

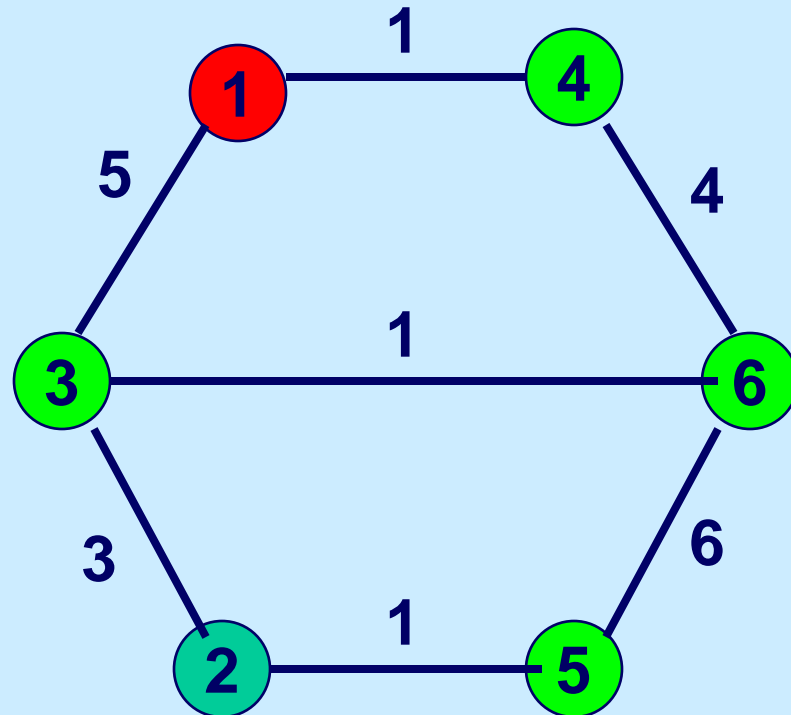
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**15.082 and 6.855J**

**Min Global Cut Animation**

# Initialize

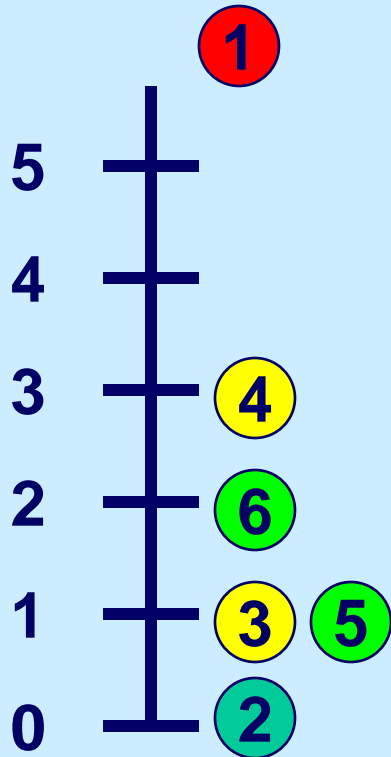
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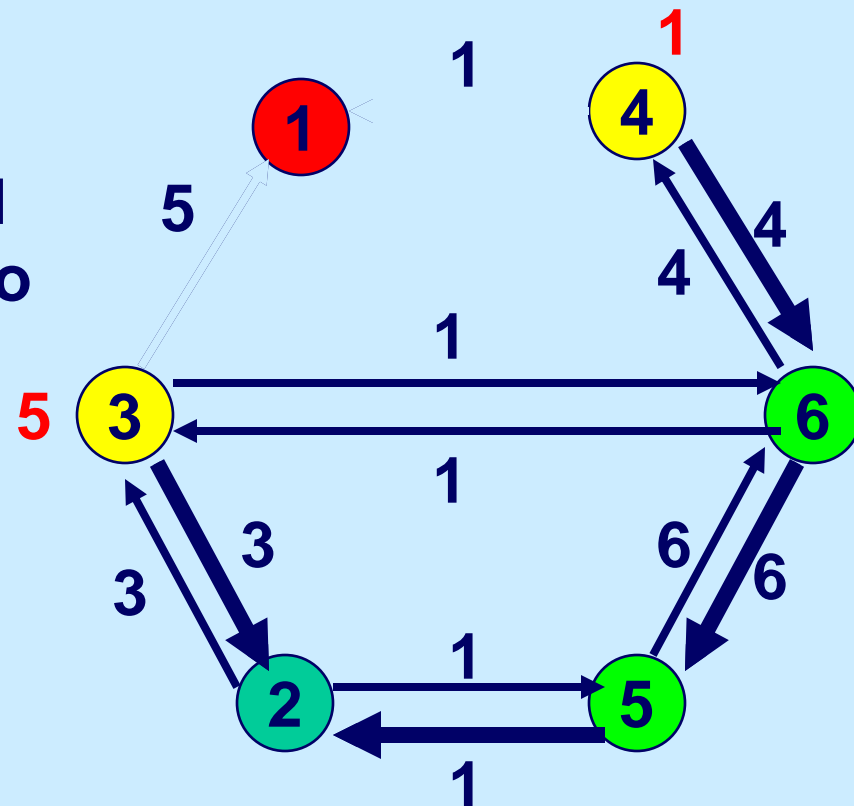
Saturate the arcs out of node 1.

Update the residual network

# Initialize



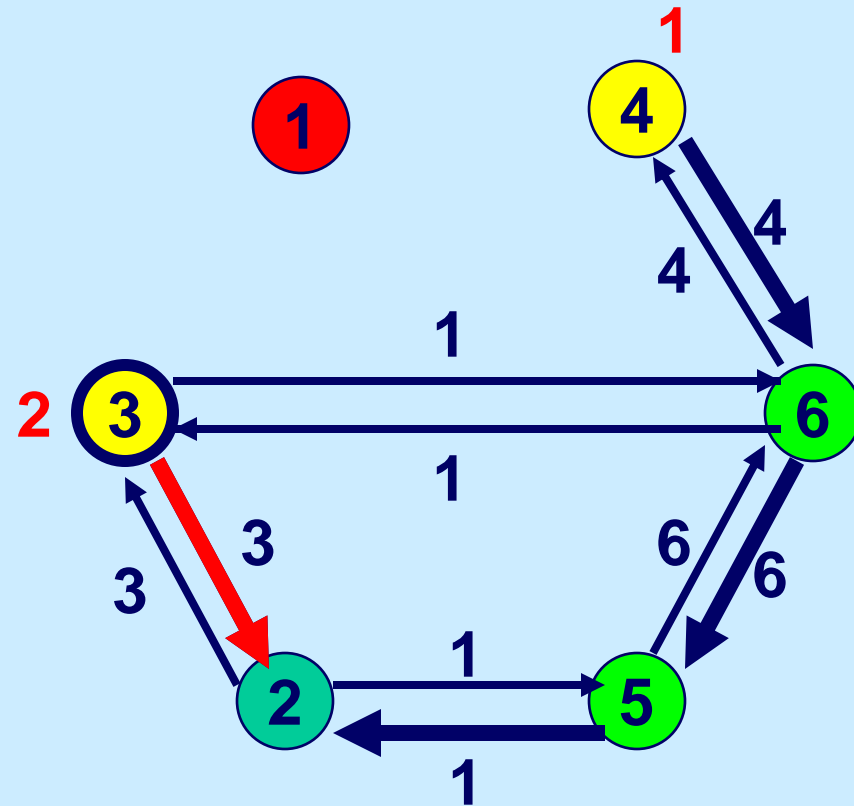
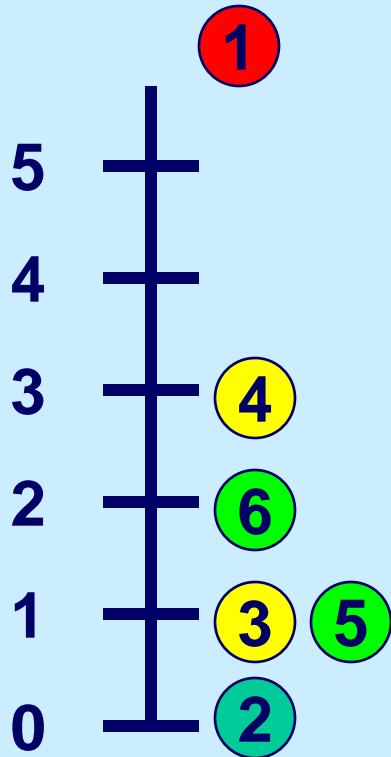
We will never push from node 1 again or into node 1.



Compute distances to node 2

Determine admissible arcs

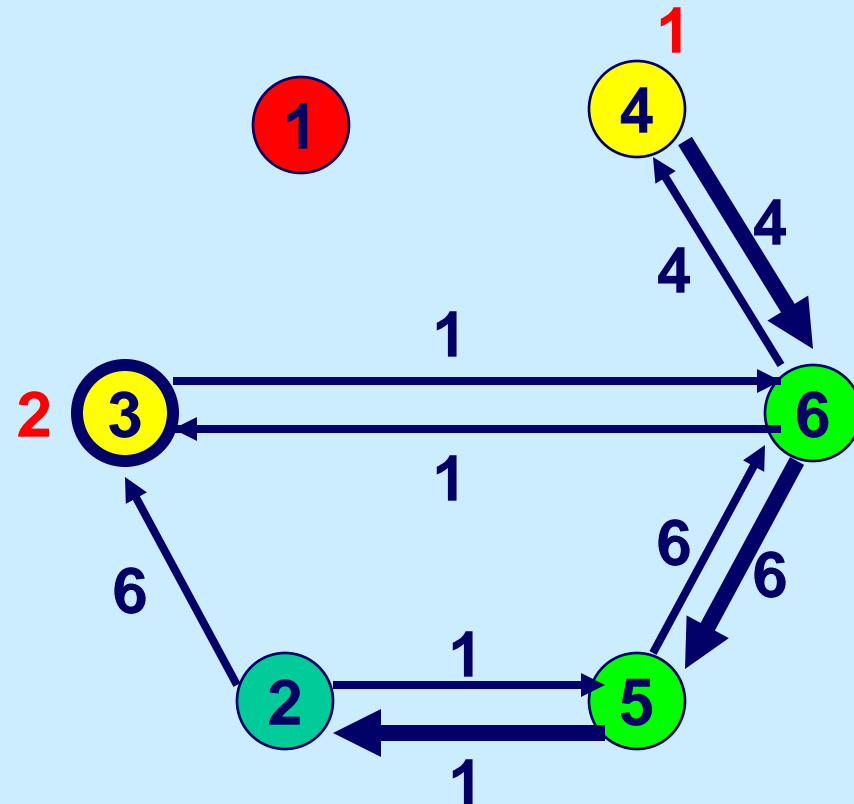
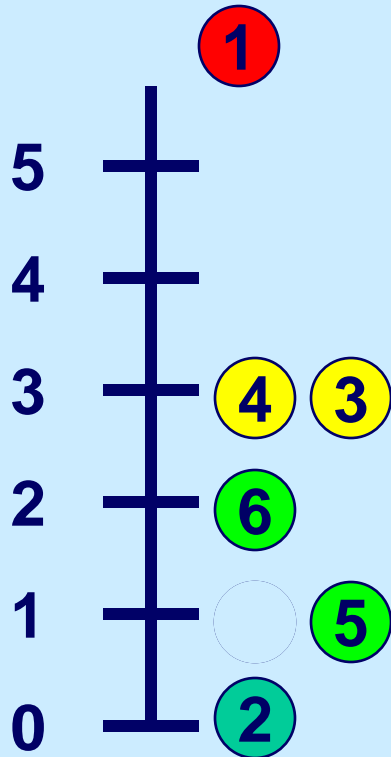
# Push/Relabel



