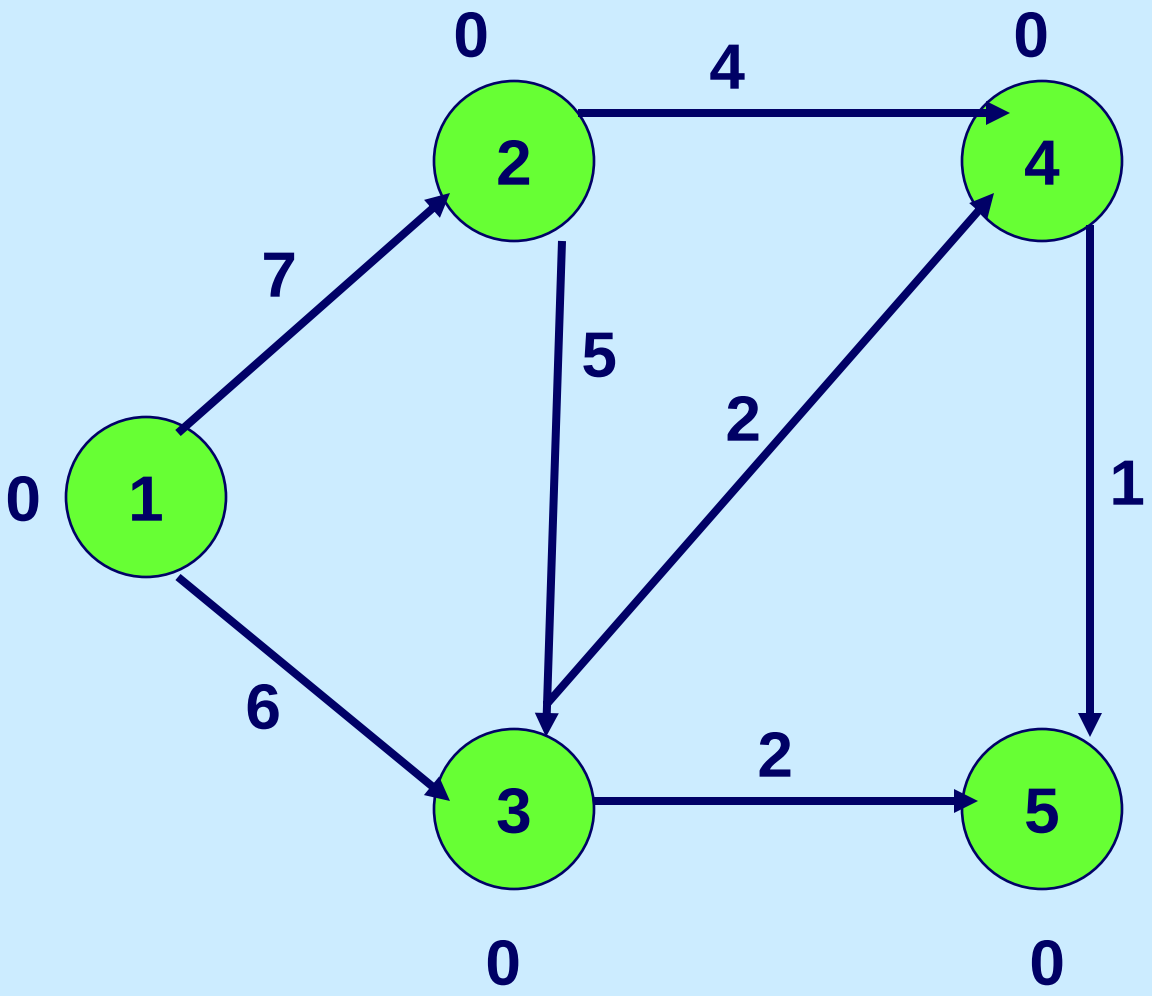
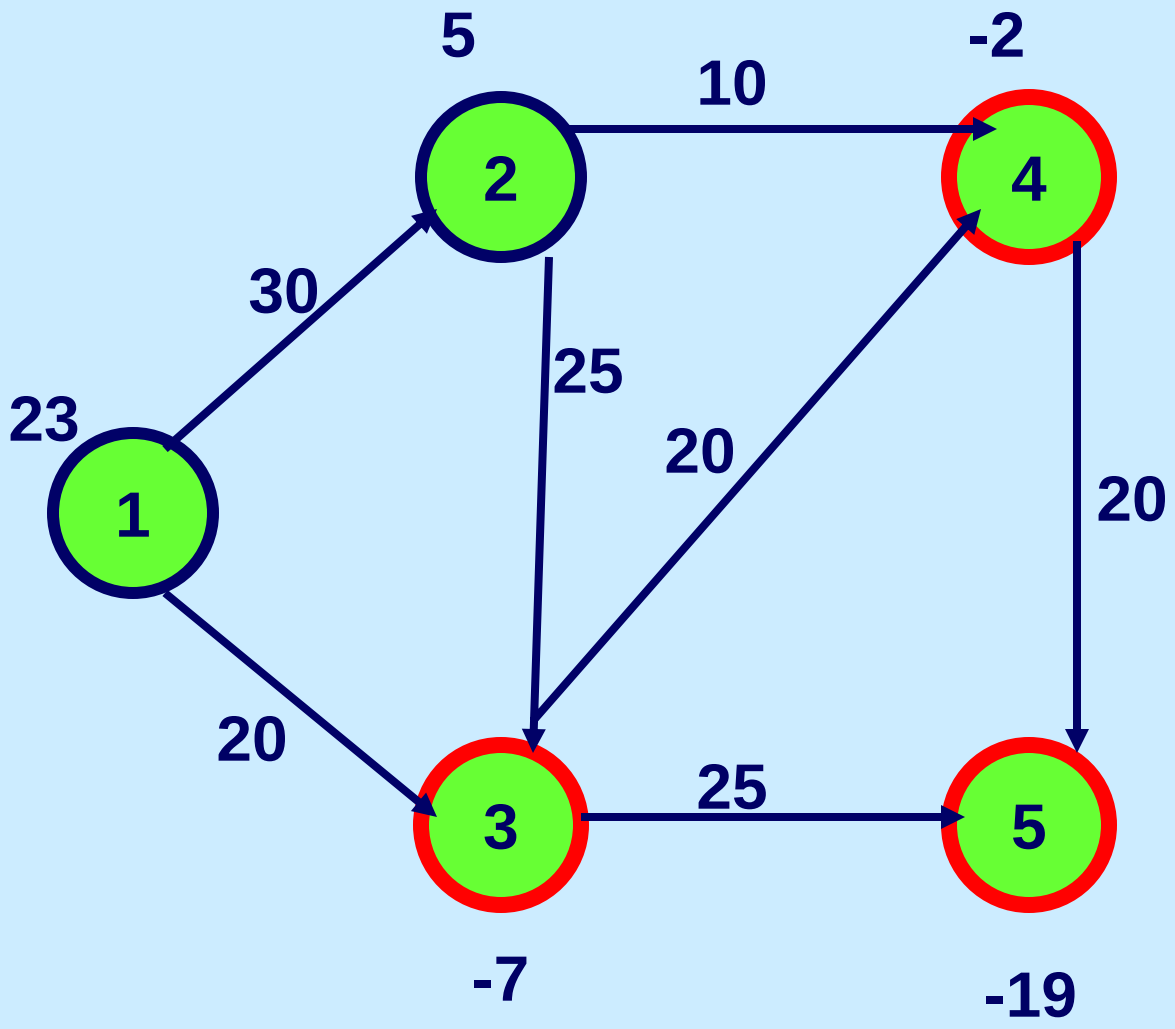
15.082J and 6.855J

