

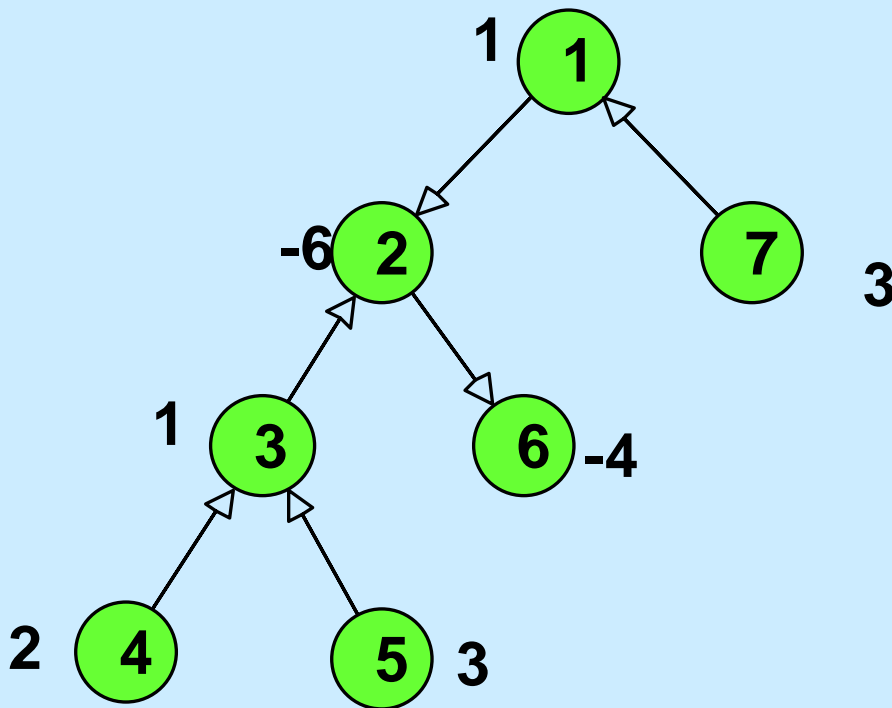
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**15.082 and 6.855J**

**Network Simplex Animations**

# Calculating A Spanning Tree Flow

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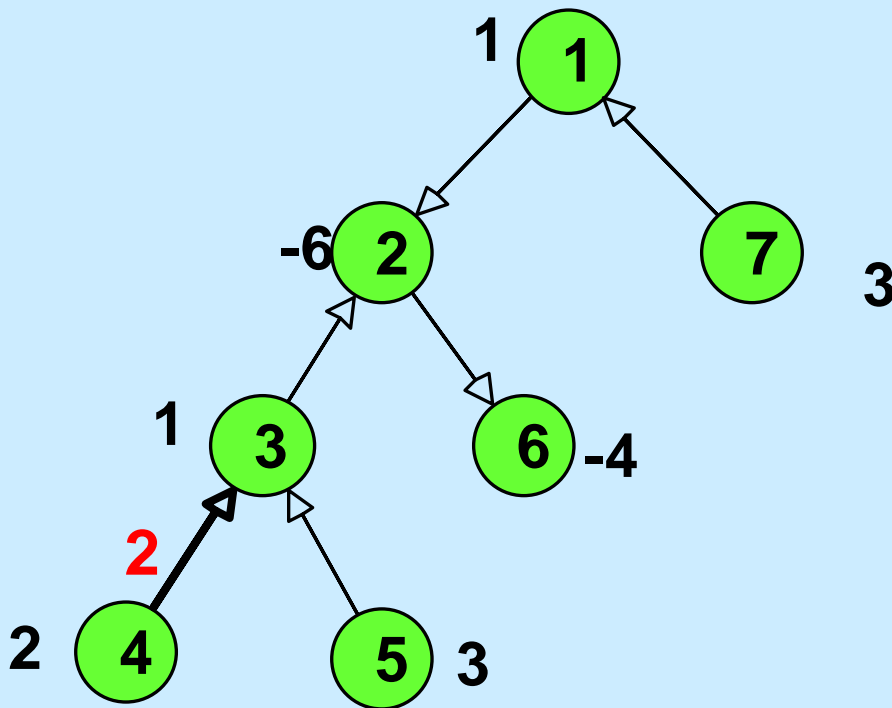


A tree with supplies and demands.  
(Assume that all other arcs have a flow of 0)

