do NOT use differential calculus to determine g(n,m). (Trust us, differential calculus will be far too time consuming.) Rather, let k*(n,m) be the unique value of k for which $n^3/k = knm$. Let g'(n,m) = f(n,m,k*(n,m)). Then show that $g(n,m) \le g'(n,m) \le 2\, g(n,m)$.

   Remark. Sometimes it's possible to develop an algorithm whose running time depends on a parameter k. This problem shows that one can determine the best choice of the parameter k without relying on differential calculus, but by just setting terms equal.

7. (10 points) The document processing program TeX uses an optimization procedure to decompose a paragraph into several lines so that when lines are left- and right-adjusted, the appearance of the paragraph will be the most attractive. Suppose that a paragraph consists of n words and that each word is assigned a sequence number. Let c_{ij} denote the attractiveness of a line if it begins with the word i and ends with the word j-1. The program TeX uses formulas to compute the value of each c_{ij}. Given the c_{ij}'s show how to formulate the problem of decomposing a paragraph into several lines of text in order to maximize the total attractiveness of all lines as a shortest path problem. Be sure to explain the connection between the shortest path and the most attractive paragraph.

8. (10 points, 5 points per part)

a. Suppose that the amount of flow in the maximum flow from source to sink is m log n, and that all capacities are integral. How long would it take to find this maximum flow using the Ford-Fulkerson augmenting path algorithm? Briefly justify your answer.

b. Suppose, as in part a, that the amount of maximum flow from source to sink is m log n, and that all capacities are integral. How long would it take to find this maximum flow using the shortest augmenting path algorithm? Briefly justify your answer.

9. (10 points) Suppose that the minimum s-t cut in the network G has a capacity of 2000 and has 20 arcs. Suppose that one adds 10 units of capacity to each arc of G, creating a transformed network G'. Which of the following is true for the transformed network:

i. The capacity of the minimum cut of G' is exactly 2200.
ii. The capacity of the minimum cut of G' is at least 2200, and possibly more.
iii. The capacity of the minimum cut of G' is at most 2200, and possibly less.