Successive Shortest Path Algorithm

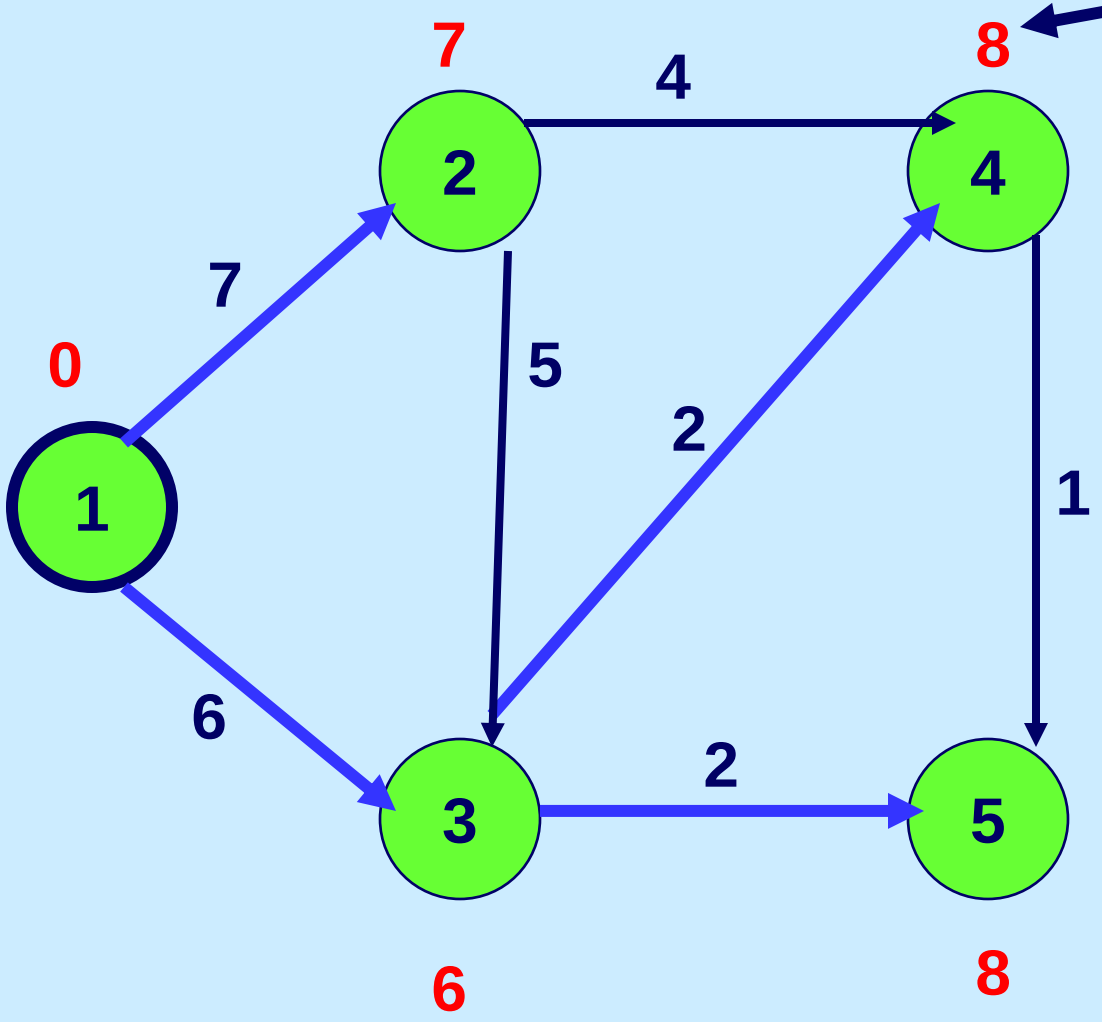
The Original Costs and Node Potentials



The Original Capacities and Supplies/Demands



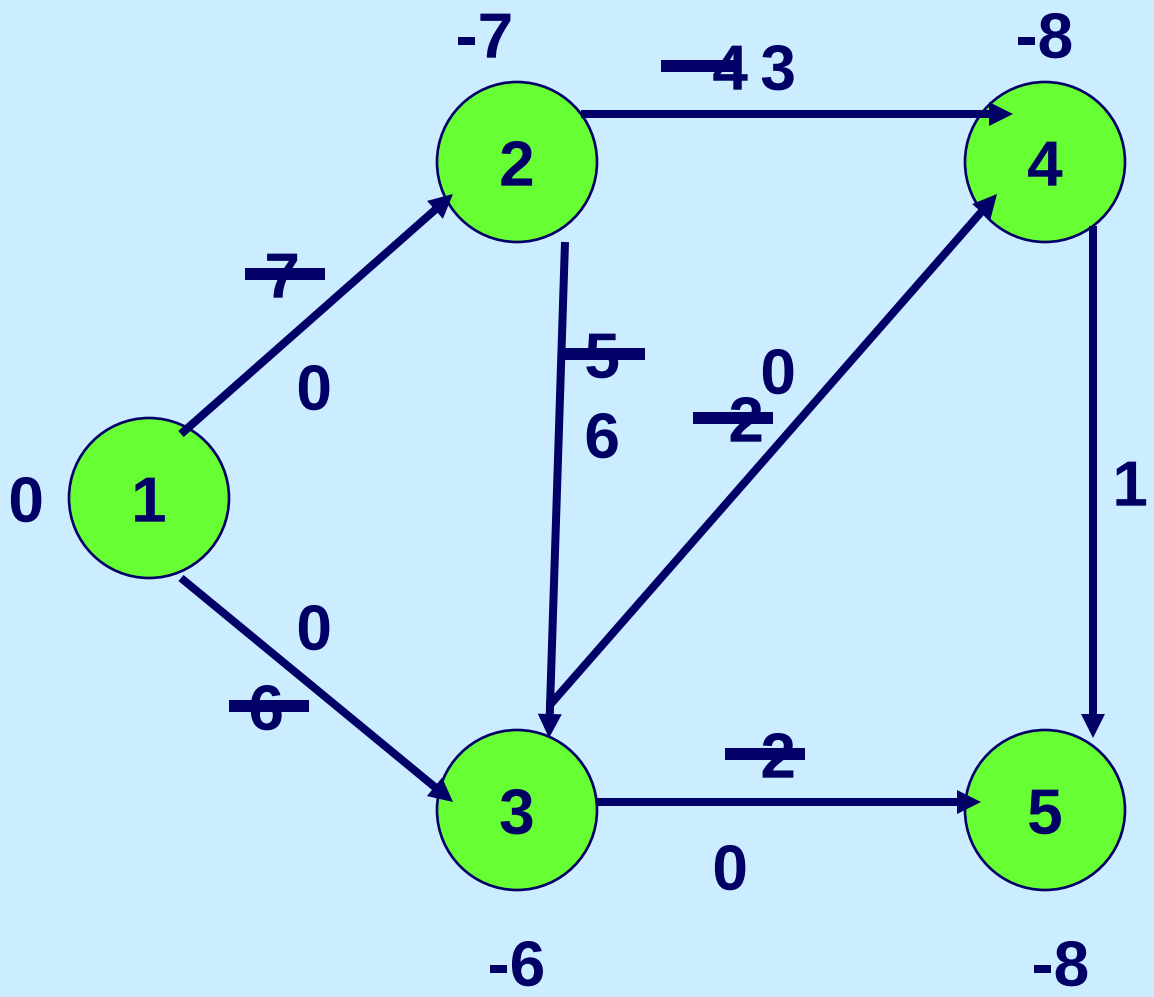
Select a supply node and find the shortest paths



shortest path distance

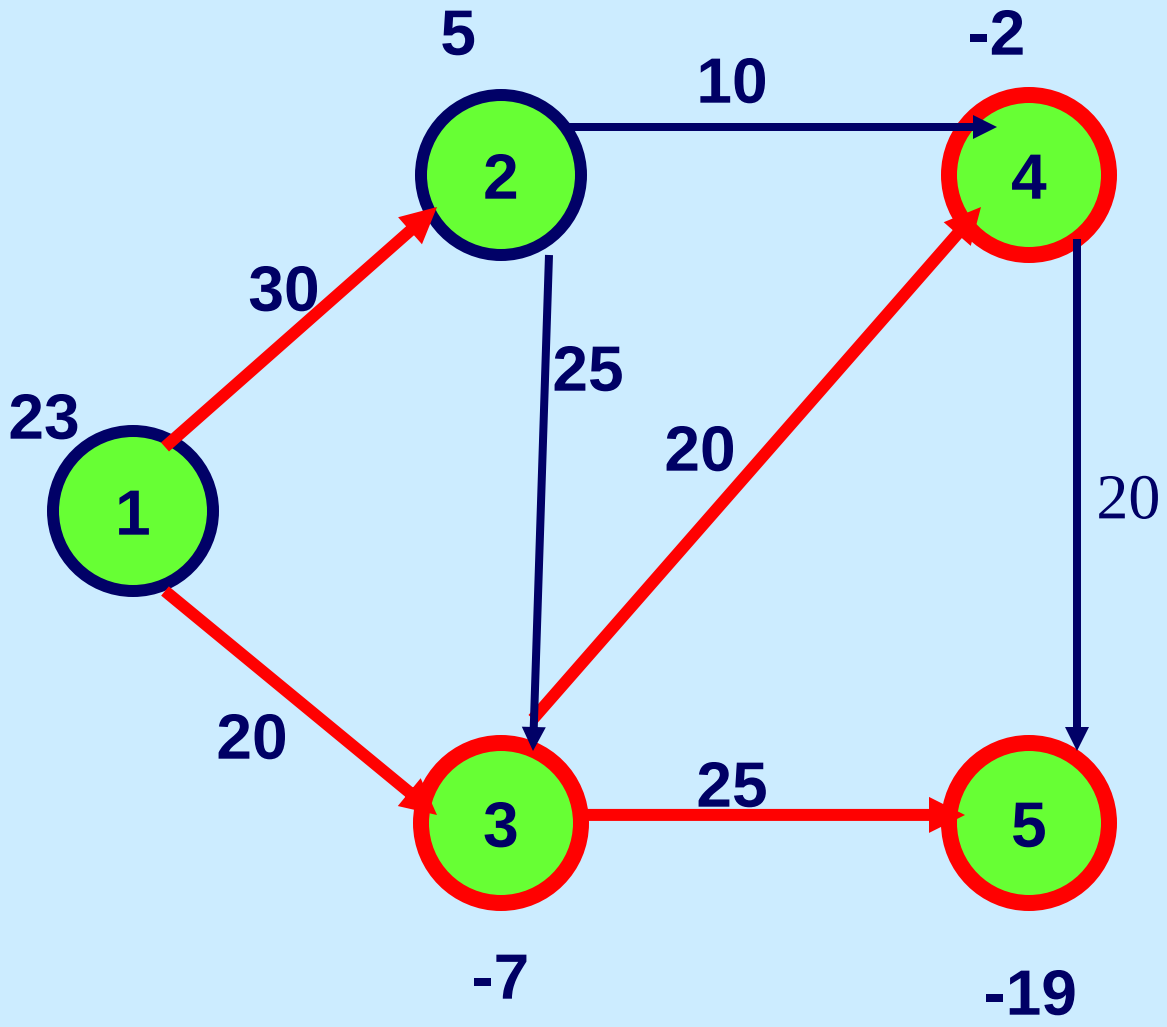
The shortest path tree is marked in bold and blue.