**What is the flow in arc (4,3)?**

# Calculating A Spanning Tree Flow

---

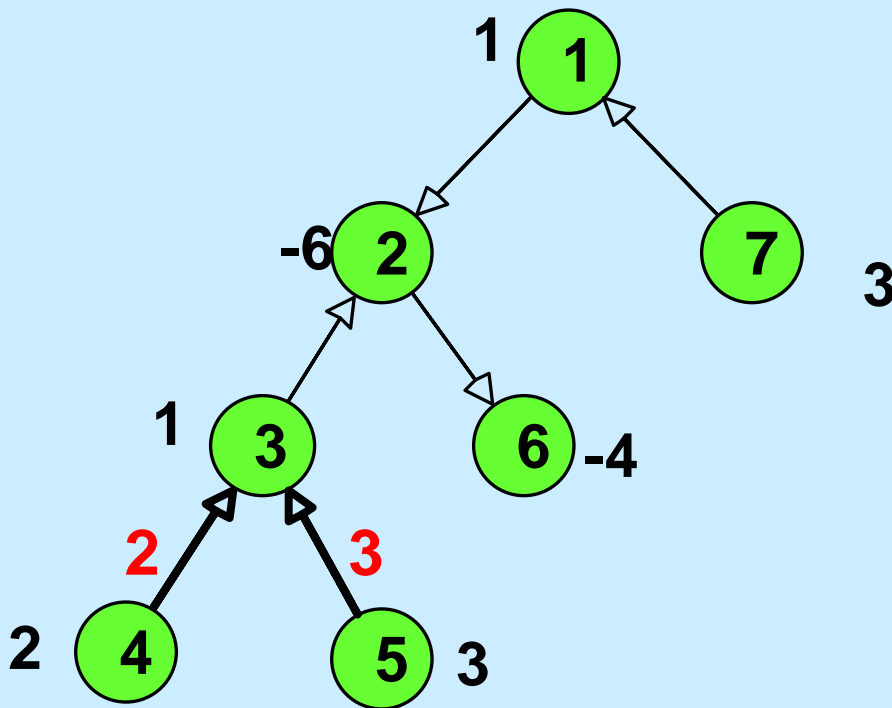


To calculate flows, iterate up the tree, and find an arc whose flow is uniquely determined.

**What is the flow in arc (5,3)?**

# Calculating A Spanning Tree Flow

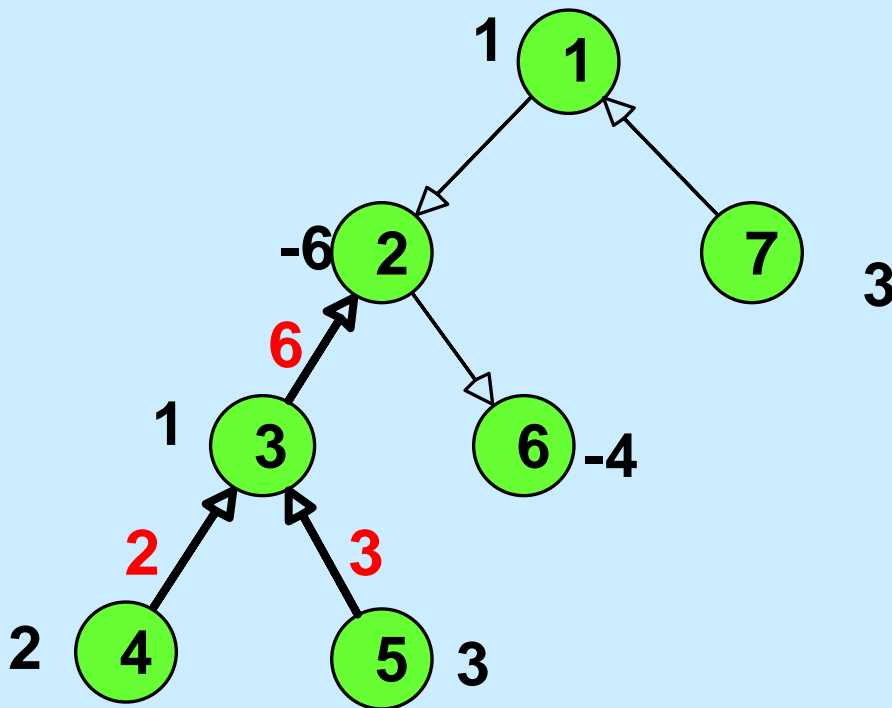
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What is the flow  
in arc (3,2)?

# Calculating A Spanning Tree Flow

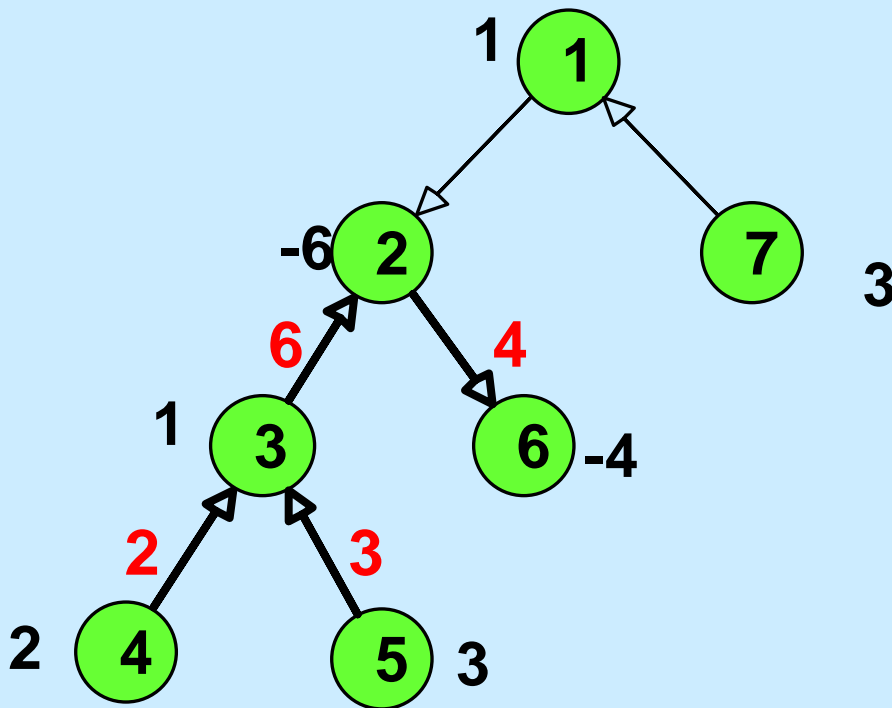
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What is the flow  
in arc (2,6)?