Select an active node

Carry out push/relabel

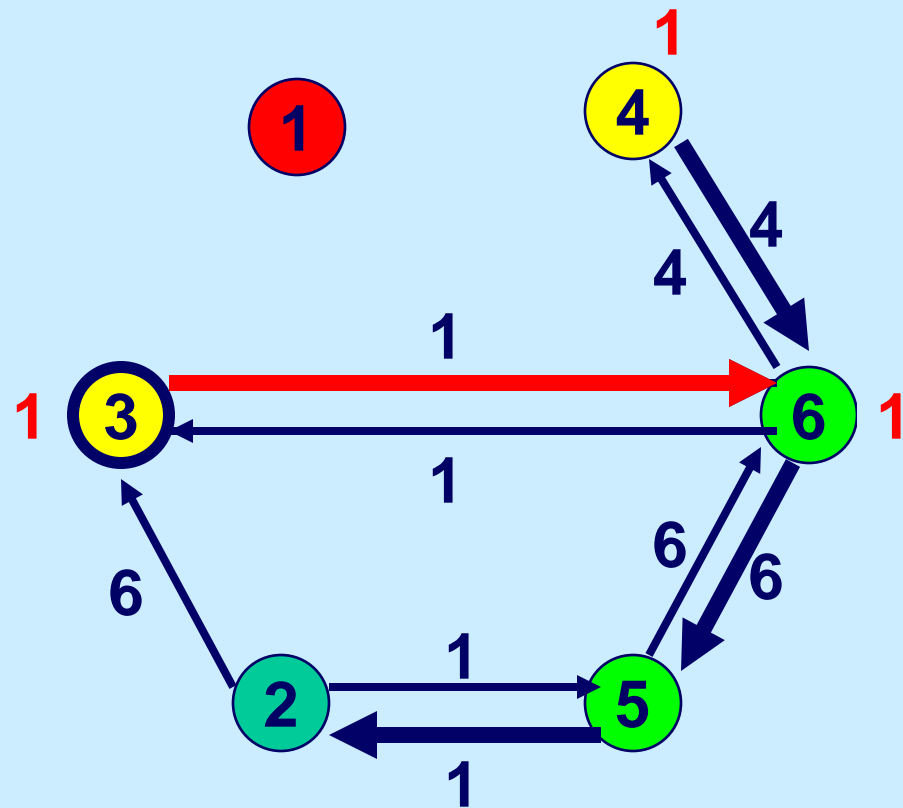
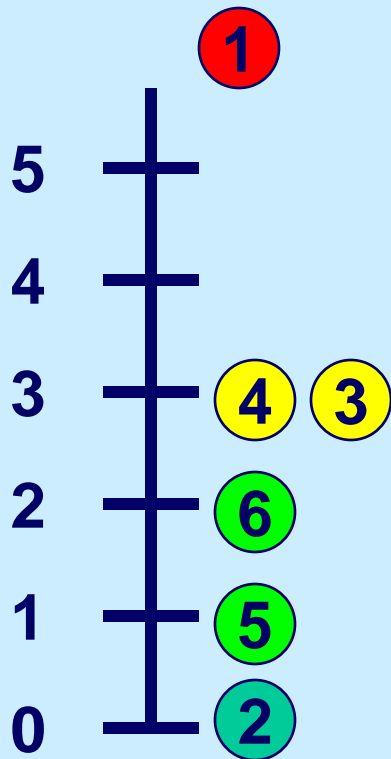
# Push/Relabel



Select an active node

Relabel. There are no admissible arcs.

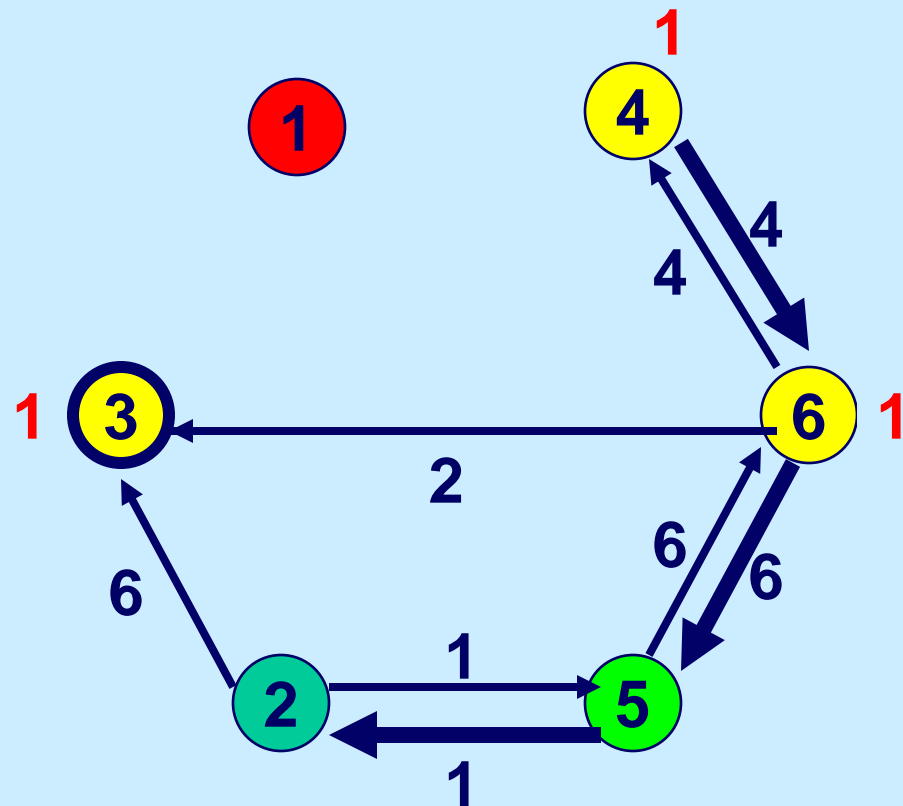
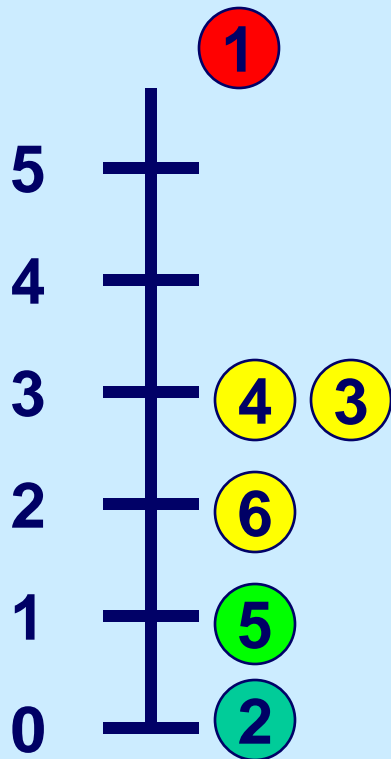
# Push/Relabel



Select an active node

Push from node 3.

# Push/Relabel



Select an active node

Relabel node 3. Rule: no empty levels permitted.

# Gaps

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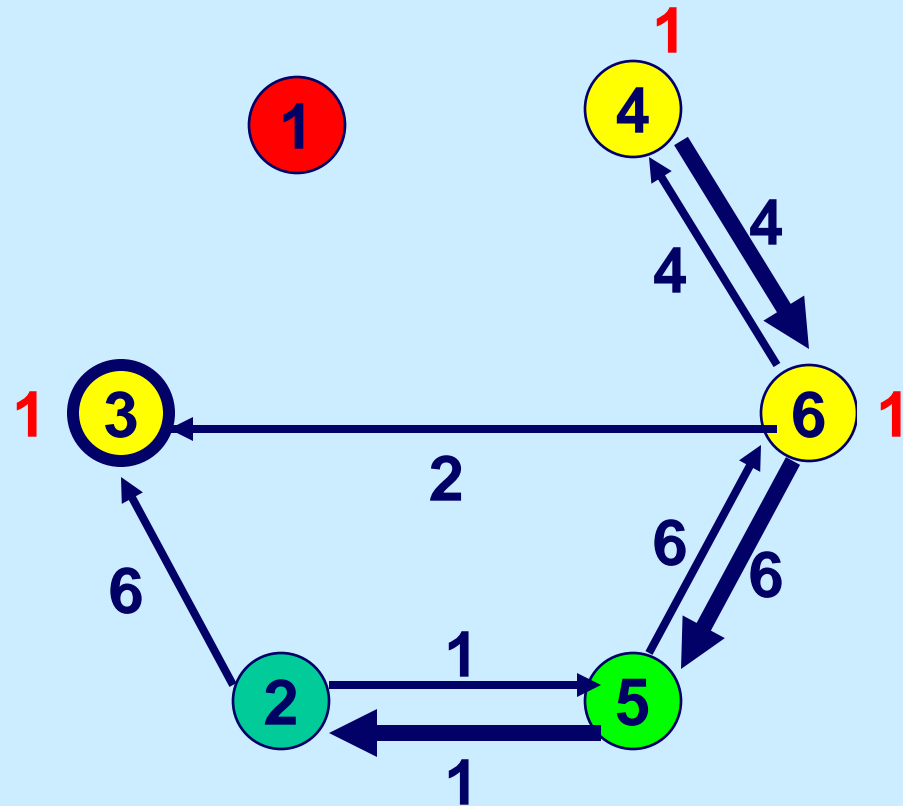
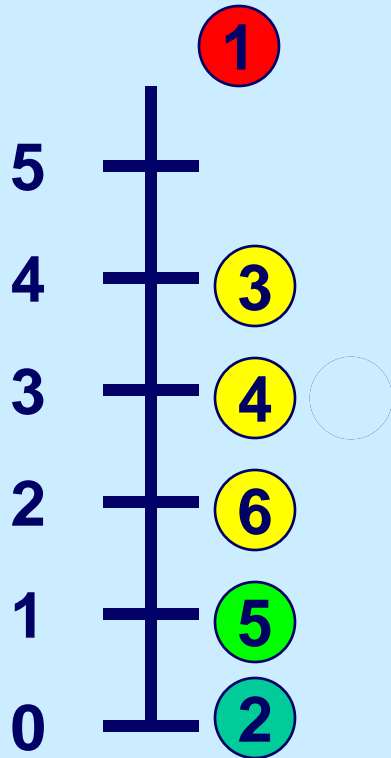
Let  $t$  denote the current sink node.

Let  $d_{\max}$  be the maximum distance label of a node other than the source node.

An *empty level* is a value  $k$  with  $d(t) < k < d_{\max}$  such that there is no node  $j$  with  $d(j) = k$ .