Update the Node Potentials and the Reduced Costs



Send Flow From a Supply Node to a Demand Node Along Shortest Paths

(along arcs with reduced costs of 0)

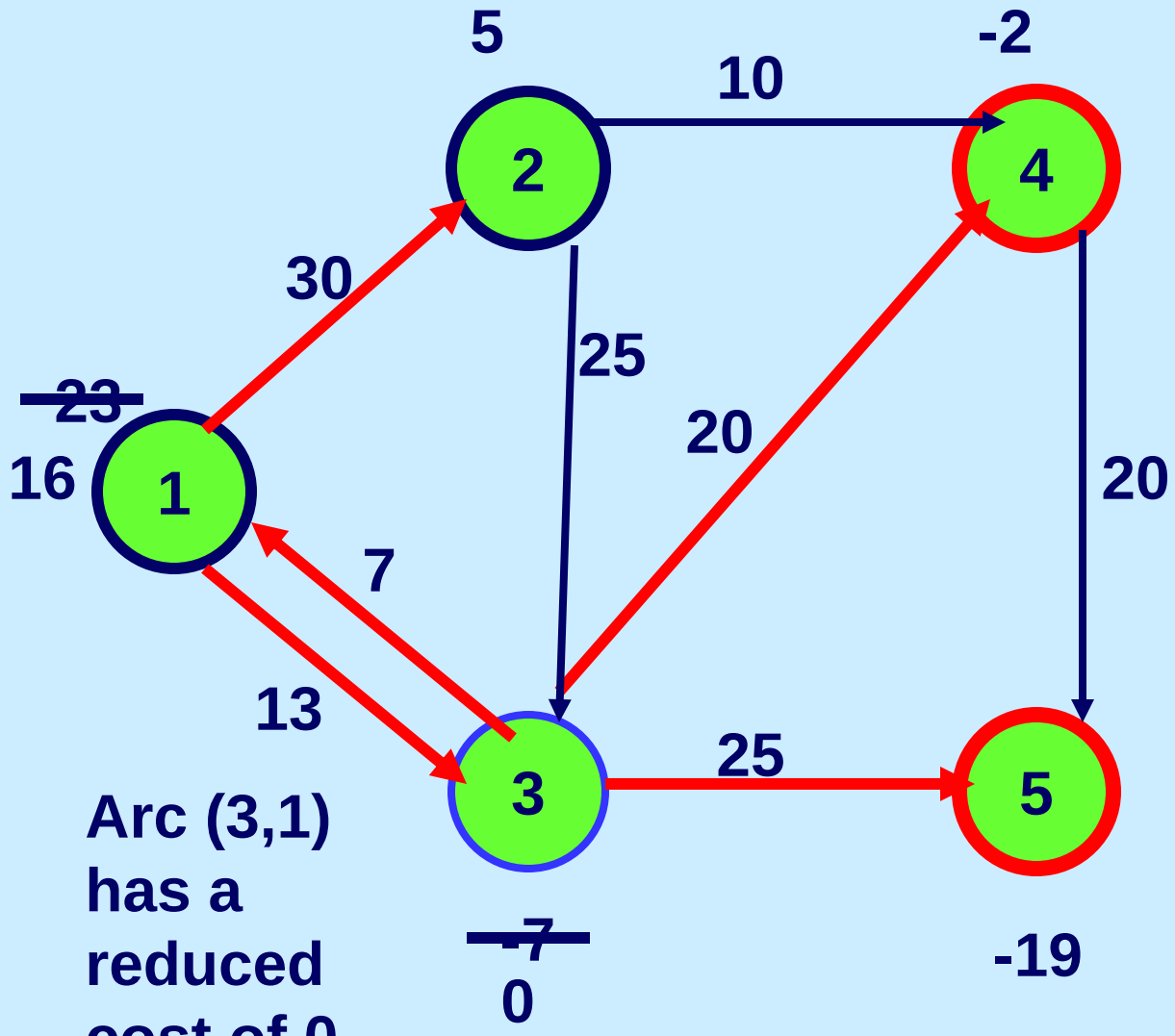


Arc numbers are residual capacities.

Red arcs have a reduced cost of 0

send 7 units from node 1 to node 3

Update the Residual Network



Arc (3,1) has a reduced cost of 0

If an arc is added to G(x), then it has a reduced cost of 0, and it is red.

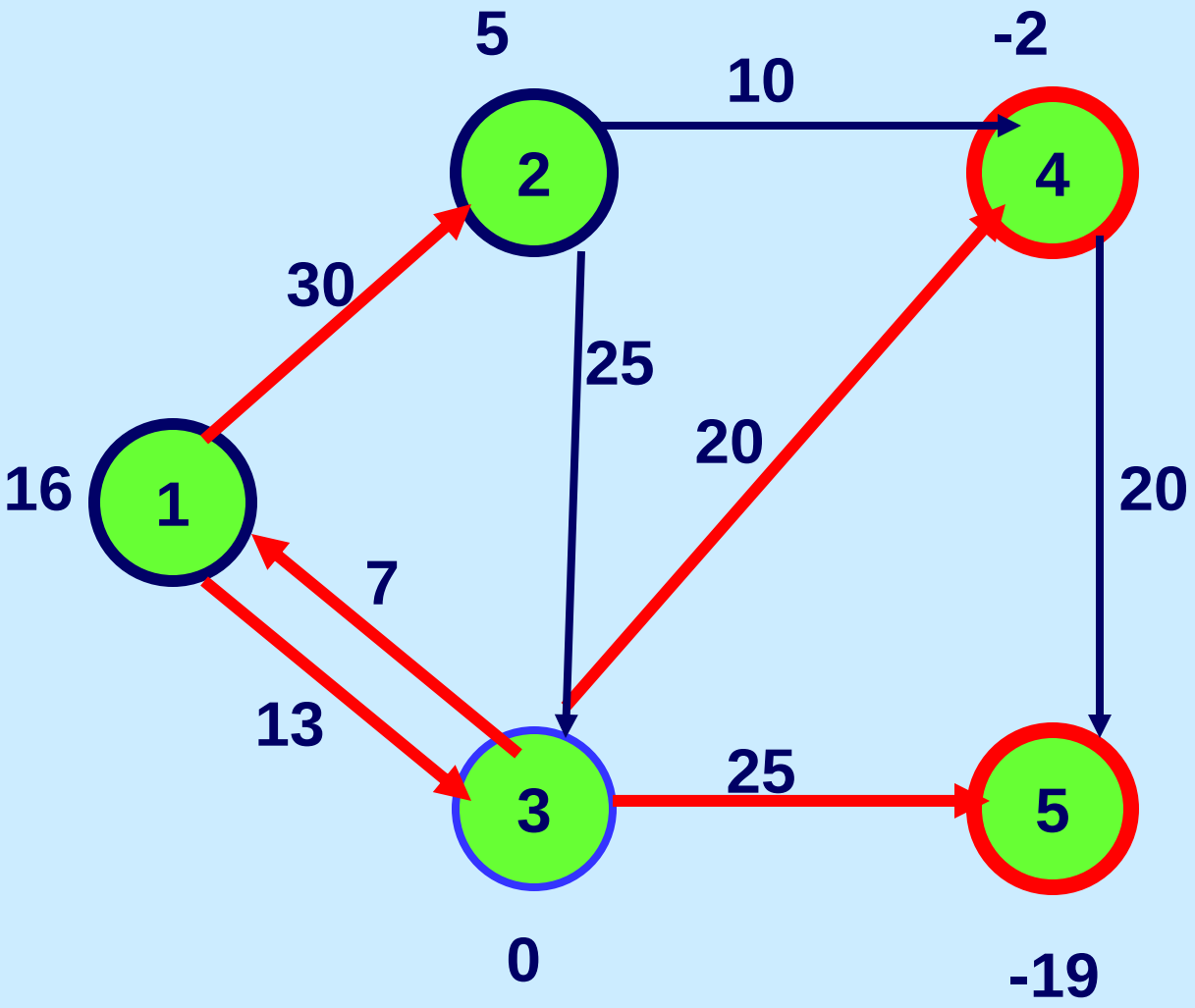
Arcs in the residual network will always have a non-negative reduced cost

A comment

At this point, one would choose a source node, and then find the shortest path from the source node to all other nodes, and then update the residual network.

However, there are still paths of 0 reduced cost in the residual network, and it makes sense to use them. This heuristic is quite useful in practice.

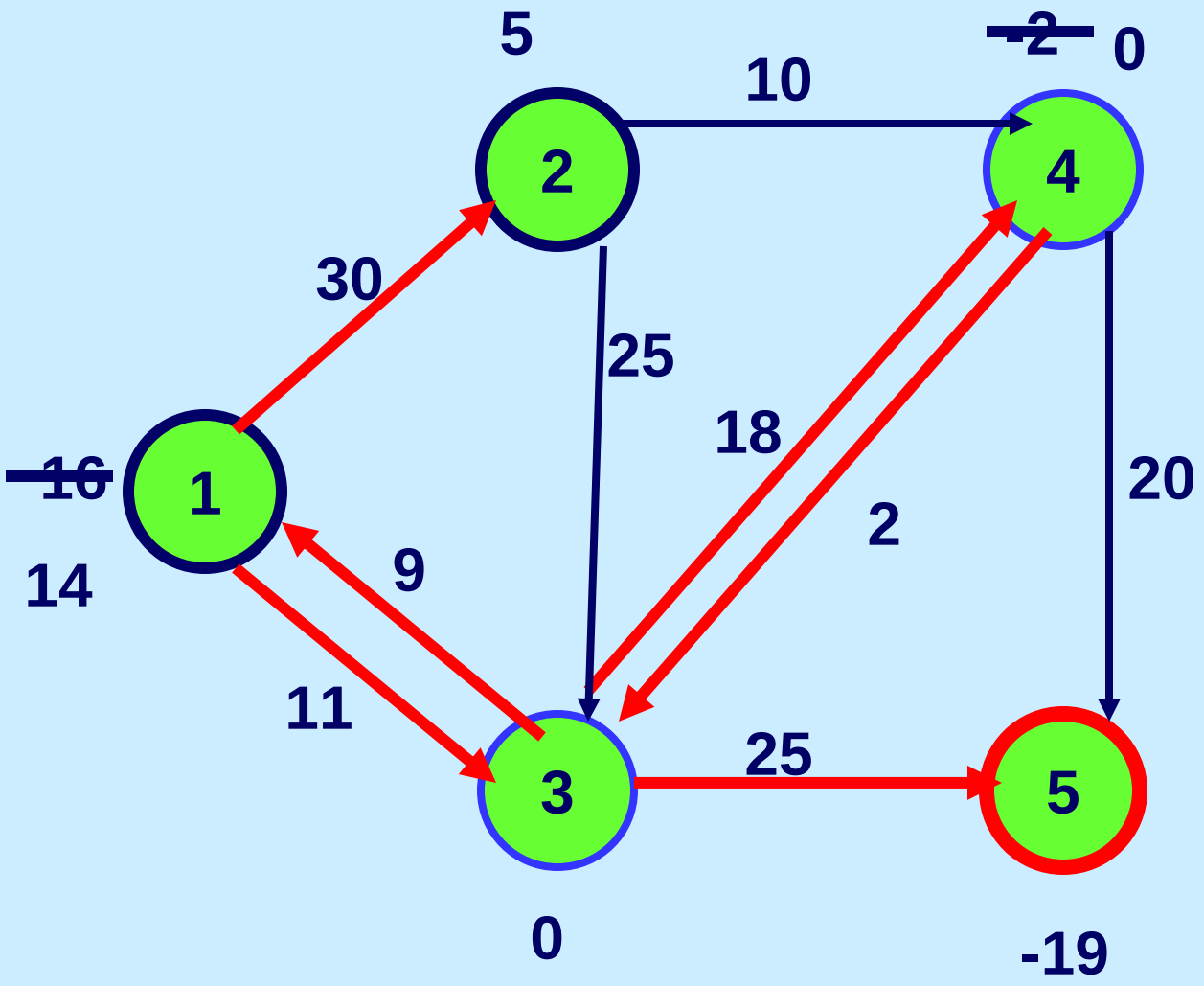
Send Flow From a Supply Node to a Demand Node Along Shortest Paths