# Calculating A Spanning Tree Flow

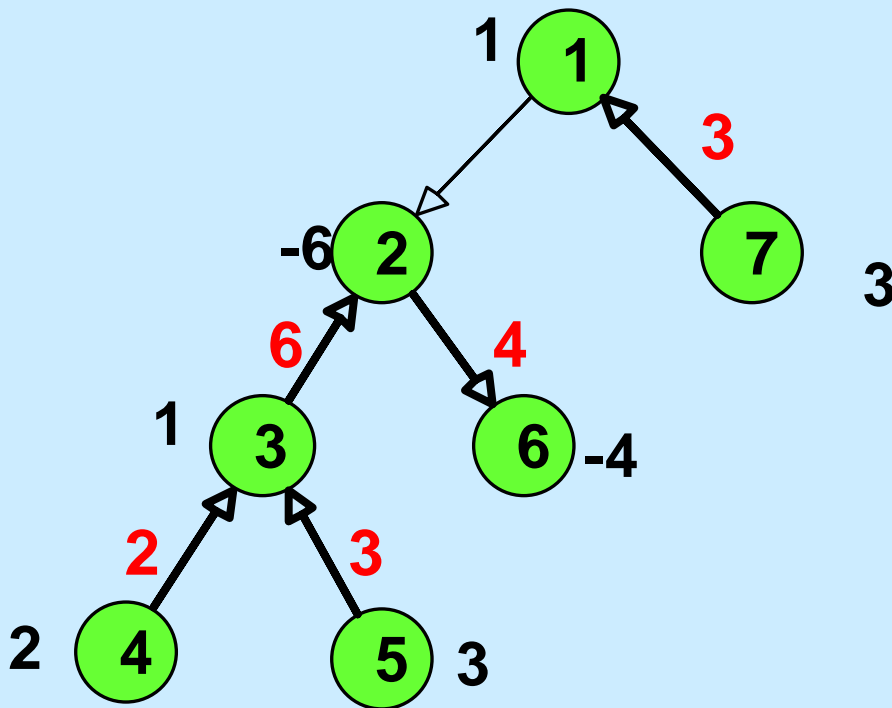
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What is the flow  
in arc (7,1)?

# Calculating A Spanning Tree Flow

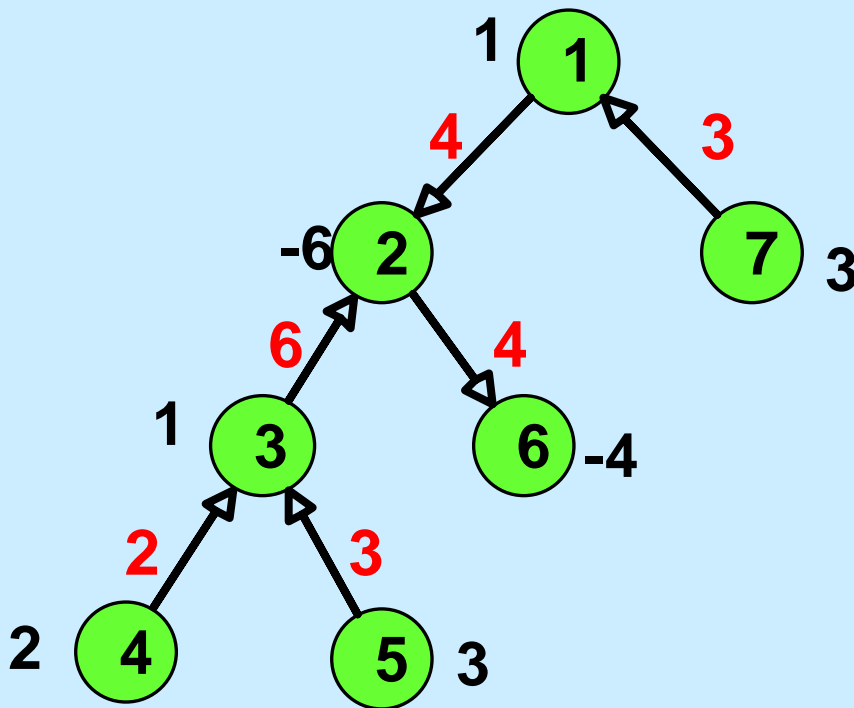
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What is the flow  
in arc (1,6)?

# Calculating A Spanning Tree Flow

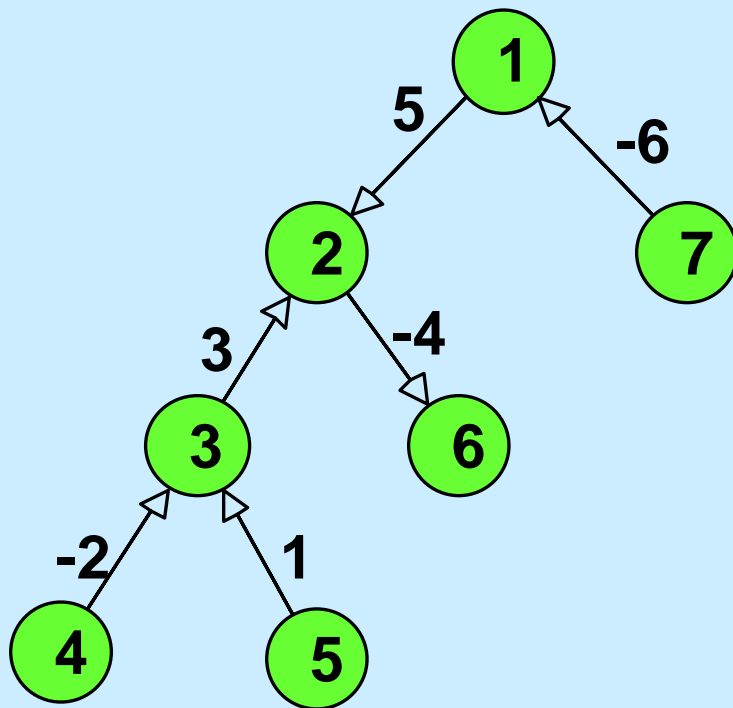
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**Note:** there are two different ways of calculating the flow on (1,2), and both ways give a flow of 4. Is this a coincidence?