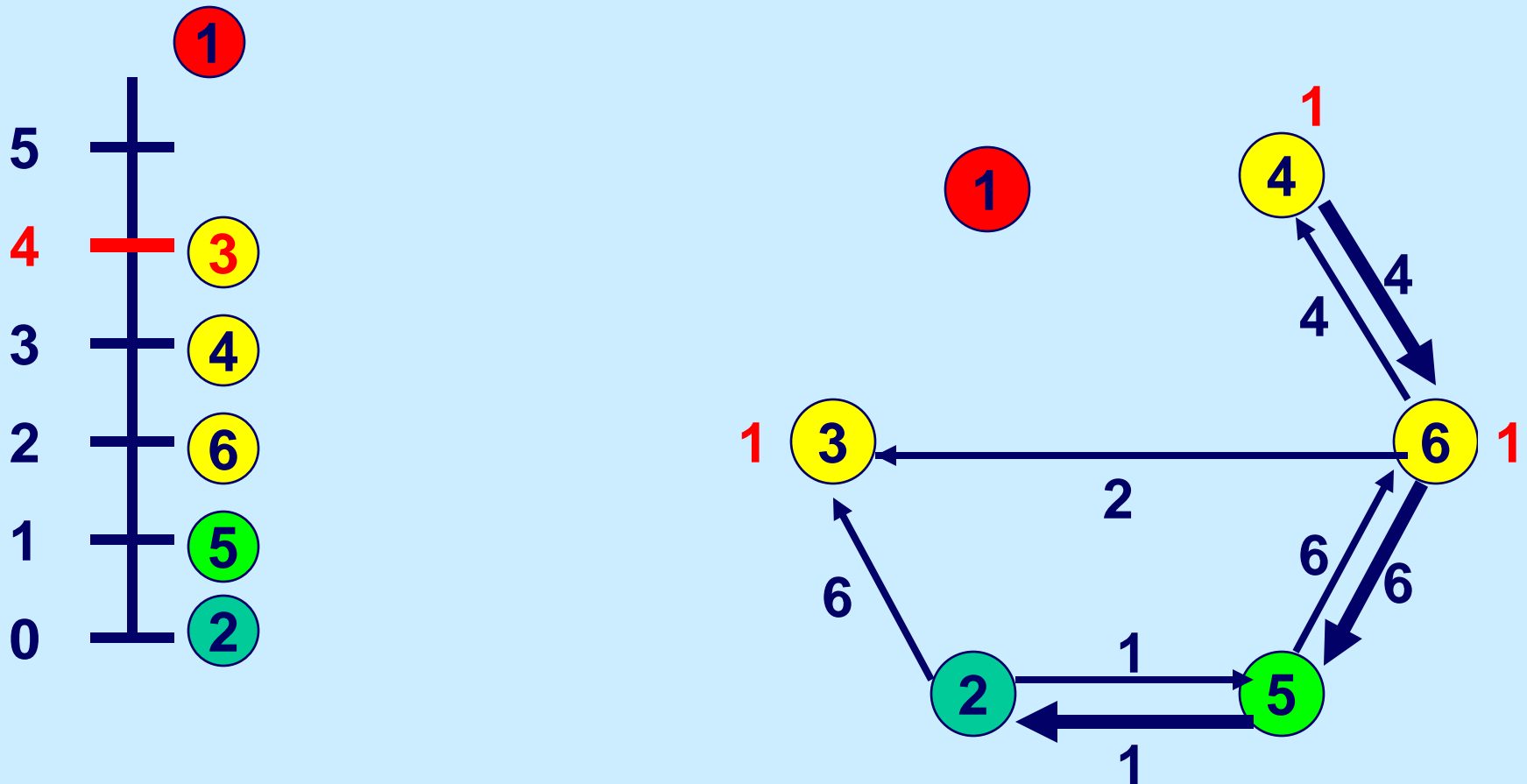
If we increase  $d(3)$  to 7, we create a gap. In such a case, we increase  $d(j)$  to  $d_{\max} + 1$ .

# Push/Relabel



Relabel node 3 so no empty level is created.

# Cut Level



A **cut level** is a level with exactly one node that needs to be relabeled.

There is no path in the residual network from a node in a cut level to a node below it.

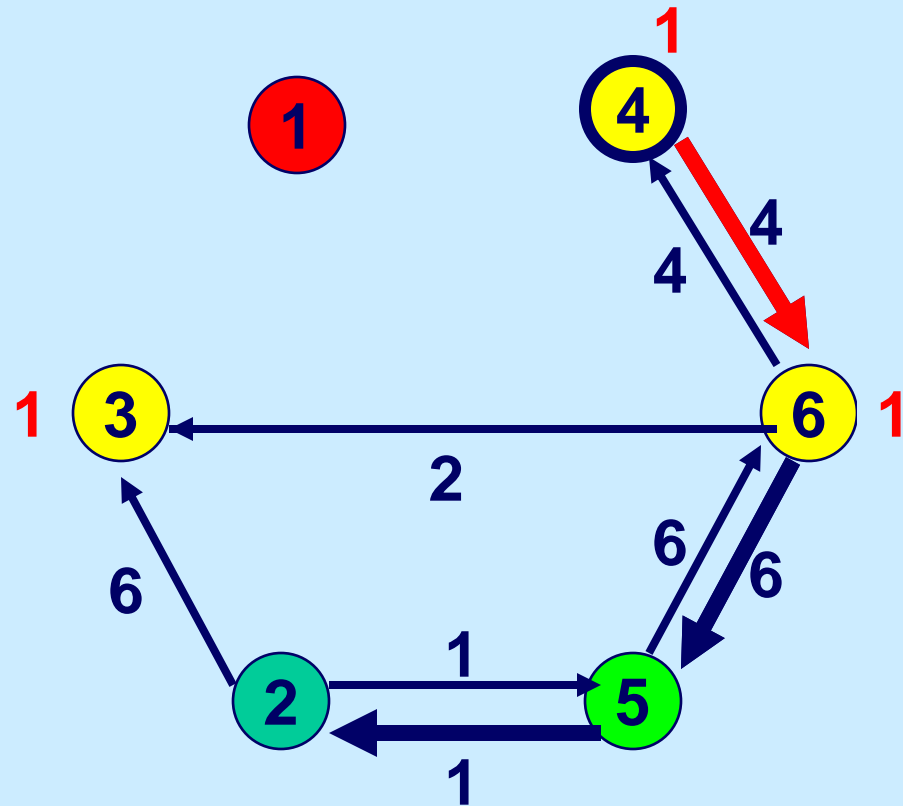
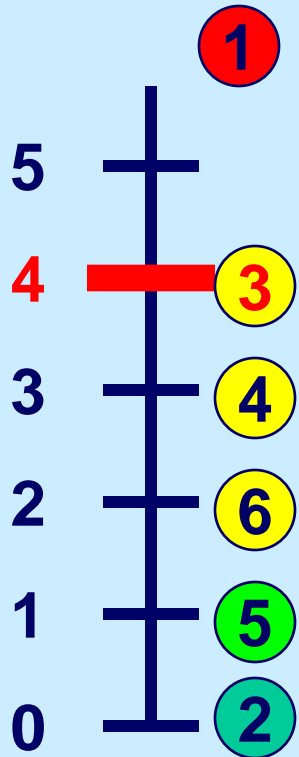
# Cut-level rule

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**Always select an active node that is below the lowest cut-level.**

**If each active node is at or above a cut level, then stop with the max preflow/ min cut.**

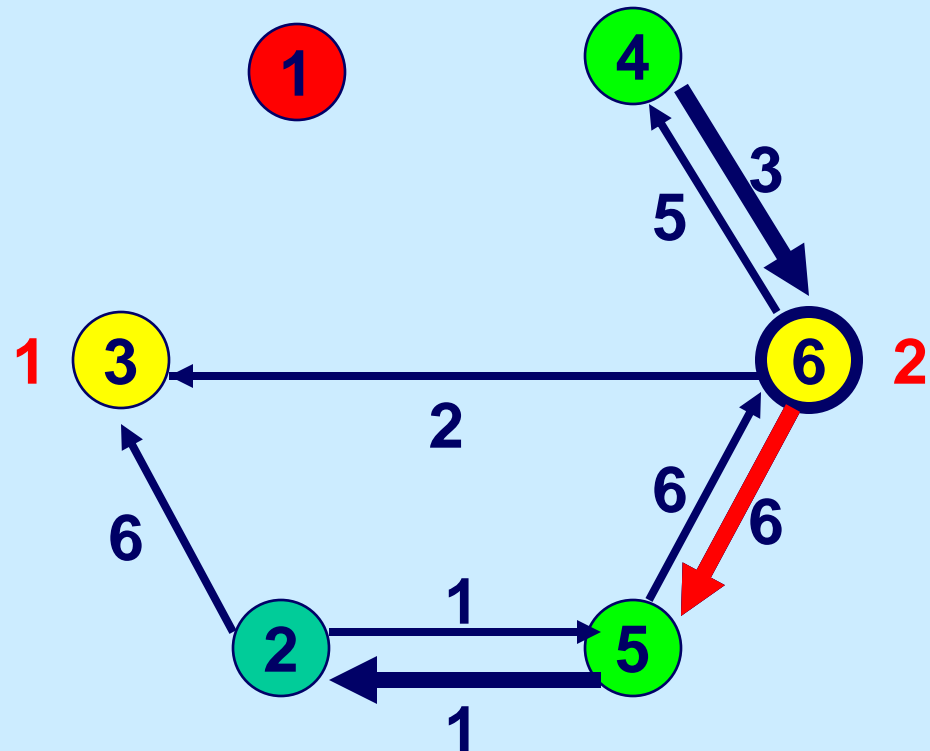
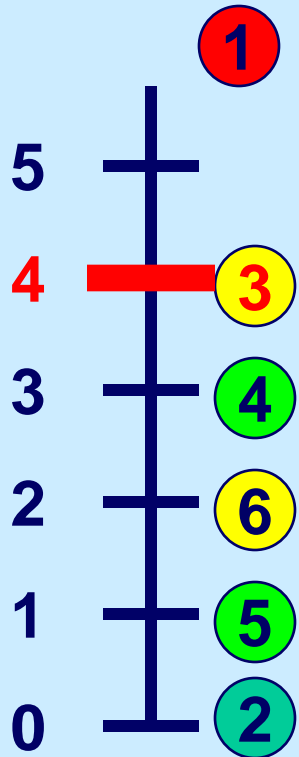
# Push/Relabel



Select an active node below the lowest cut level.

Push from node 4.

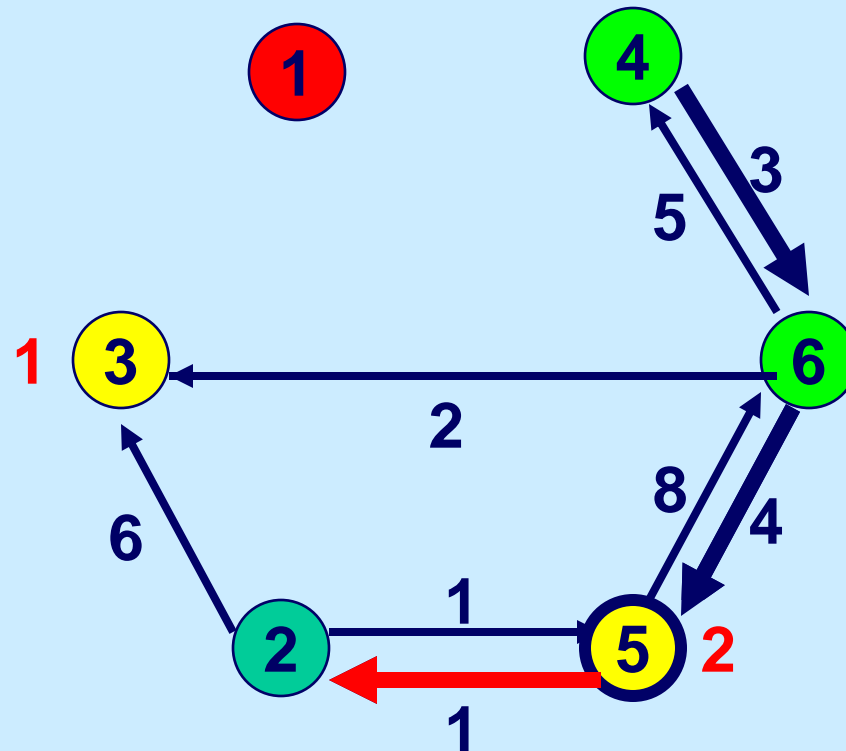
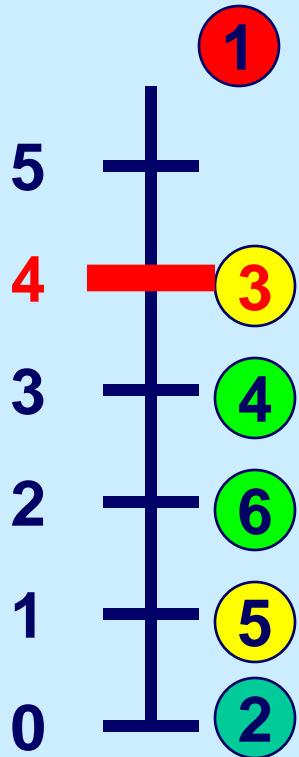
# Push/Relabel



Select an active node below the lowest cut level.