Recall that red arcs have a reduced cost of 0

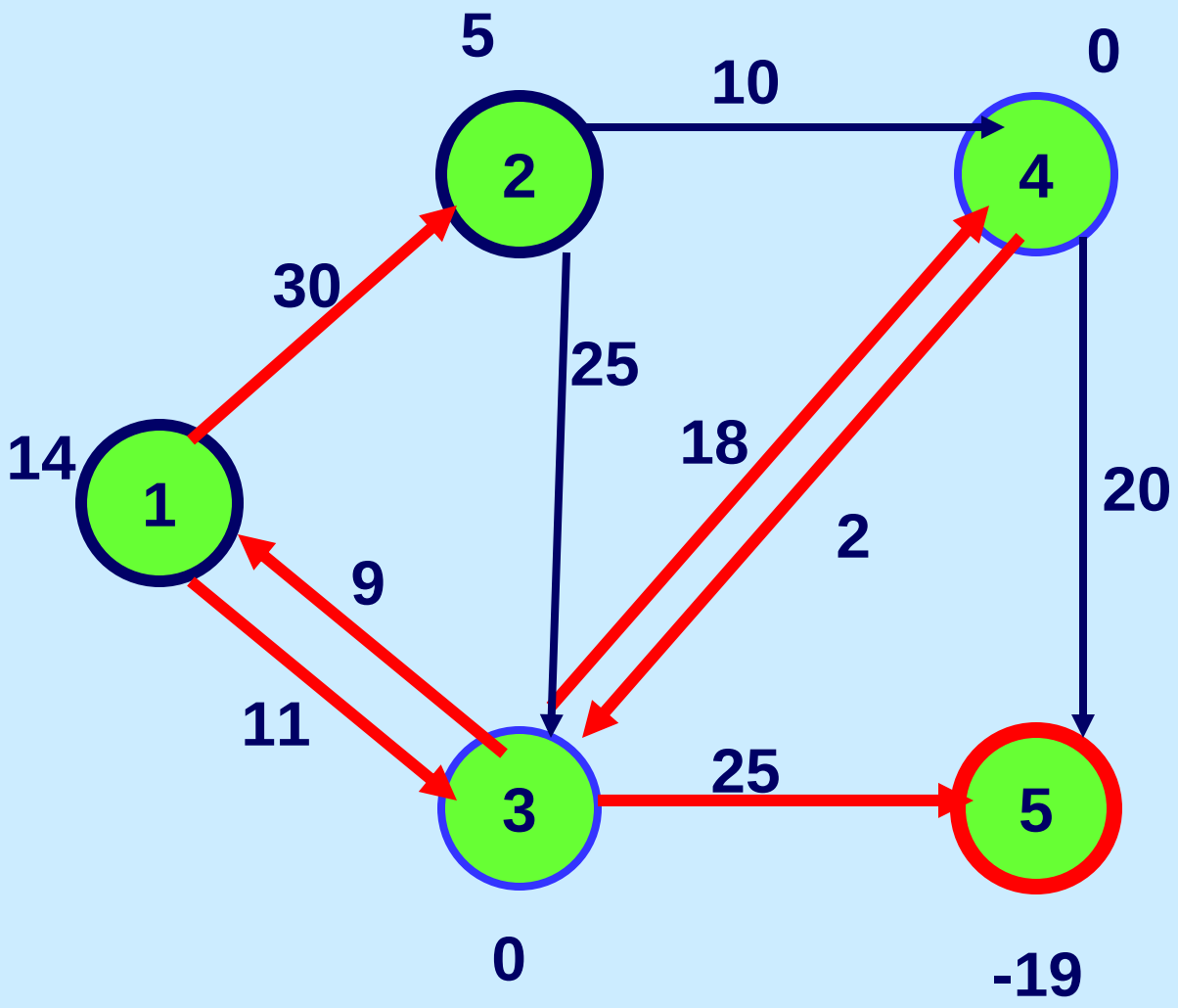
Send 2 units of flow from node 1 to node 4

Update the Residual Network



2 units of flow were sent from node 1 to node 4 on 1-3-4

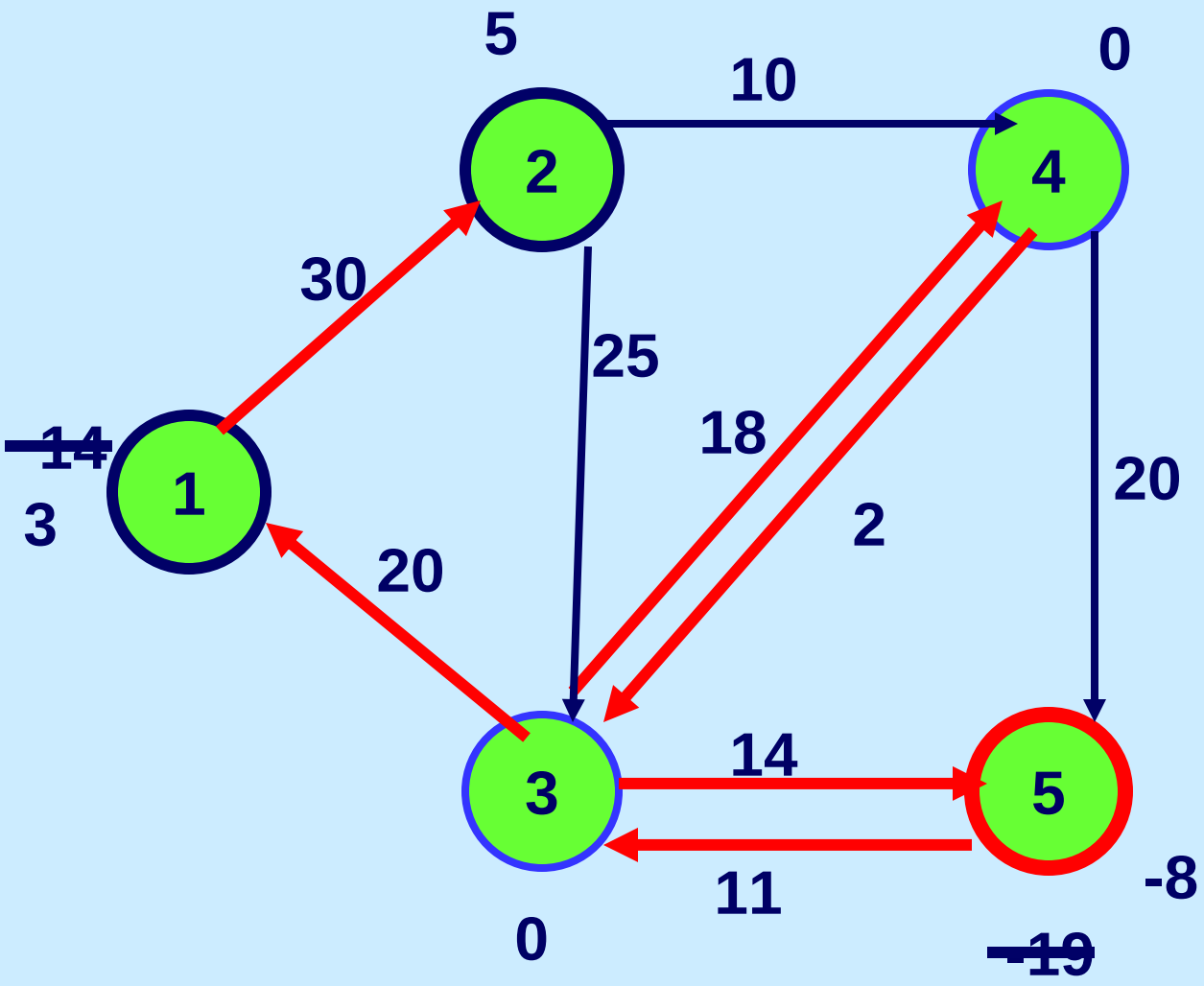
Send Flow From a Supply Node to a Demand Node Along Shortest Paths



Send flow from node 1 to node 5

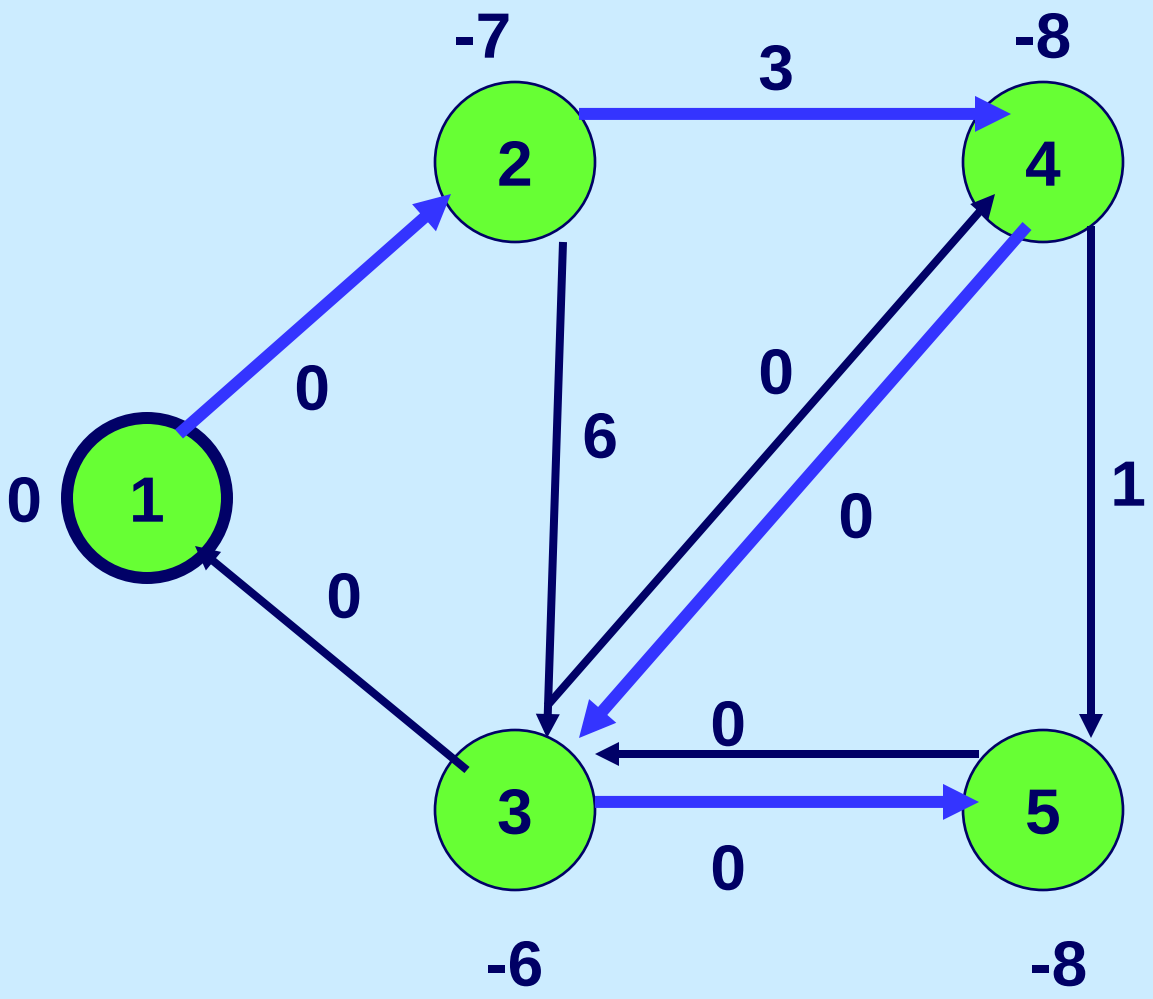
How much flow should be sent?