# Calculating Simplex Multipliers for a Spanning Tree

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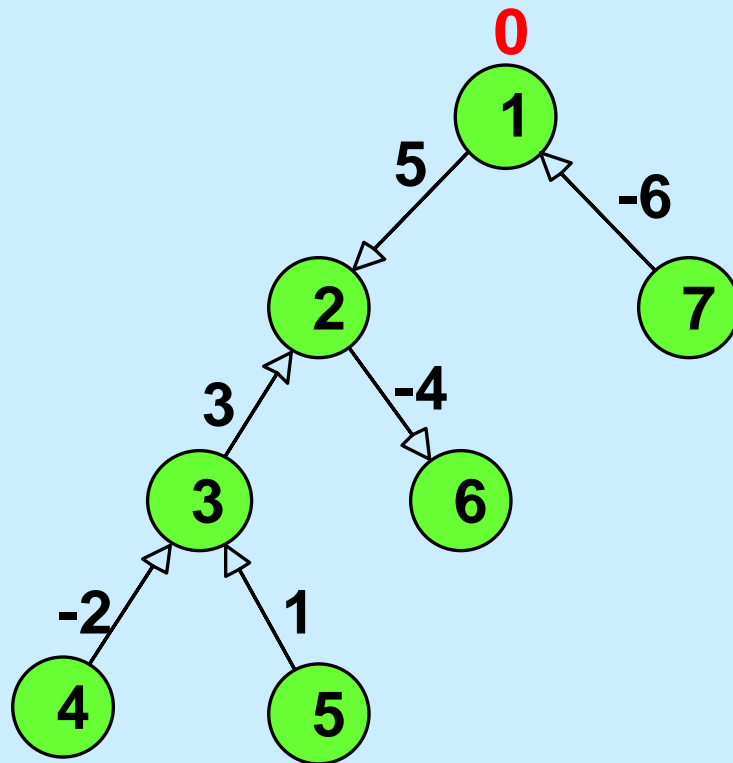
Here is a spanning tree with arc costs. How can one choose node potentials so that reduced costs of tree arcs is 0?

**Recall: the reduced cost of  $(i,j)$  is**

$$c_{ij} - \pi_i + \pi_j$$

# Calculating Simplex Multipliers for a Spanning Tree

---



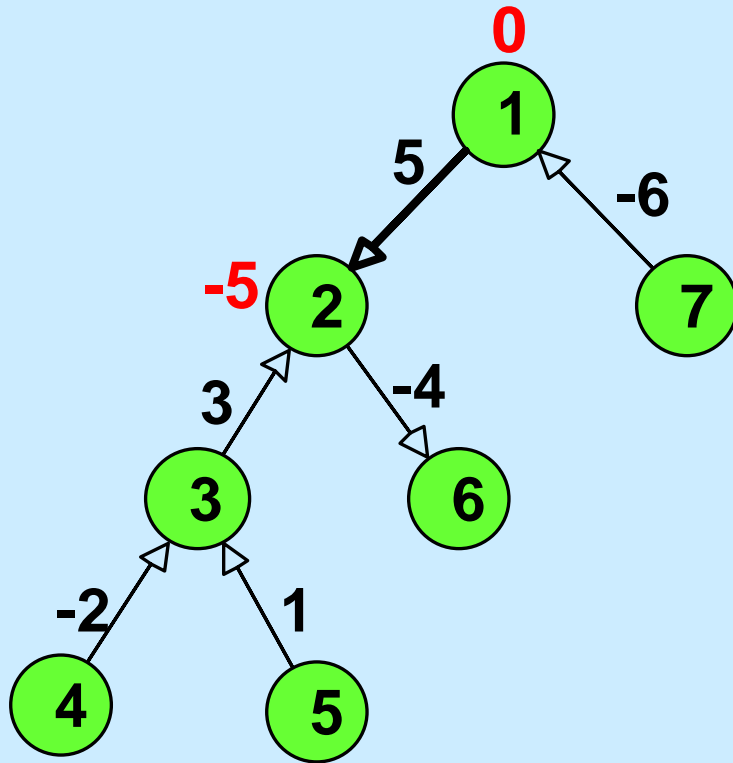
There is a redundant constraint in the minimum cost flow problem.

One can set  $\pi_1$  arbitrarily. We will let  $\pi_i = 0$ .