Push from node 6.

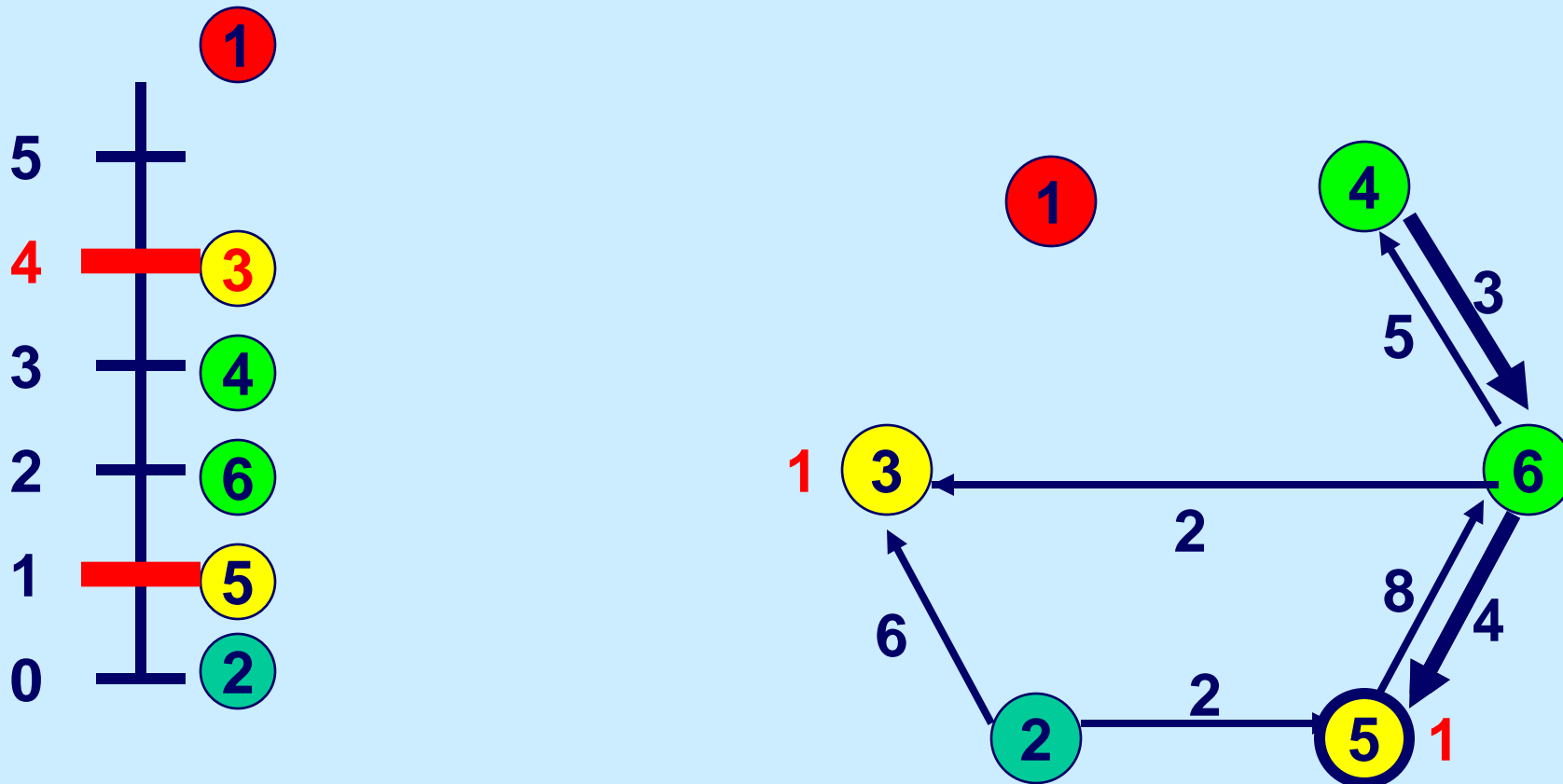
# Push/Relabel



Select an active node below the lowest cut level.

Push from node 5.

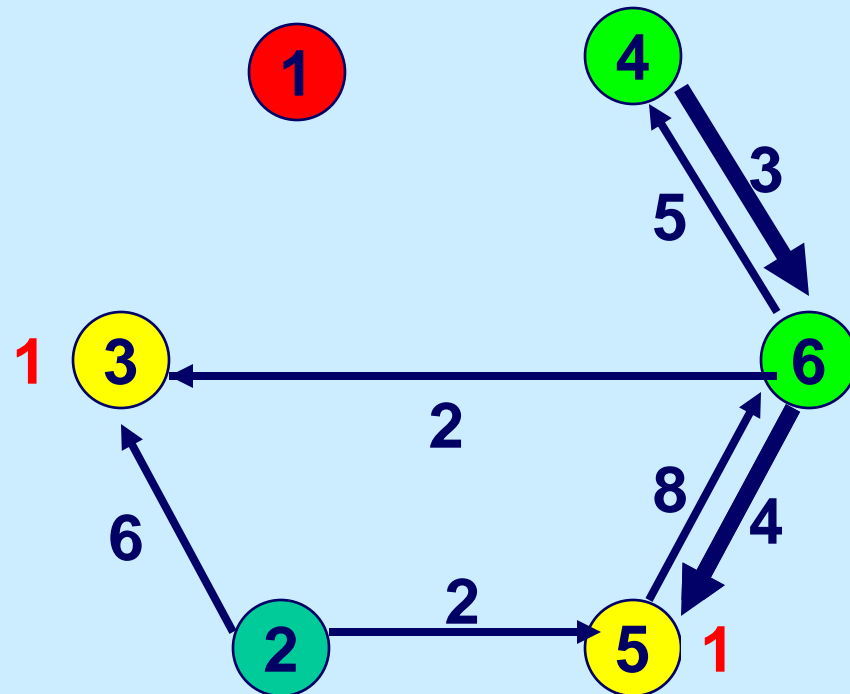
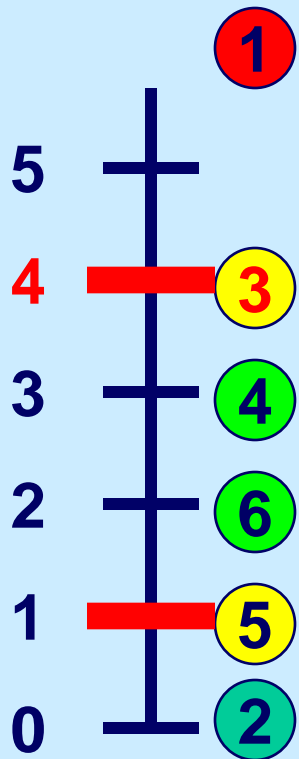
# Push/Relabel



Select an active node below the lowest cut level.

Relabel node 5 if it will not create an empty level.

# End of finding the first cut

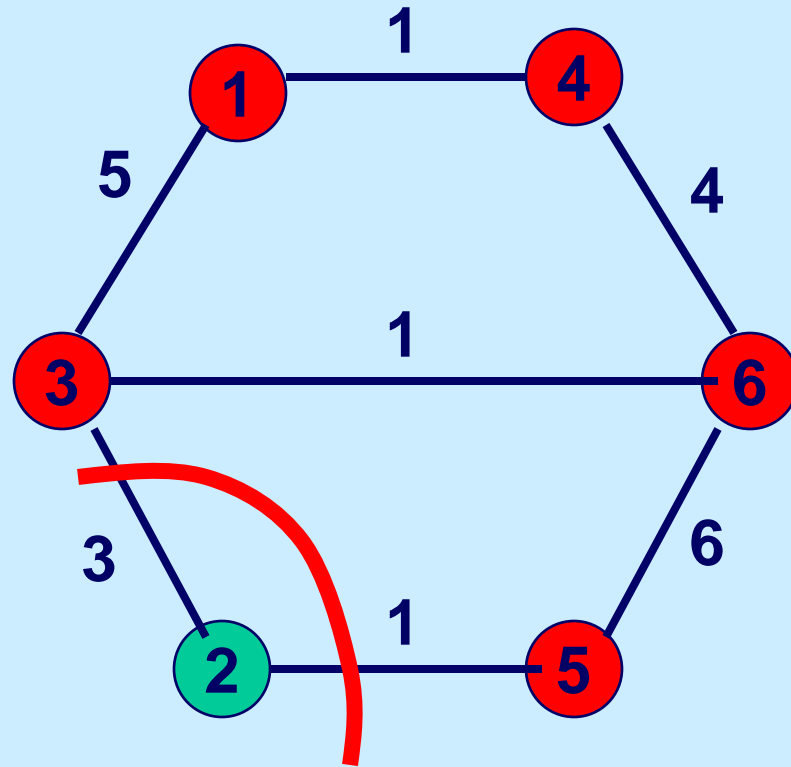
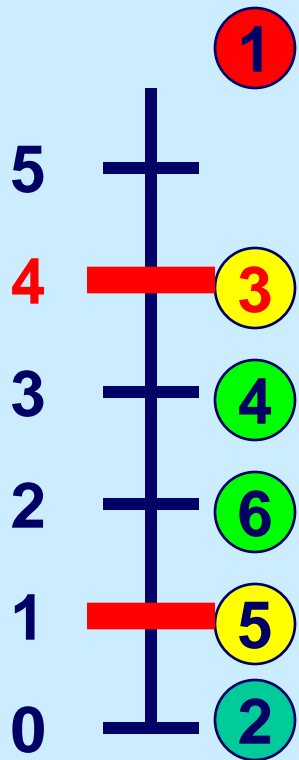


There is no active node below the lowest cut level.

The max preflow and min cut have been found.