Update the Residual Network



11 units of flow were sent from node 1 to node 5

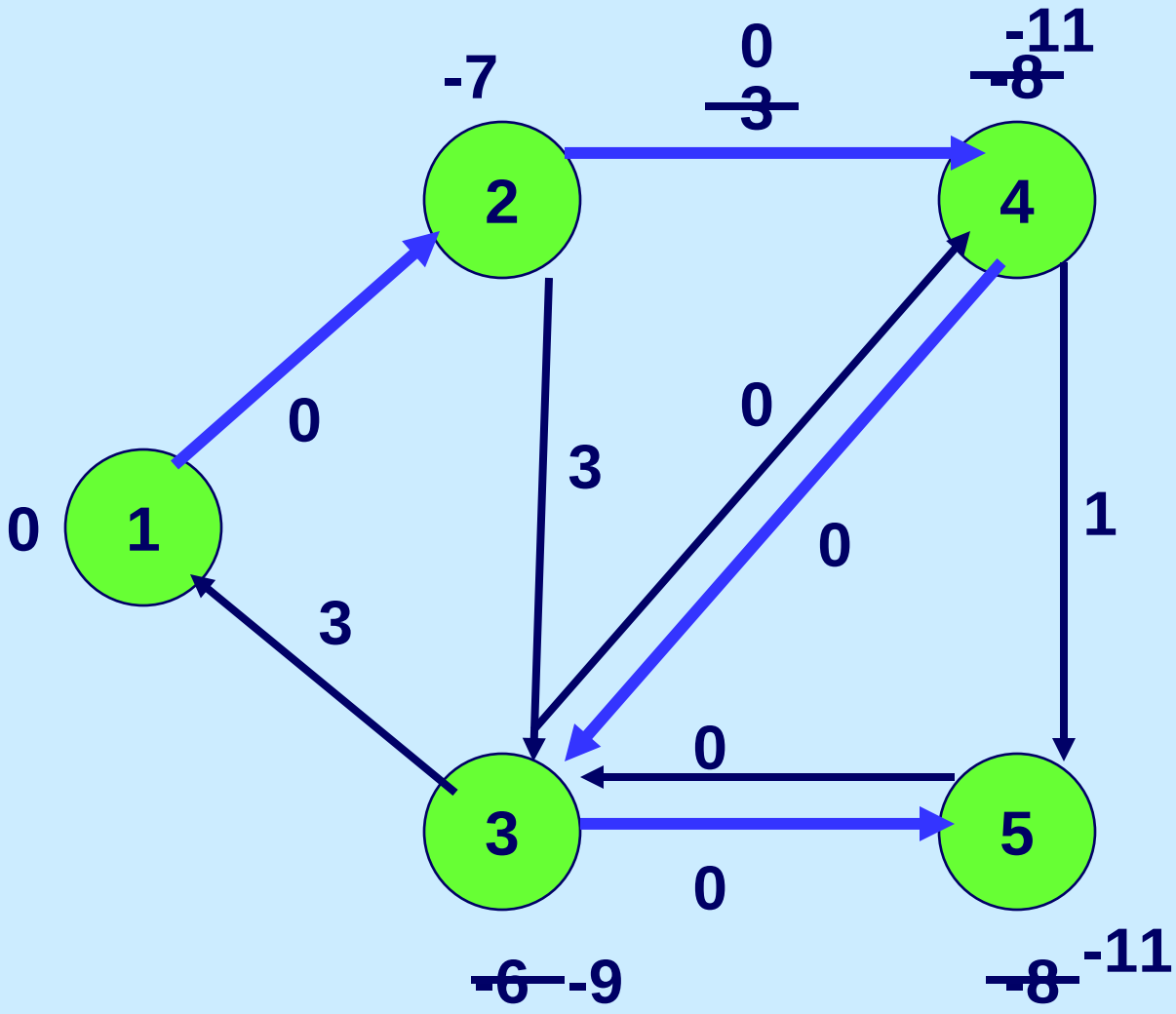
Select a supply node and find the shortest paths



The shortest path tree is marked in bold and blue.

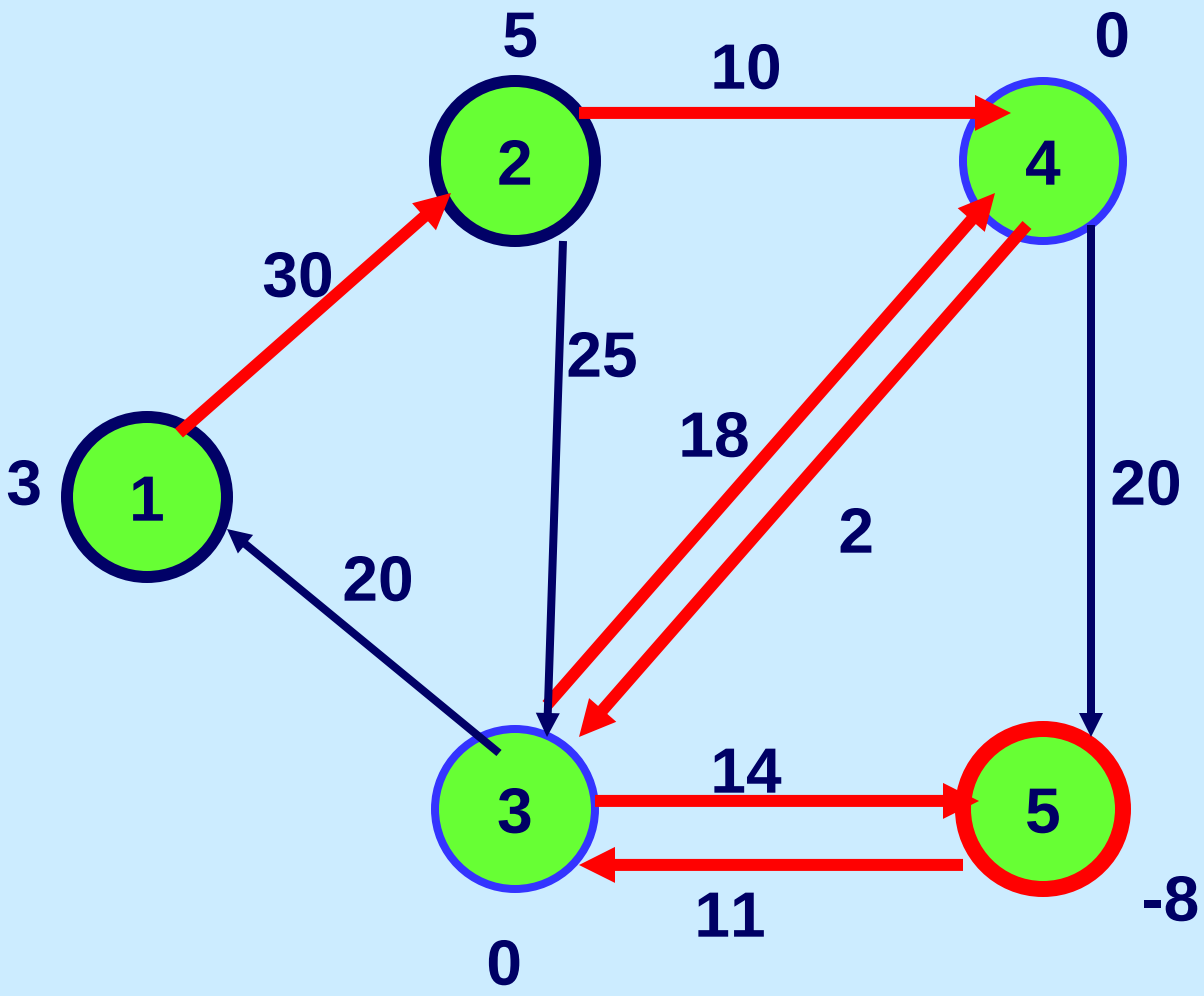
The values on the nodes are the current node potentials

Update the node potentials and the reduced costs



To obtain new node potentials, subtract the shortest path distances from the old potentials.