**What is the simplex multiplier for node 2?**

# Calculating Simplex Multipliers for a Spanning Tree

---



The reduced cost  
of (1,2) is

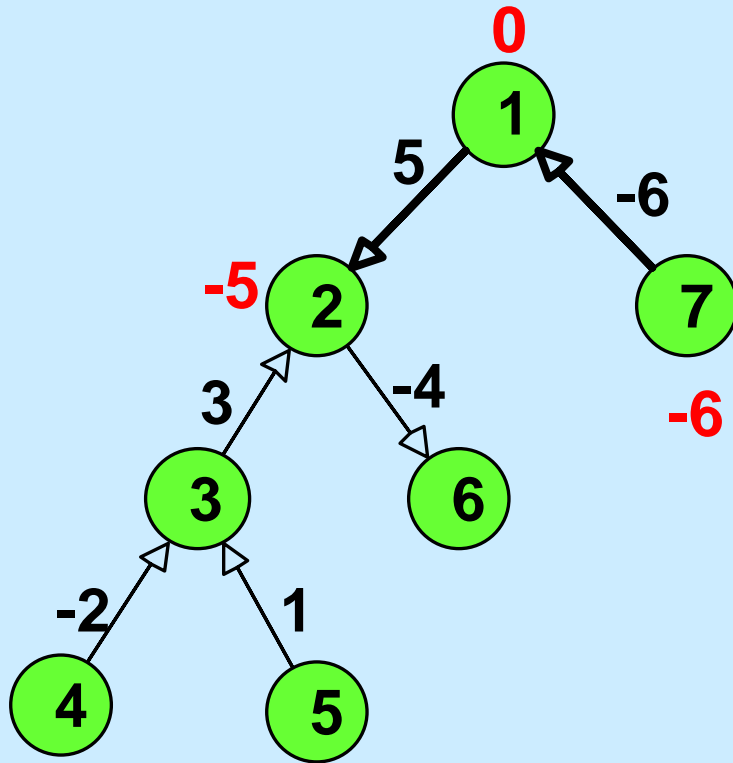
$$c_{12} - \pi_1 + \pi_2 = 0.$$

$$\text{Thus } 5 - 0 + \pi_2 = 0.$$

**What is the  
simplex multiplier  
for node 7?**

# Calculating Simplex Multipliers for a Spanning Tree

---



The reduced cost  
of (1,2) is

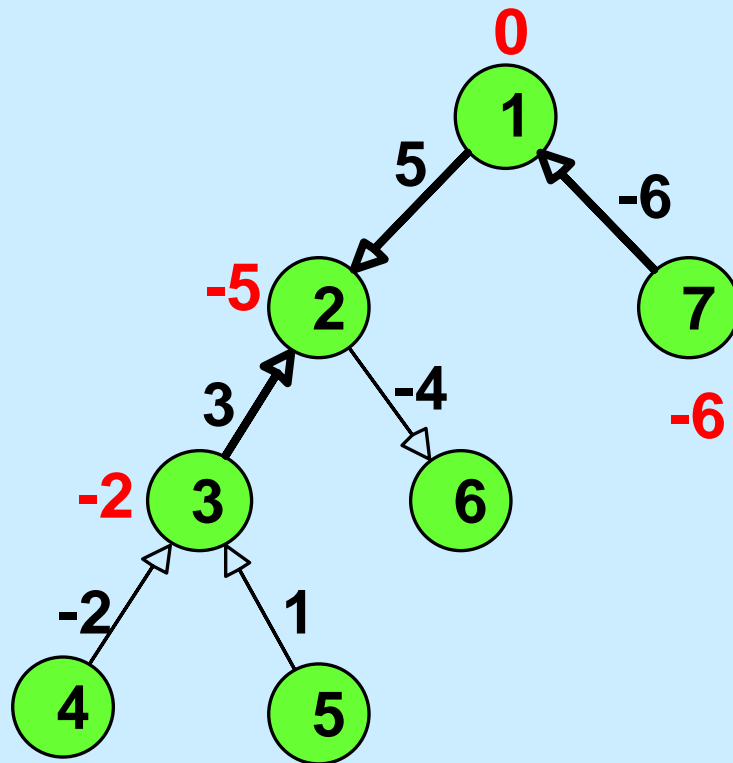
$$c_{71} - \pi_7 + \pi_1 = 0.$$

Thus  $-6 - \pi_2 + 0 = 0$ .

**What is the  
simplex multiplier  
for node 3?**

# Calculating Simplex Multipliers for a Spanning Tree

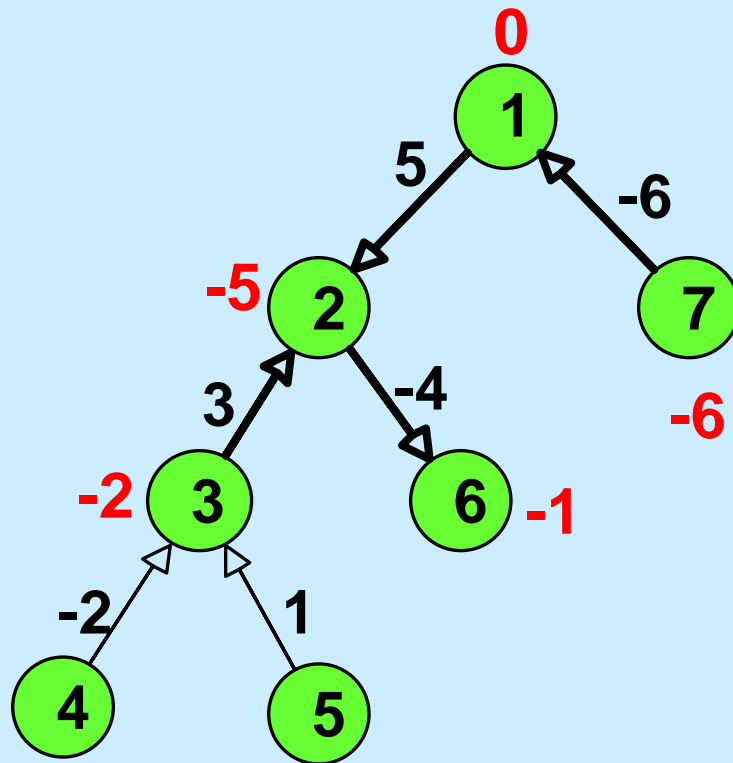
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**What is the  
simplex multiplier  
for node 6?**