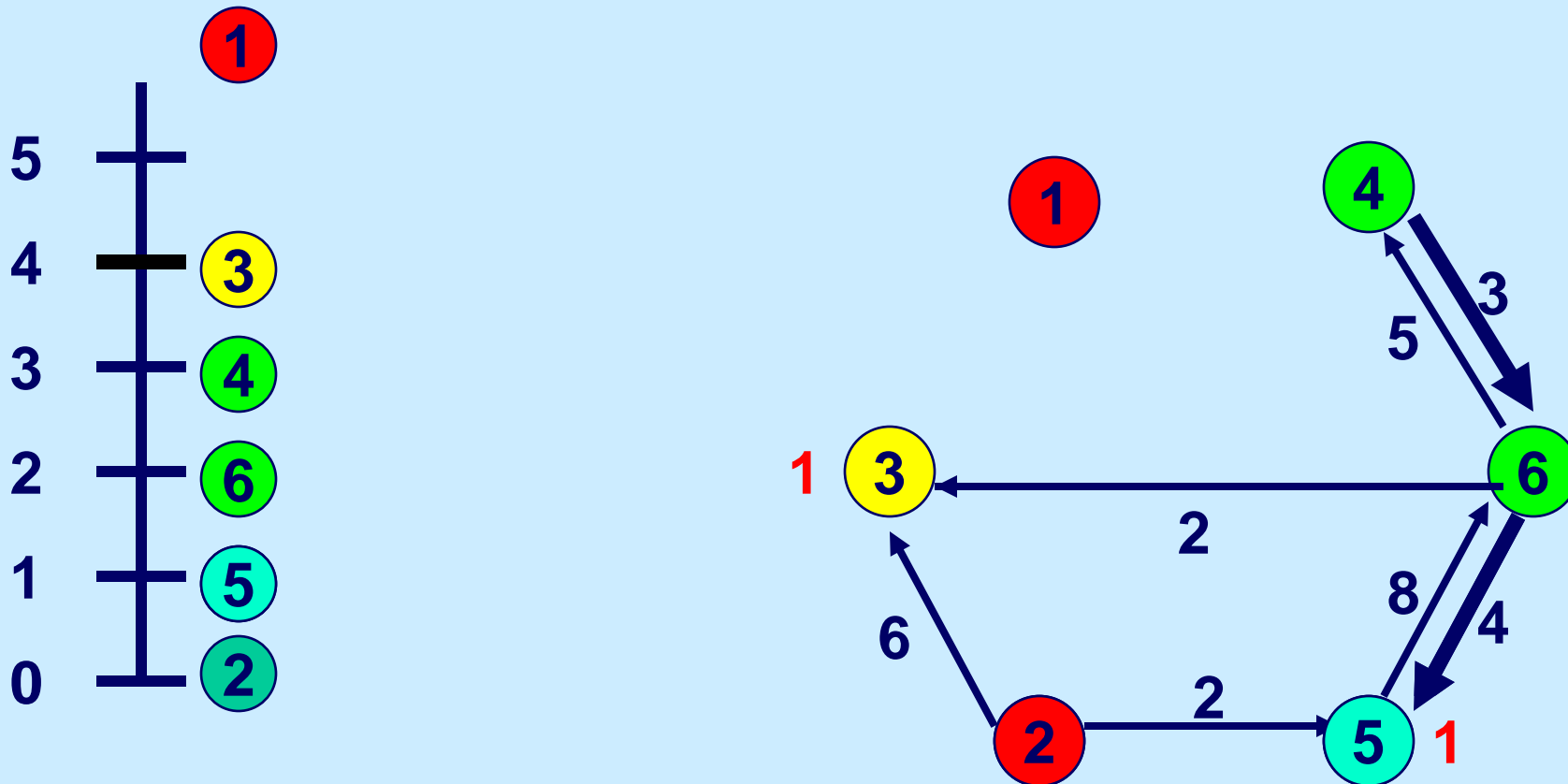
Let  $T$  be all nodes that can reach the sink node.

# End of finding the first cut



Here is the first minimum cut.

# Beginning of the second cut

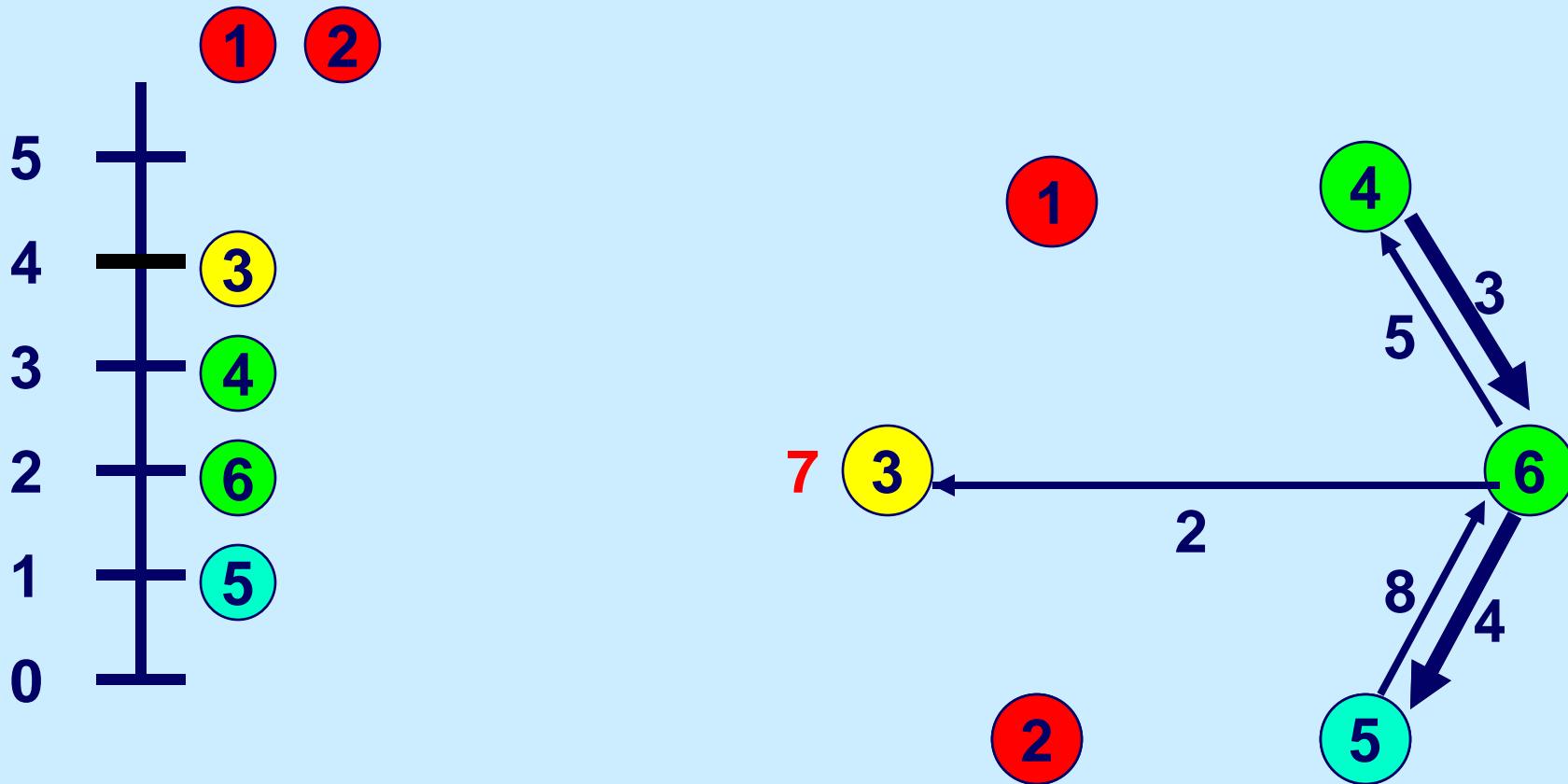


Choose a new sink node at the lowest level

Make node 2 a source node

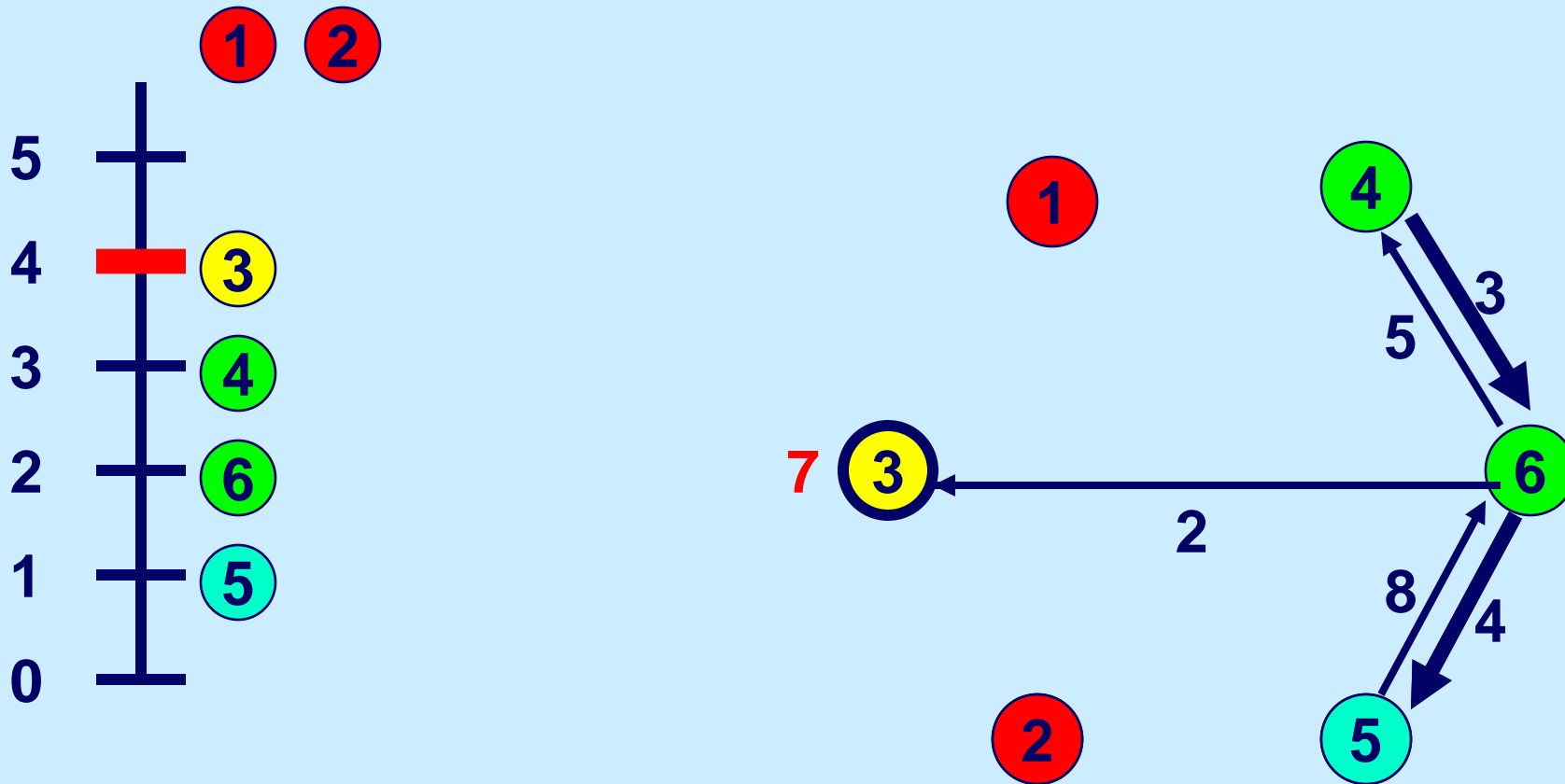
Saturate arcs out of node 2.

# Beginning of the second cut



We will not push from node 2 or into node 2 again.  
There is no need to physically merge nodes 1 and 2.