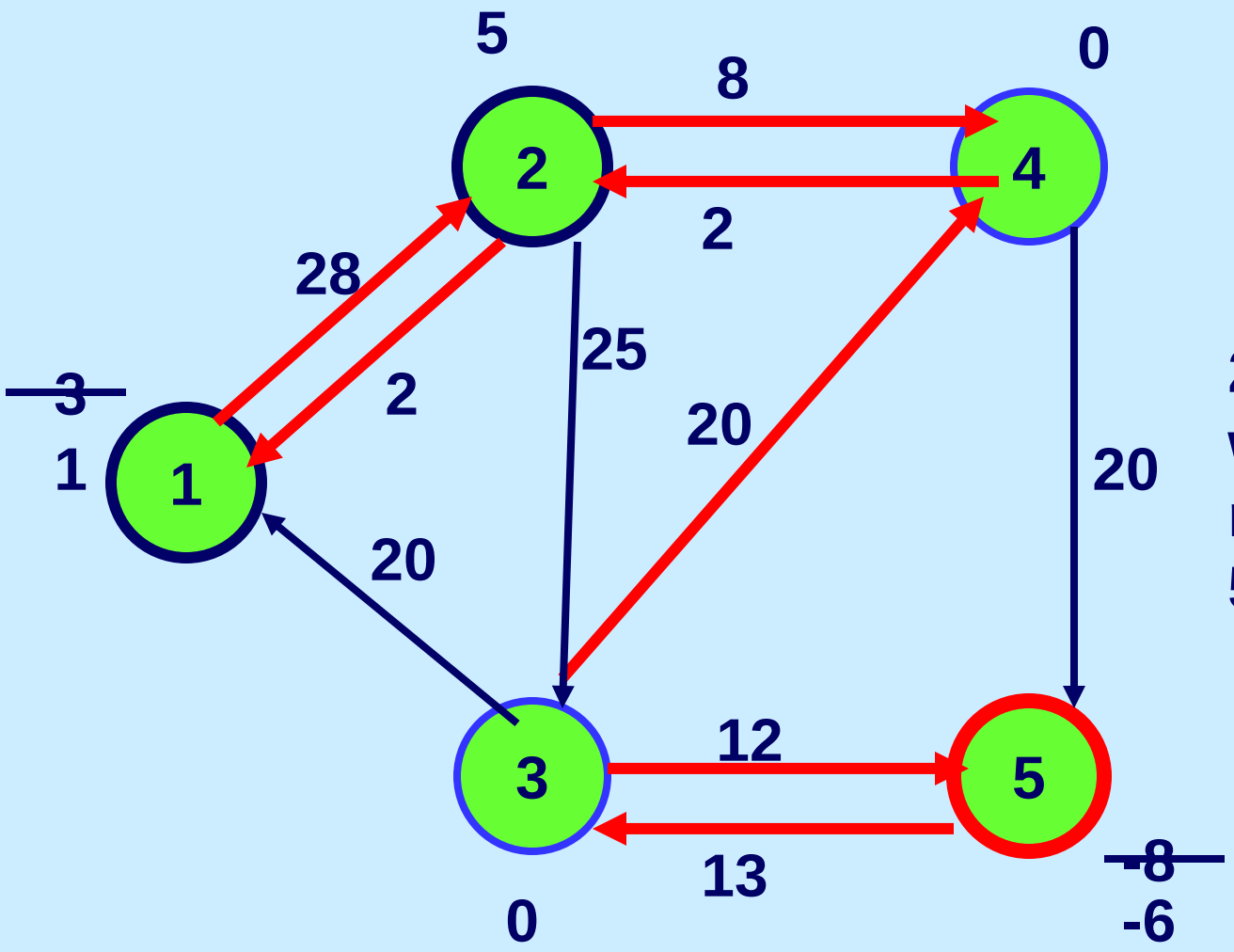
Send Flow From a Supply Node to a Demand Node Along Shortest Paths



Send flow from node 1 to node 5

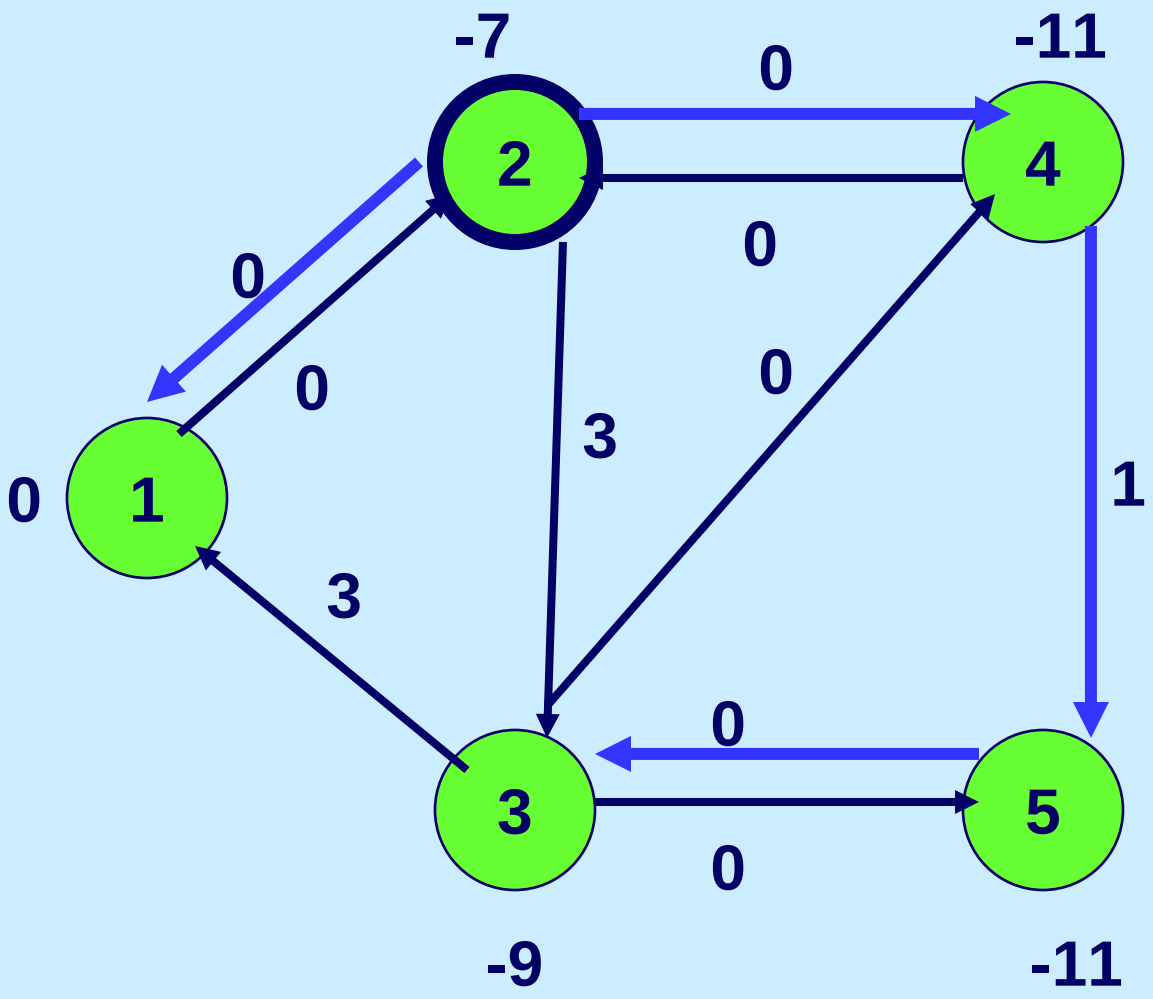
How much flow will be sent?

Update the Residual Network



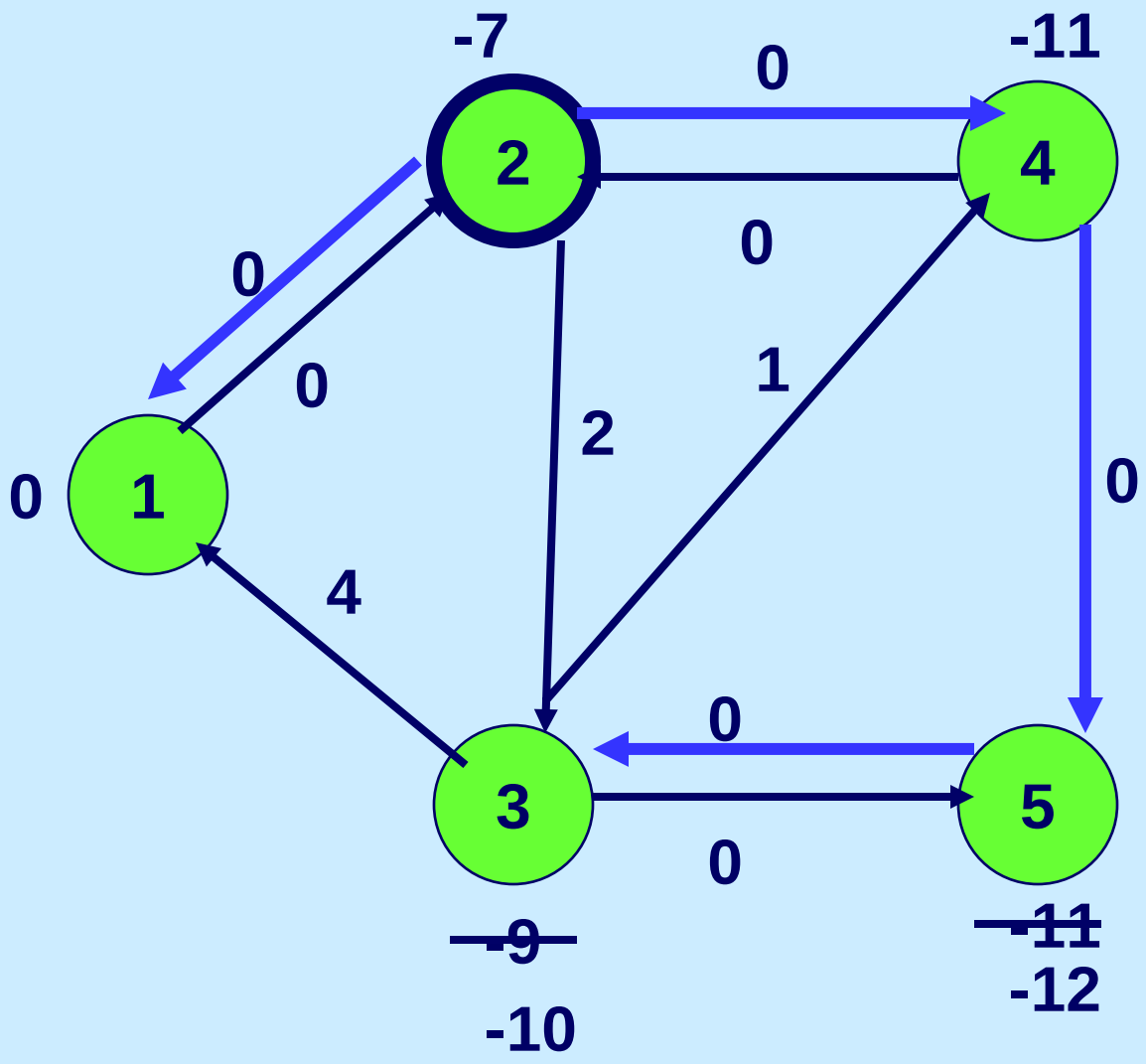
2 units of flow were sent from node 1 to node 5