# Calculating Simplex Multipliers for a Spanning Tree

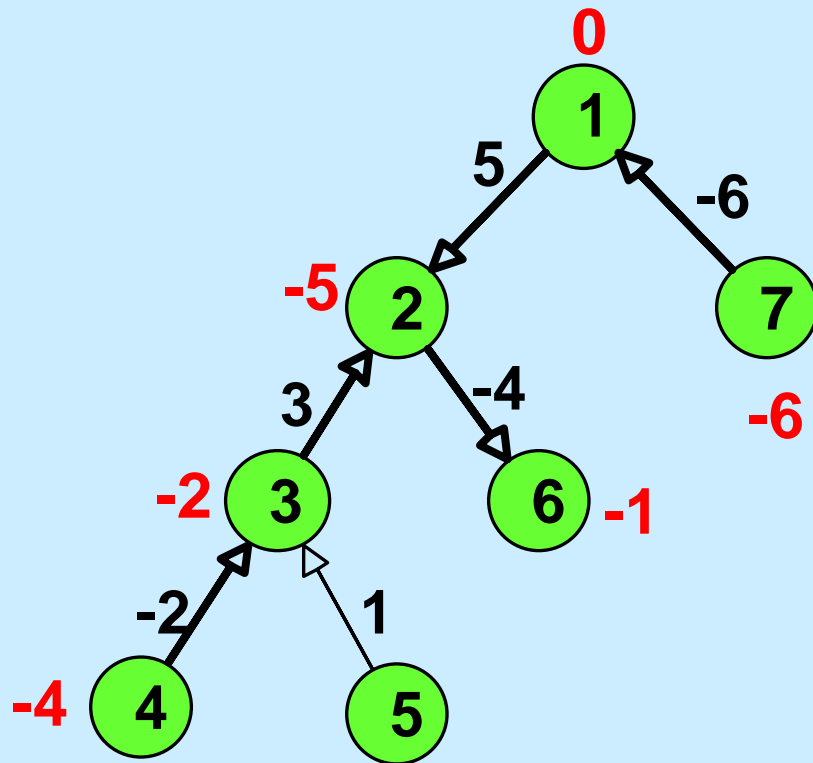
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**What is the  
simplex multiplier  
for node 4?**

# Calculating Simplex Multipliers for a Spanning Tree

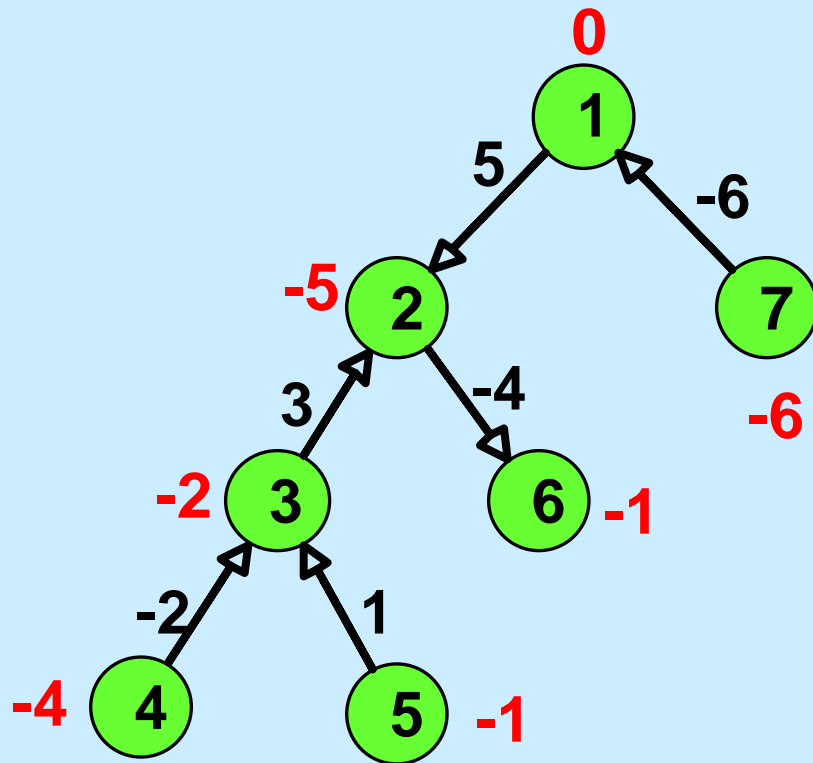
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**What is the  
simplex multiplier  
for node 5?**