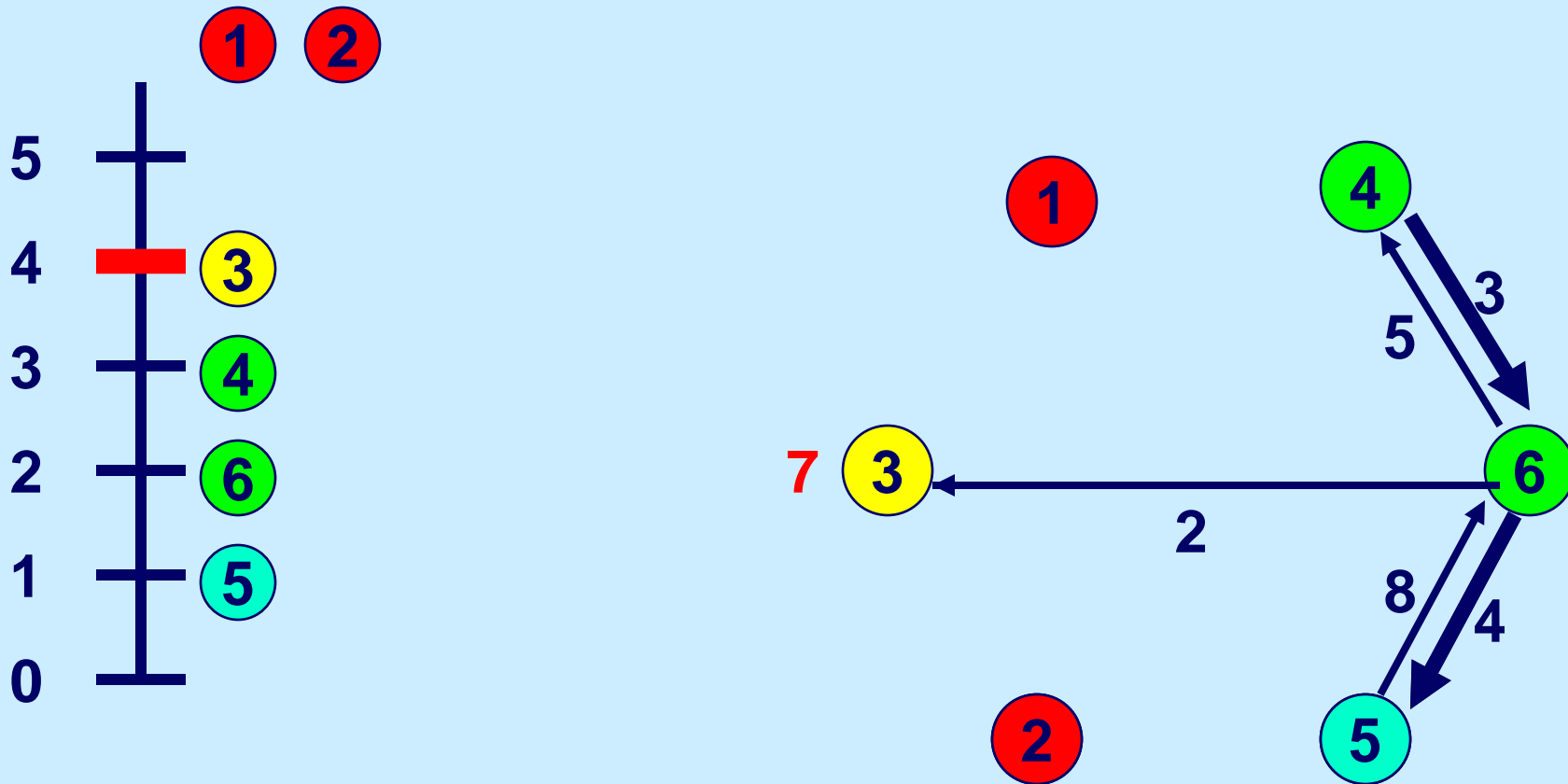
# Push/Relabel



Select an active node

Relabel node 3 if it will not create an empty level.

# Push/Relabel



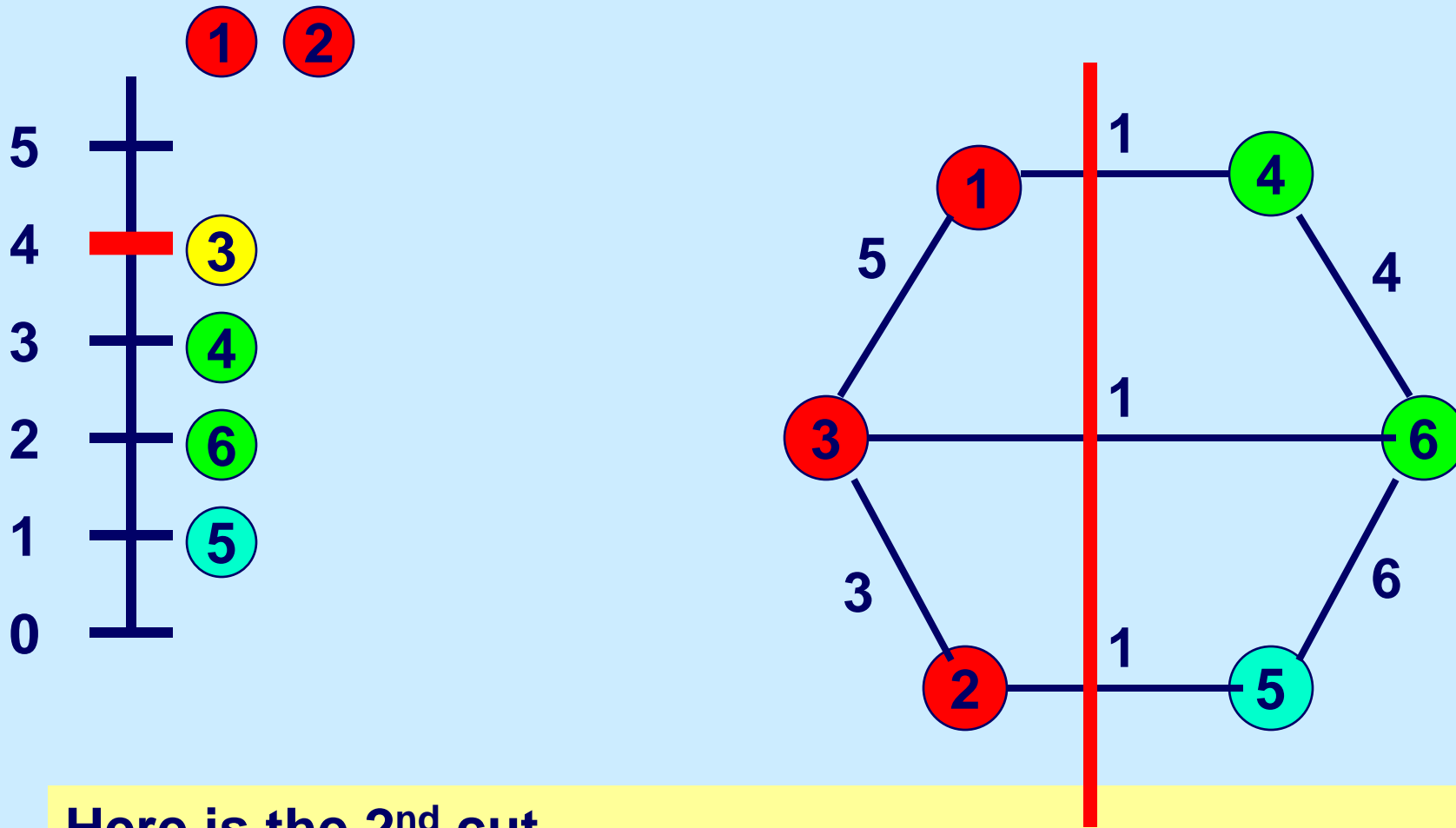
**Level 4 becomes a cut level**

**There is no active node below a cut level.**

**The second cut has been found.**

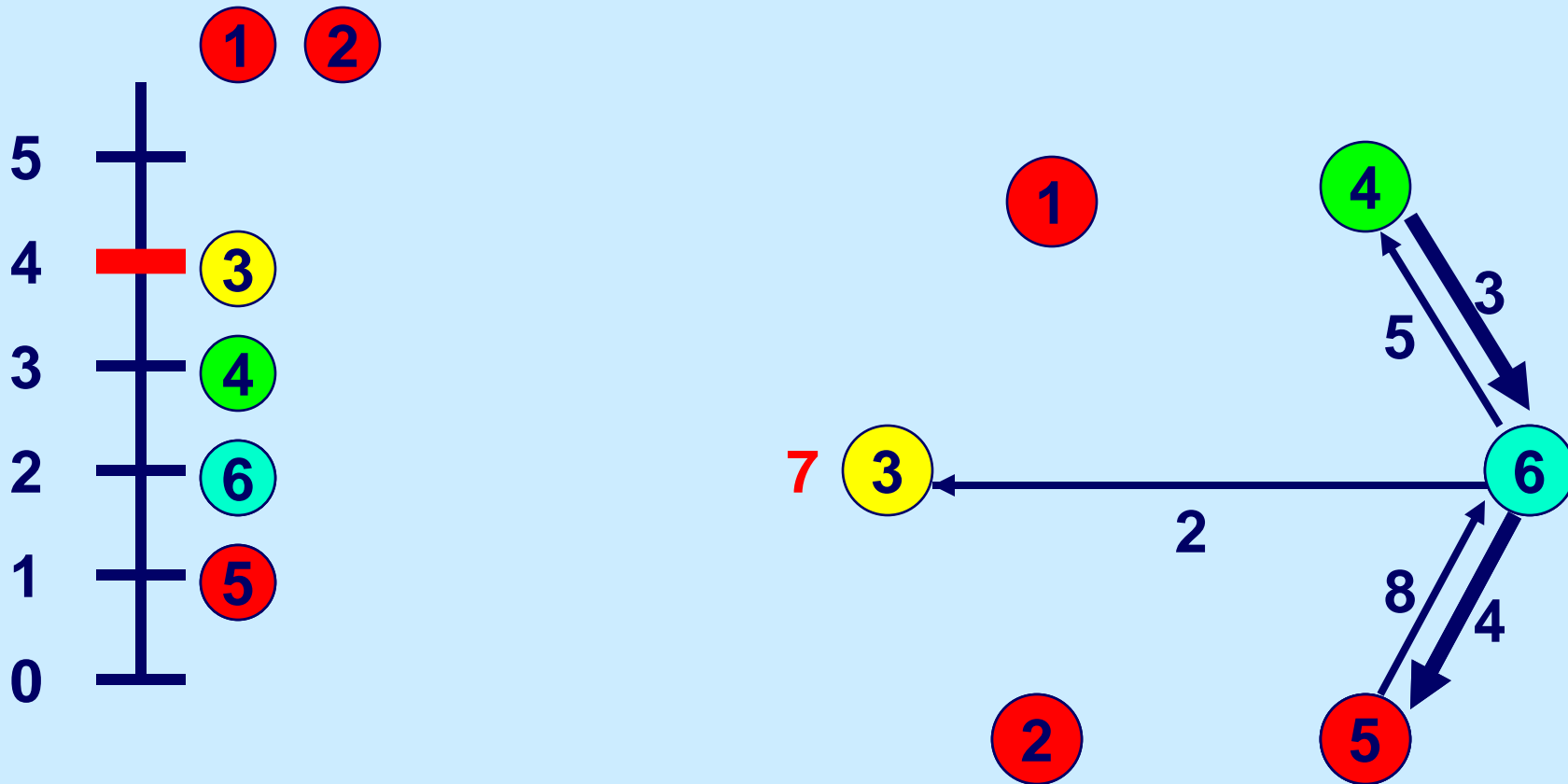
**Let  $T$  be all nodes that can reach the sink node.**

# Push/Relabel



Here is the 2<sup>nd</sup> cut.