Select a supply node and find the shortest paths



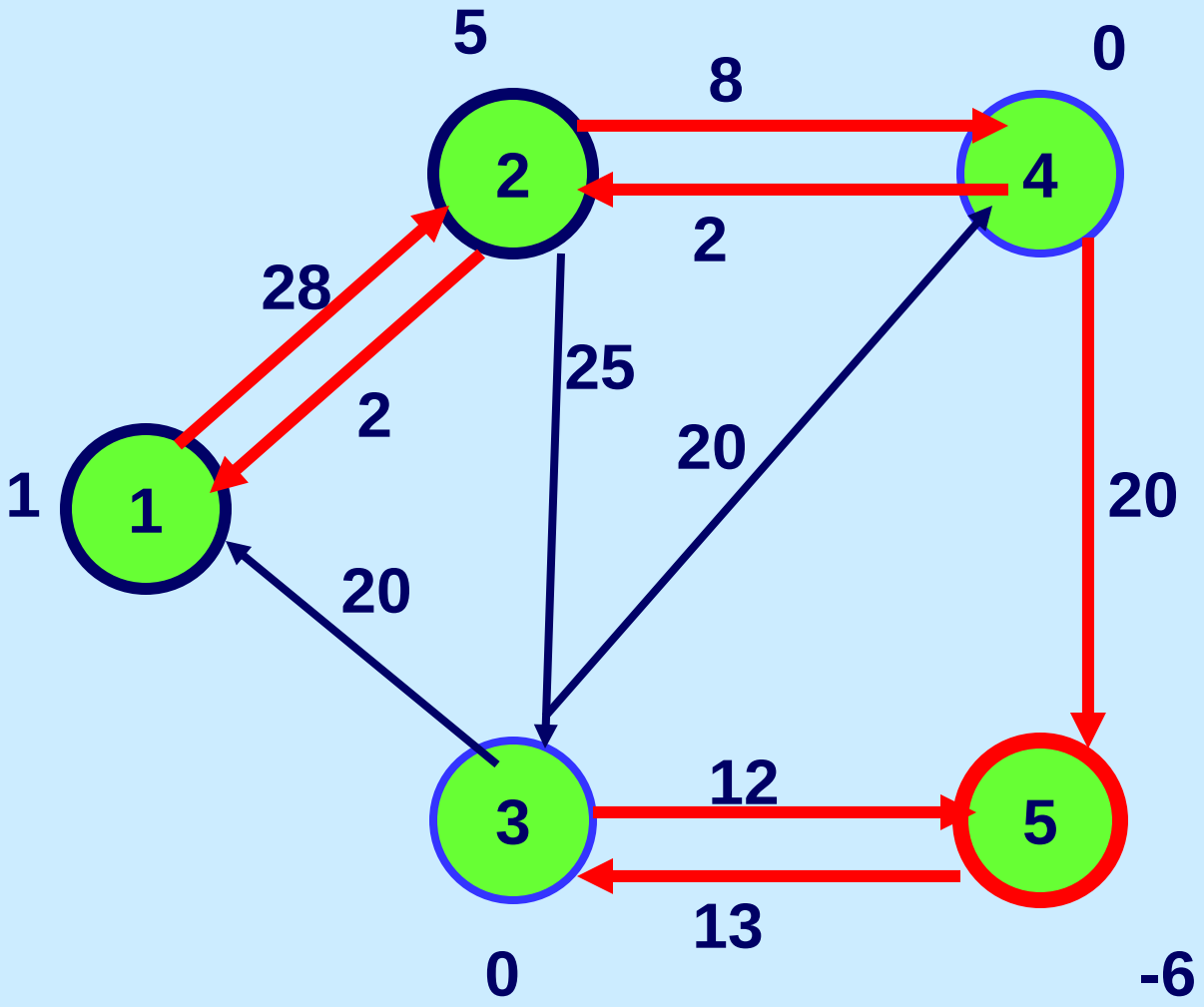
The shortest path tree is marked in bold and blue.

Update the Node Potentials and the Reduced Costs



To obtain the new node potential, subtract the shortest path distance from the old potential.

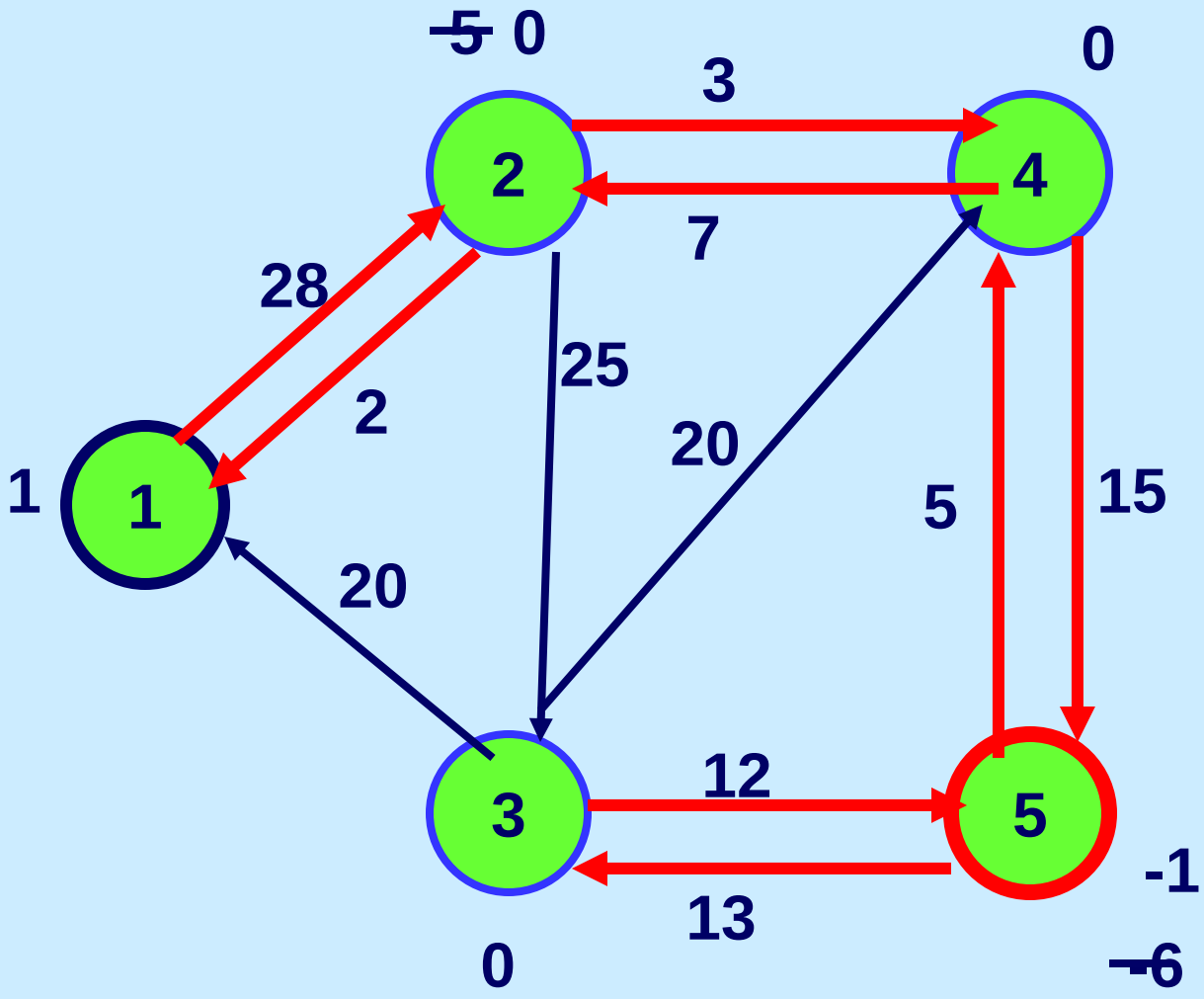
Send Flow From a Supply Node to a Demand Node Along Shortest Paths