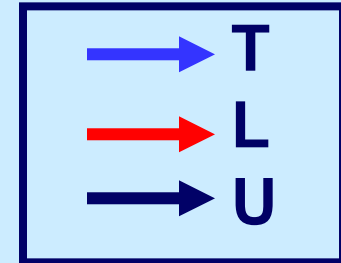
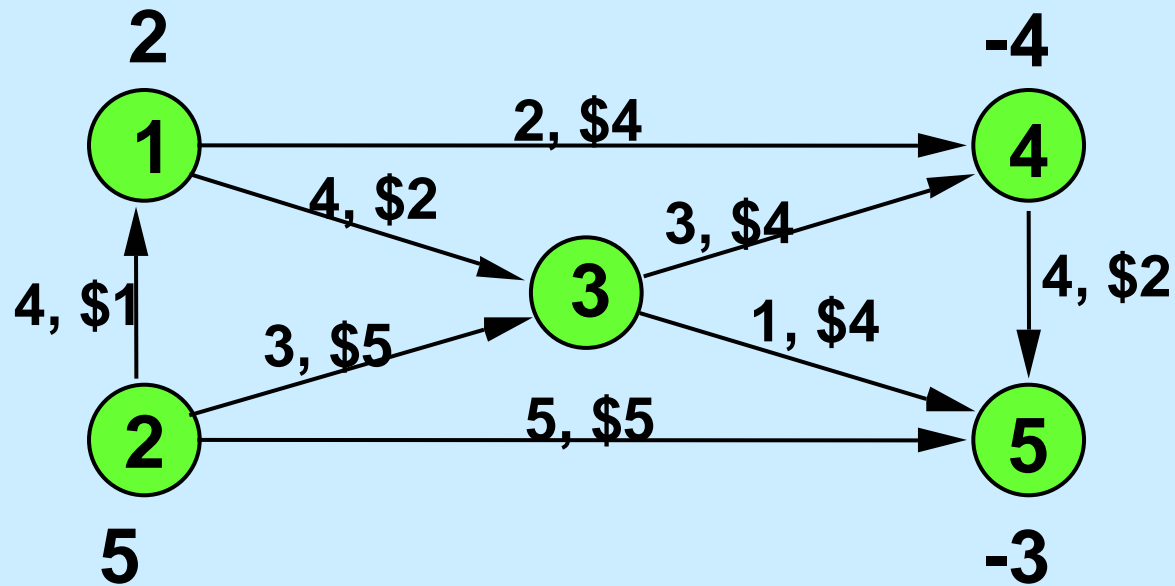
# Calculating Simplex Multipliers for a Spanning Tree

---



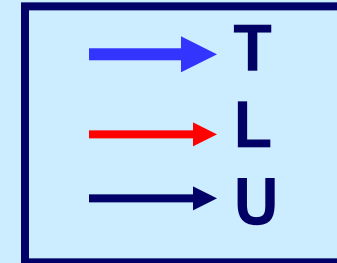
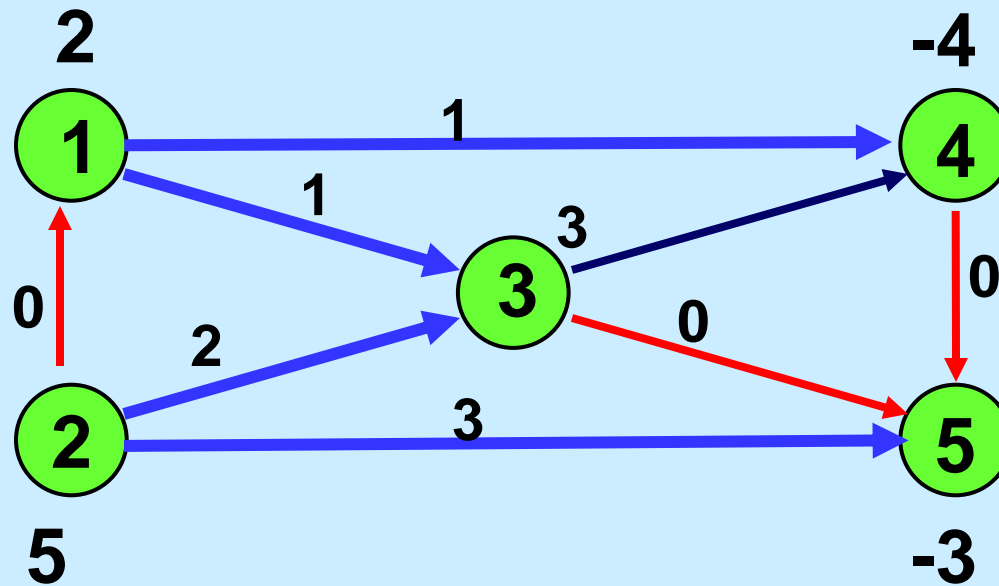
**These are the simplex multipliers associated with this tree. They do not depend on arc flows, nor on costs of non-tree arcs.**

# Network Simplex Algorithm



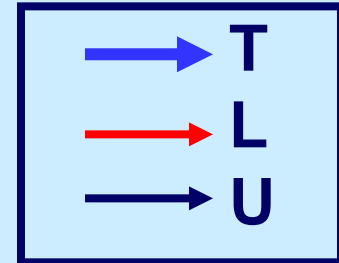
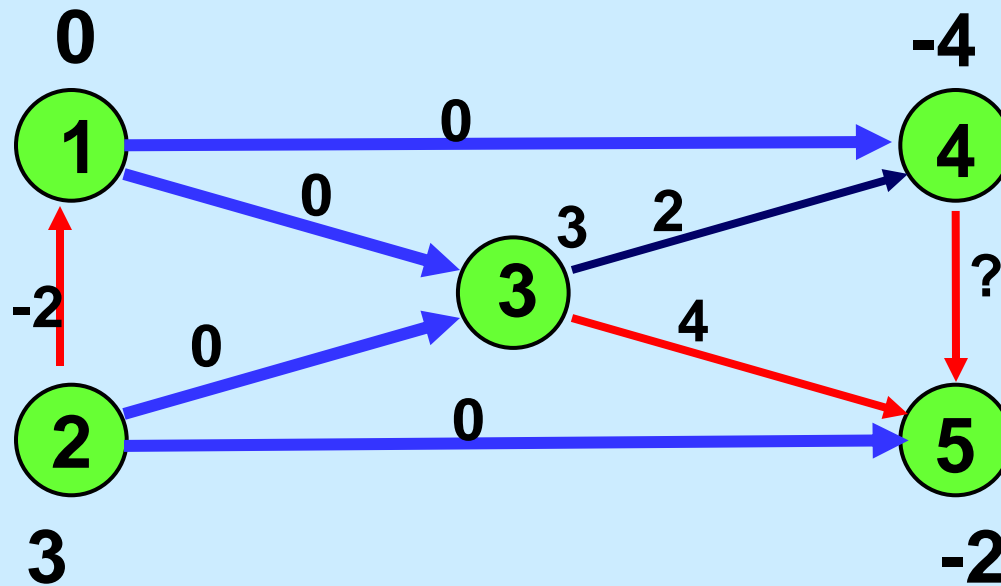
The minimum Cost Flow Problem