## Starting the 3<sup>rd</sup> cut

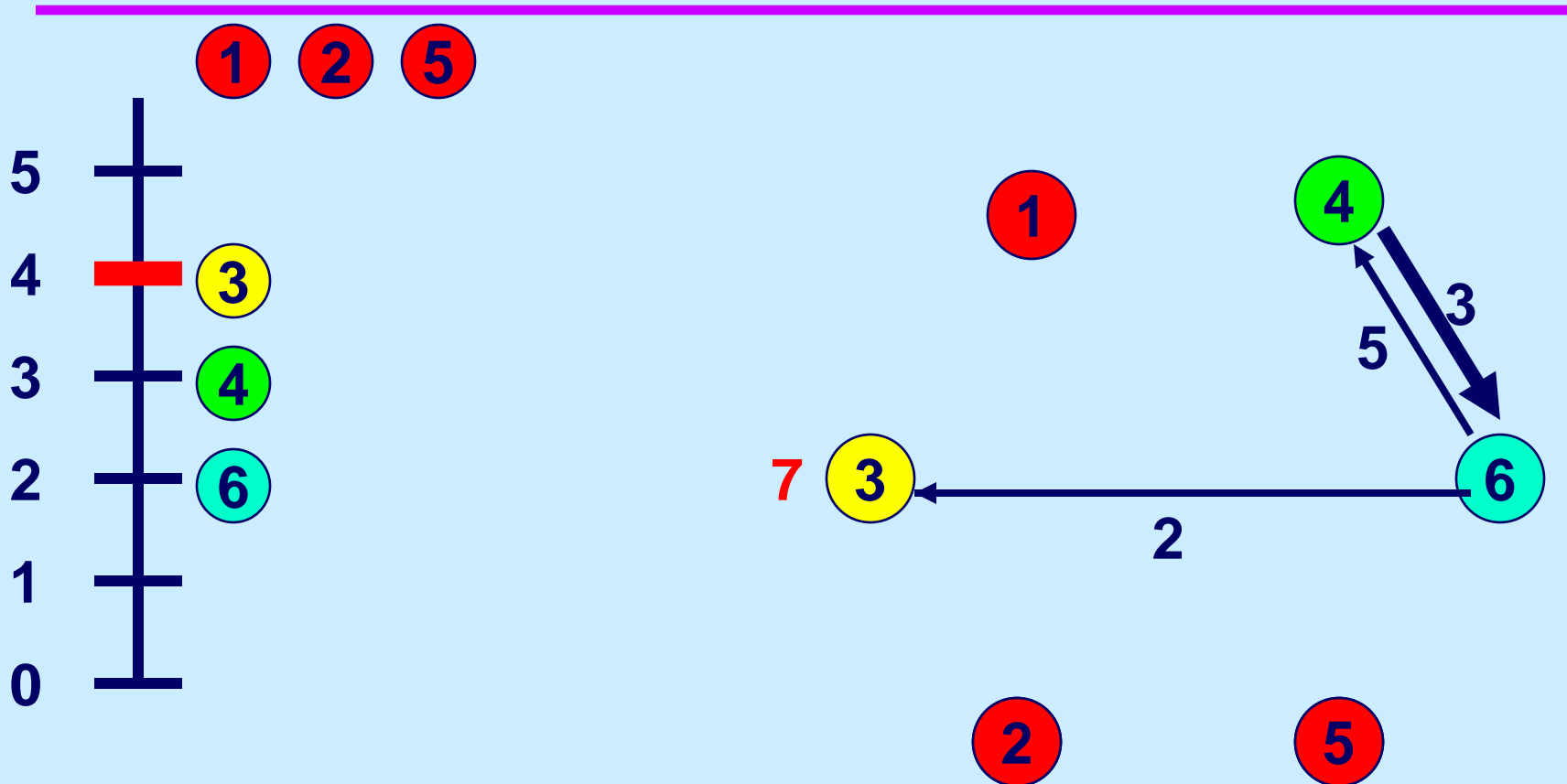


Make node 5 a source node

Node 6 becomes the new sink node

Saturate arcs out of node 5.

## Starting the 3<sup>rd</sup> cut



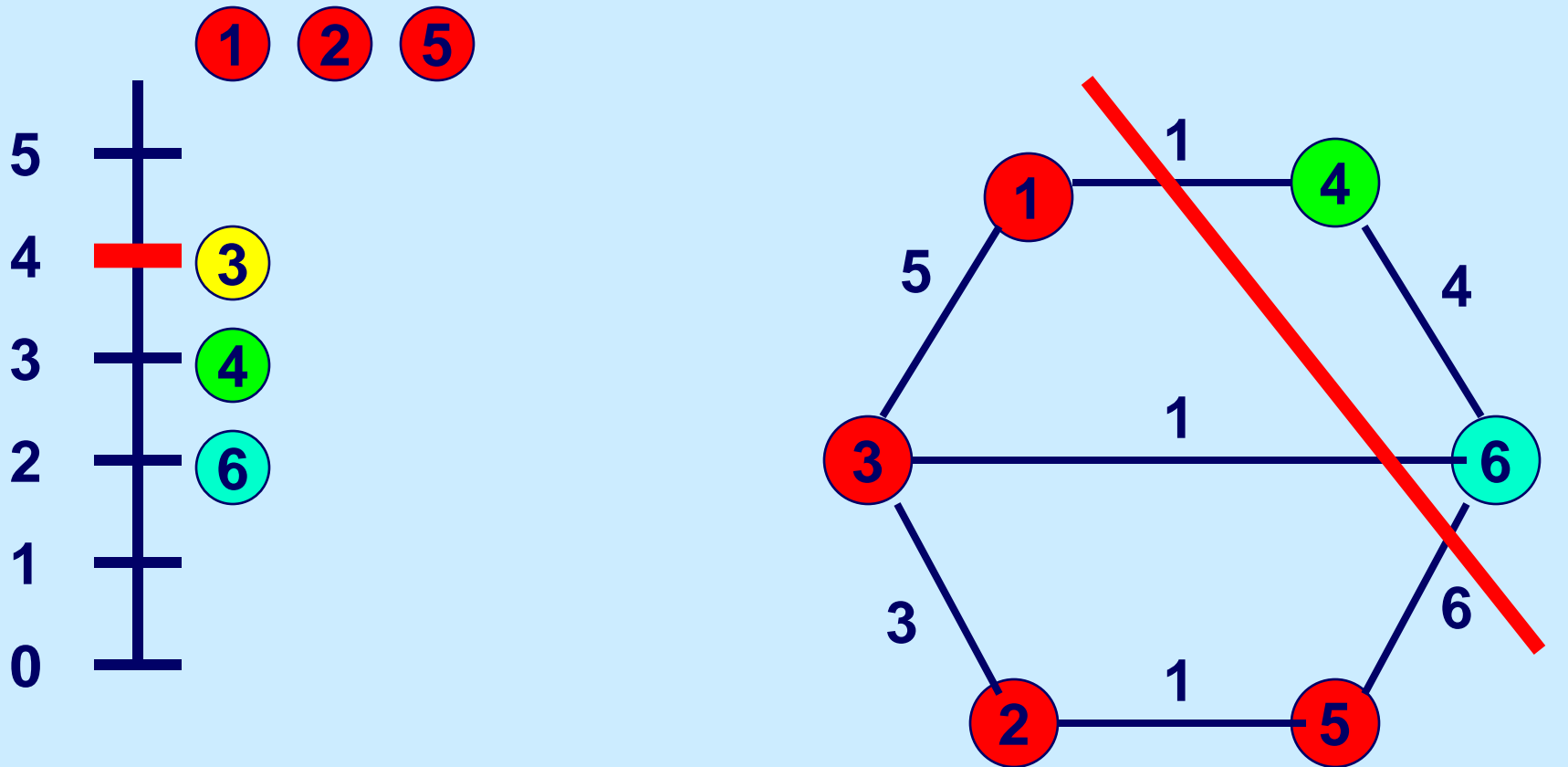
**Level 3 is still a cut level**

**There are still no active nodes below a cut level**

**We have found the 3<sup>rd</sup> cut**

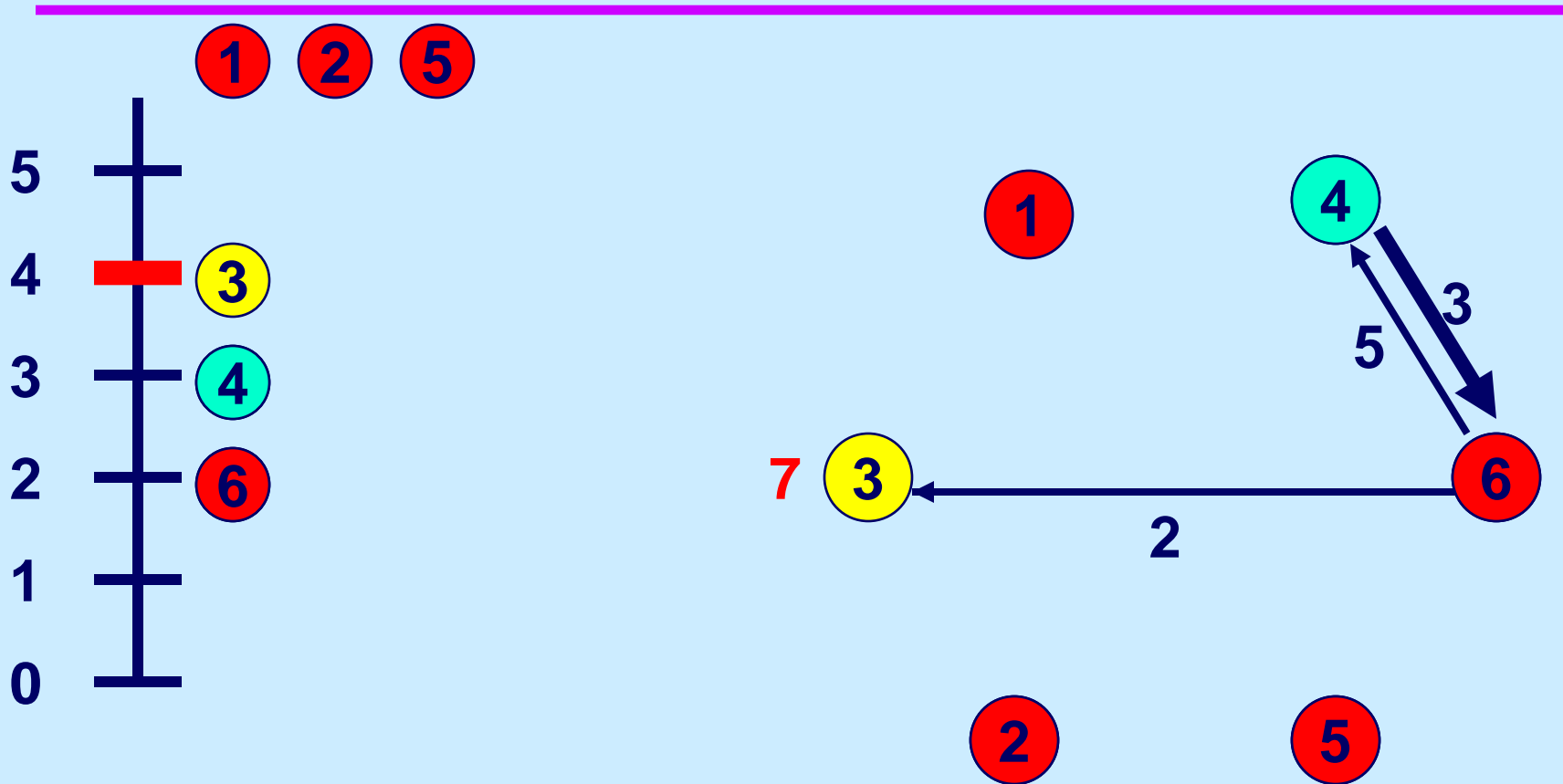
**Let T be all nodes that can reach the sink node.**

# The 3<sup>rd</sup> cut



The above cut is the best cut with 1, 2, 5 on one side and with node 6 on the other side.