Send flow from node 2 to node 5

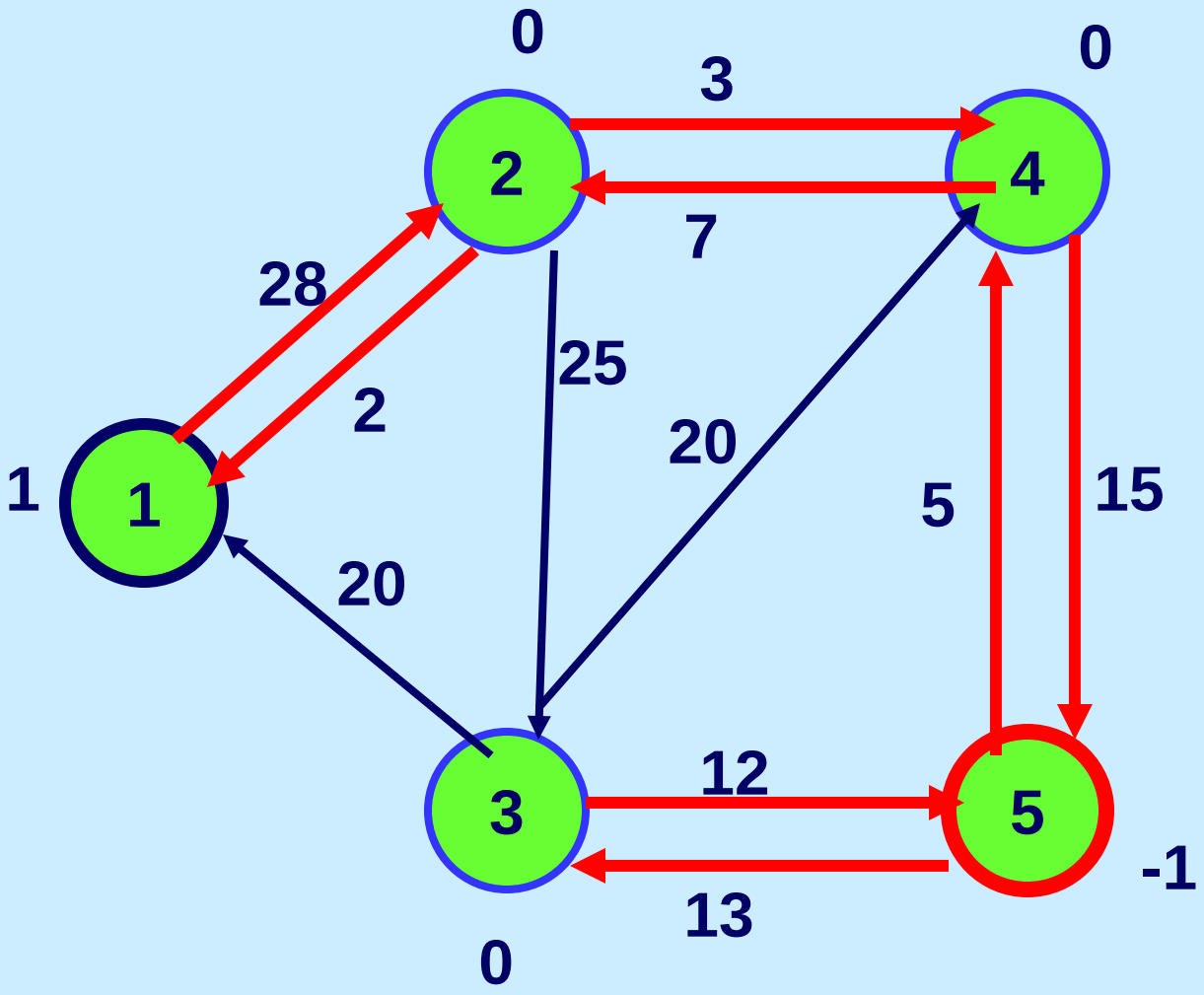
How much flow can be sent?

Update the Residual Network



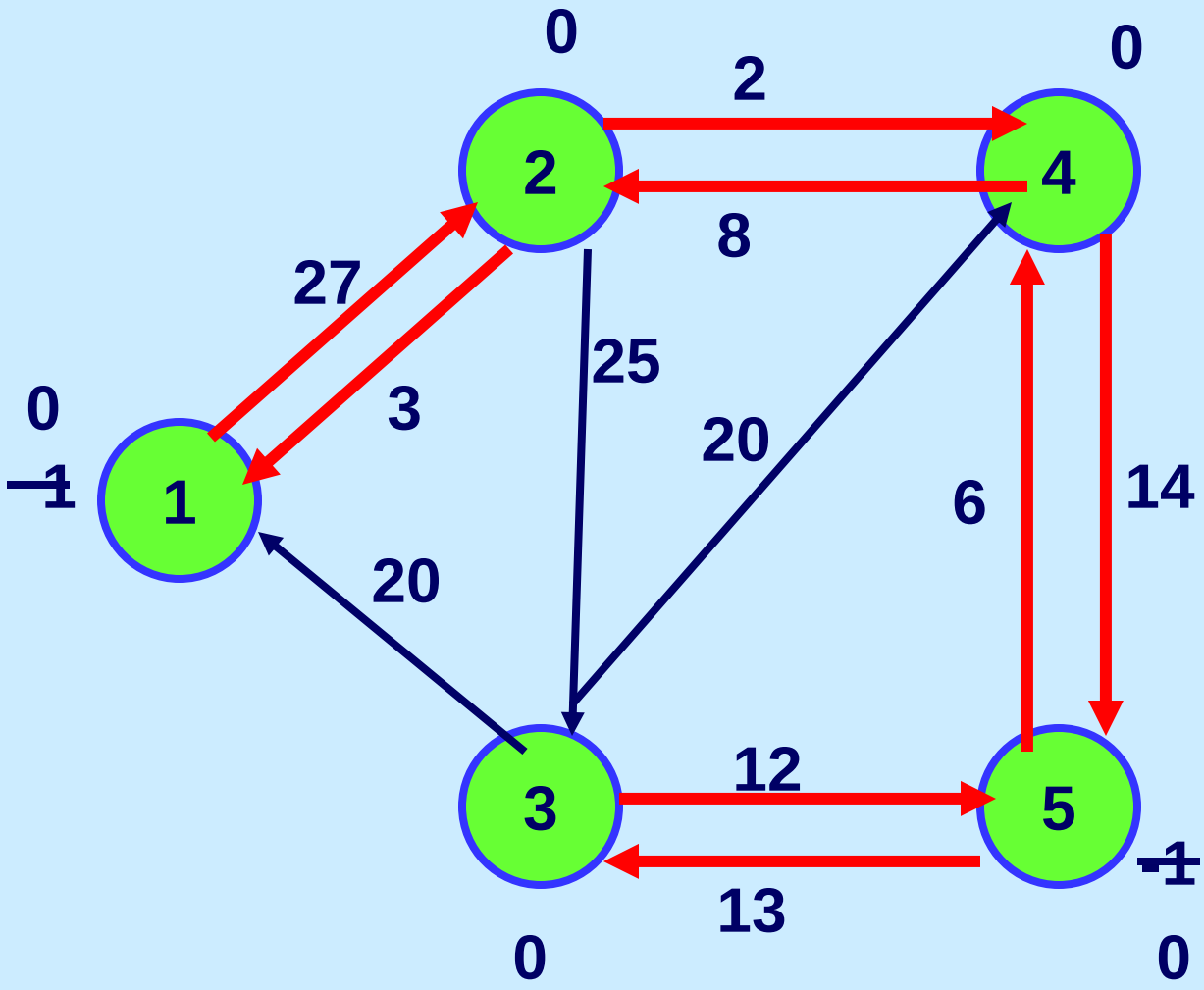
5 units of flow were sent from node 2 to node 6.

Send Flow From a Supply Node to a Demand Node



Send flow from node 1 to node 5

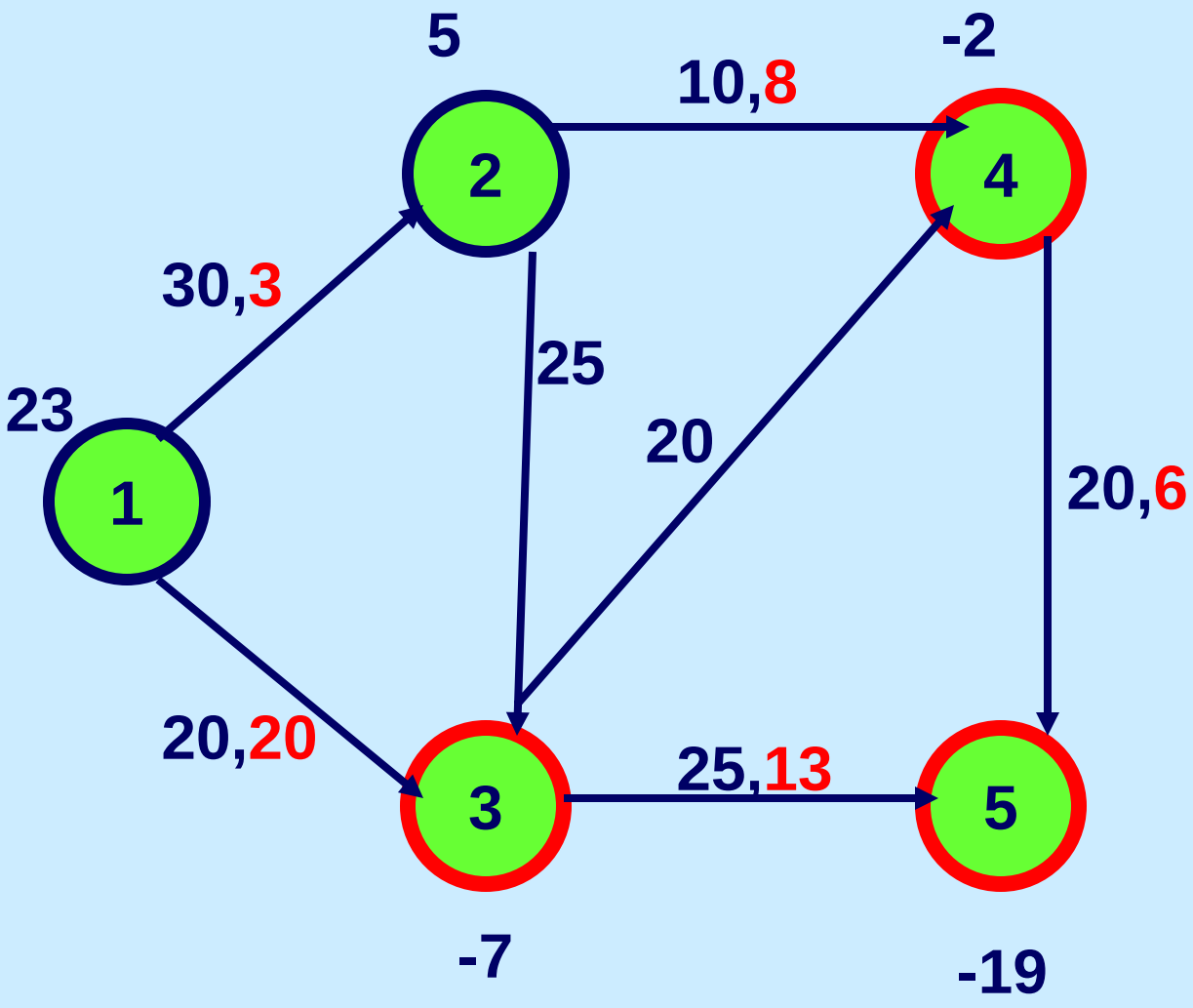
Update the Residual Network



1 unit of flow was sent from node 1 to node 5.

The resulting flow is feasible, and also optimal.

The Final Optimal Flow



The Final Optimal Node Potentials and the Reduced Costs