## Starting the 4<sup>th</sup> cut

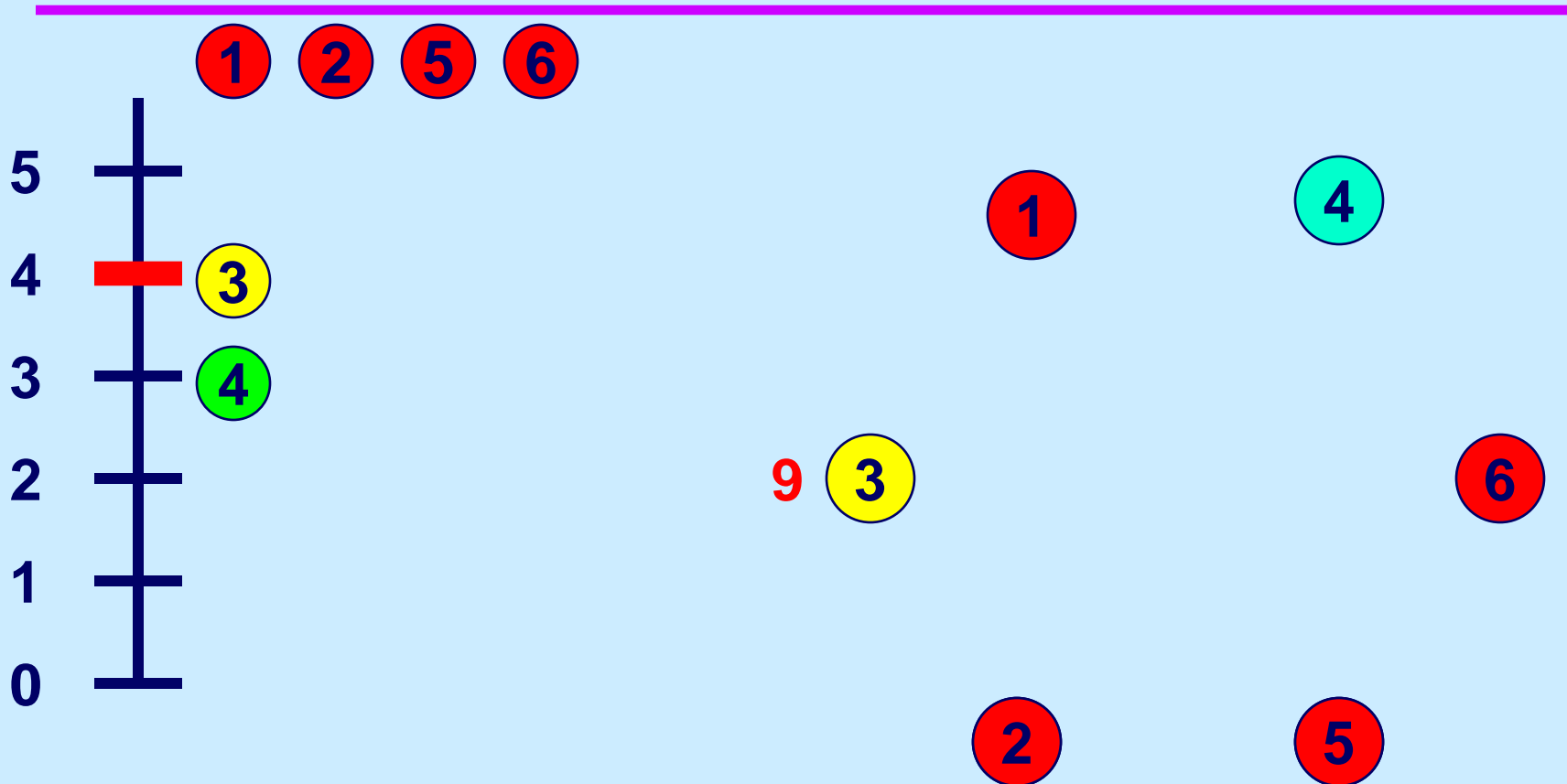


**Node 6 becomes a source node**

**Node 4 becomes the next source node**

**Saturate arcs out of node 6**

## We have found the 4<sup>th</sup> and 5<sup>th</sup> cuts

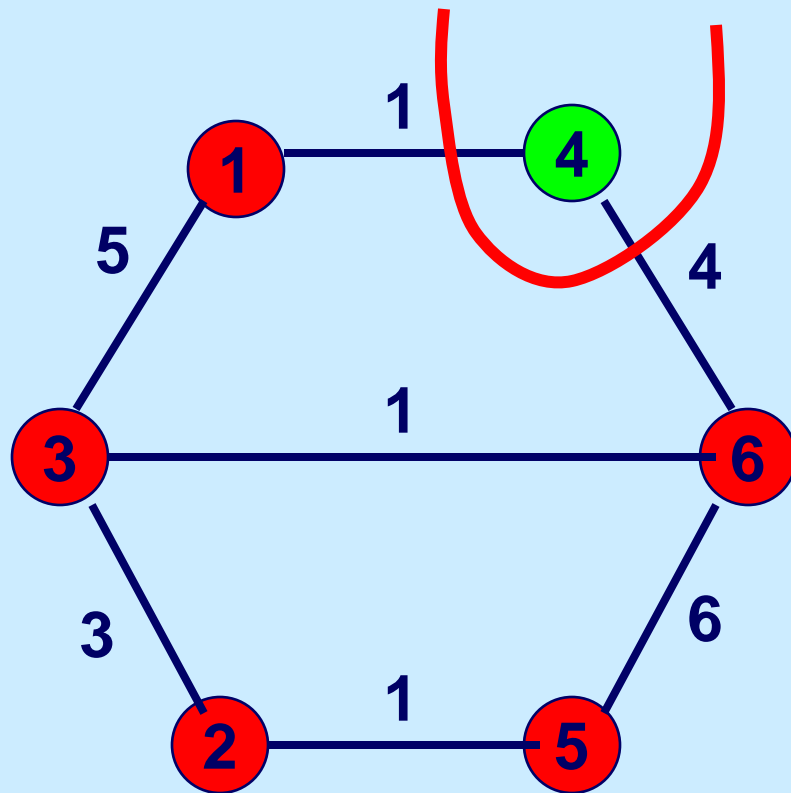


The only arcs in the network are out of source nodes

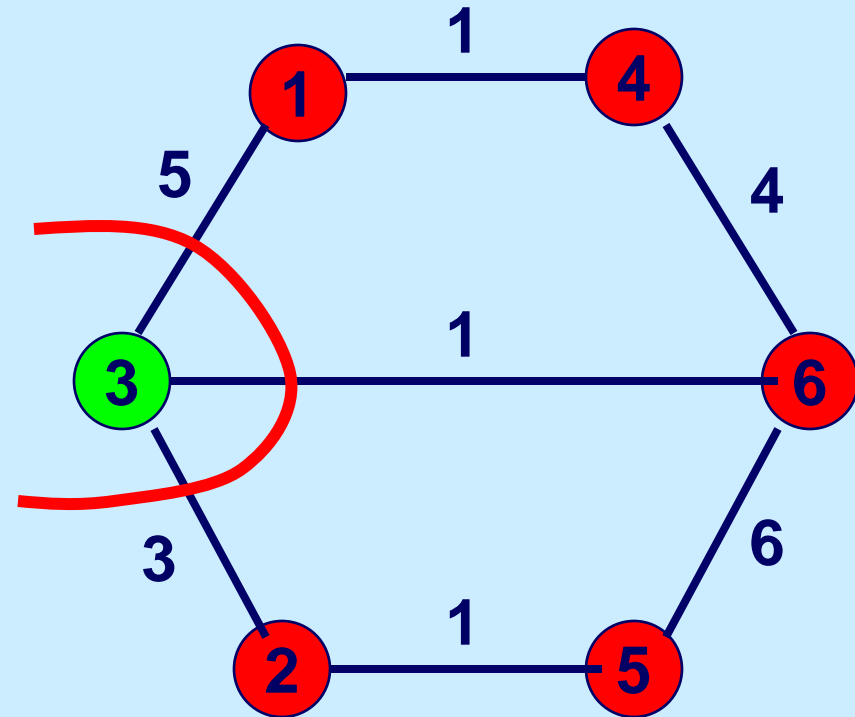
We can stop the algorithm and identify the remaining cuts.

# Here are cuts 4 and 5

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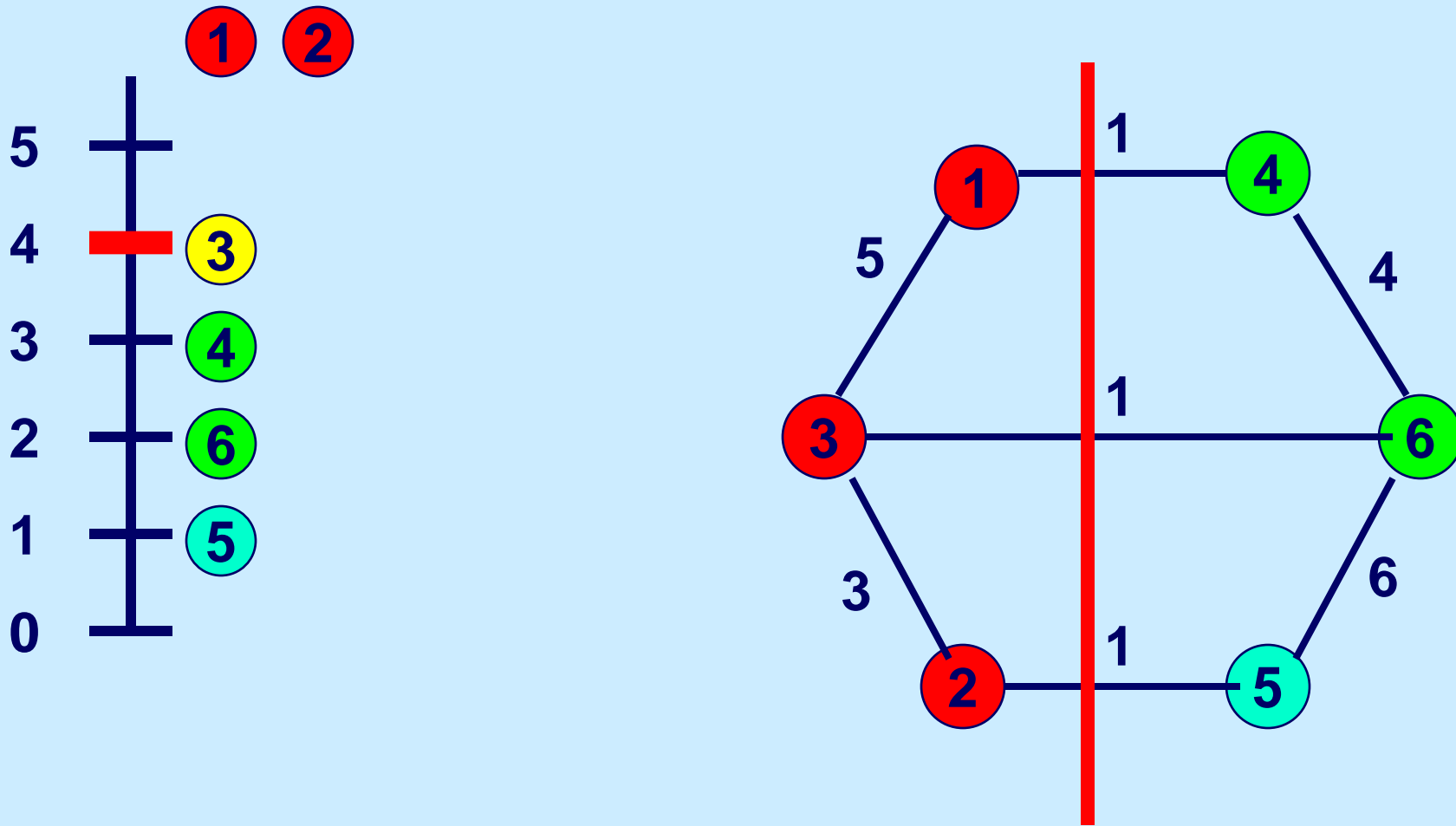


Cut 4



Cut 5

# The global min cut was cut number 2



The algorithm ends with the minimum cut with node 1 on the source side.