# Spanning tree flows



An Initial Spanning Tree Solution

# Simplex Multipliers and Reduced Costs

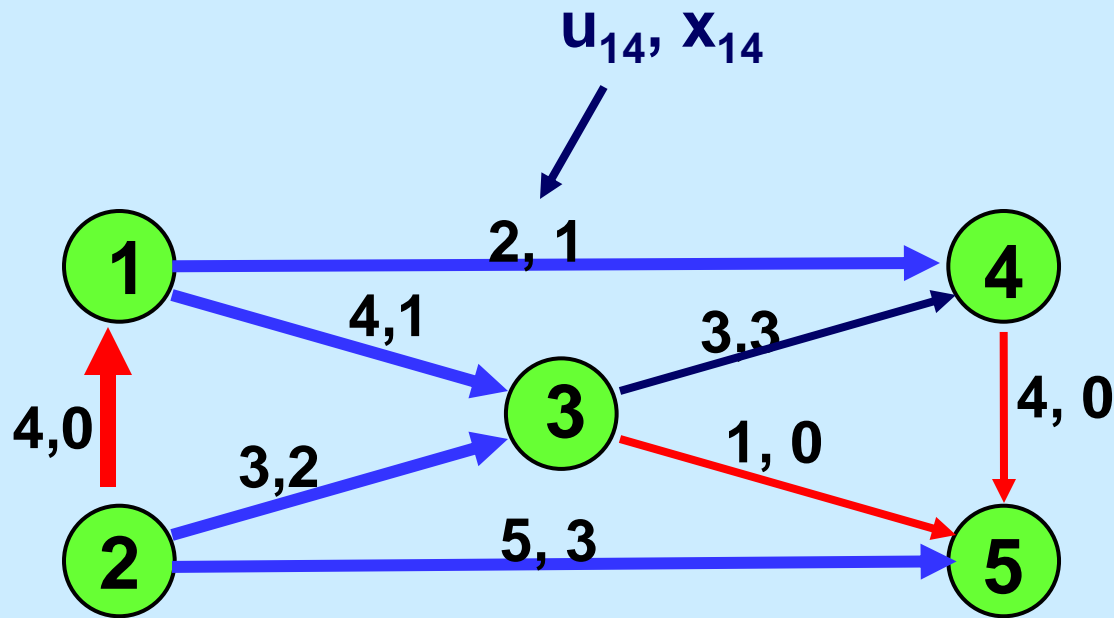


$$c_{45} = 2$$

What arcs are violating?

The initial simplex multipliers and reduced costs

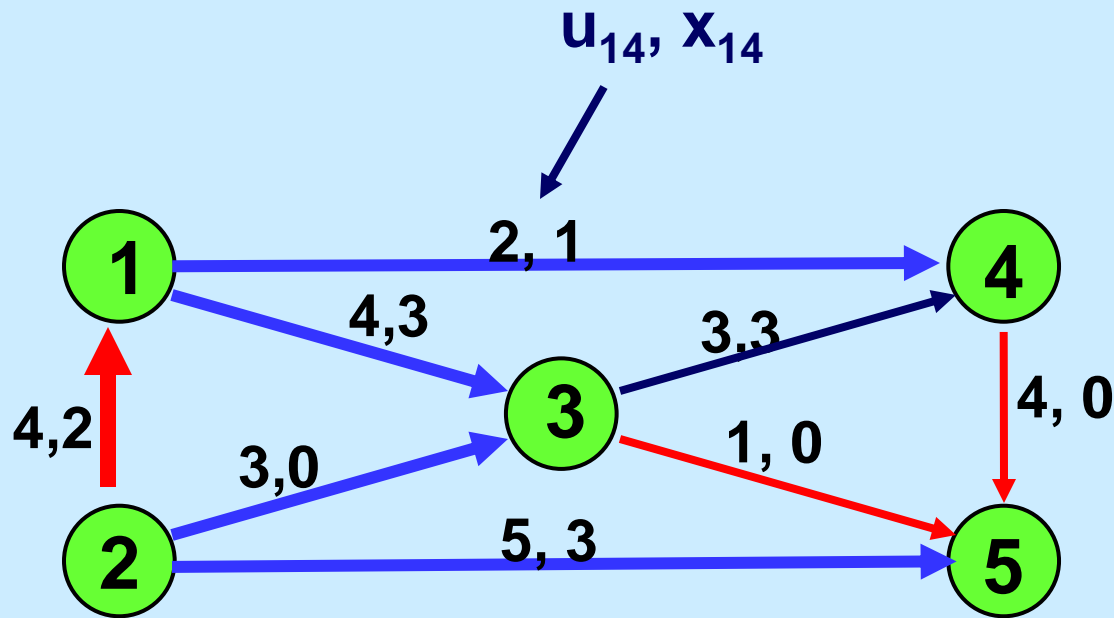
# Add a violating arc to the spanning tree, creating a cycle



Arc (2,1) is added to the tree

What is the cycle, and how much flow can be sent?

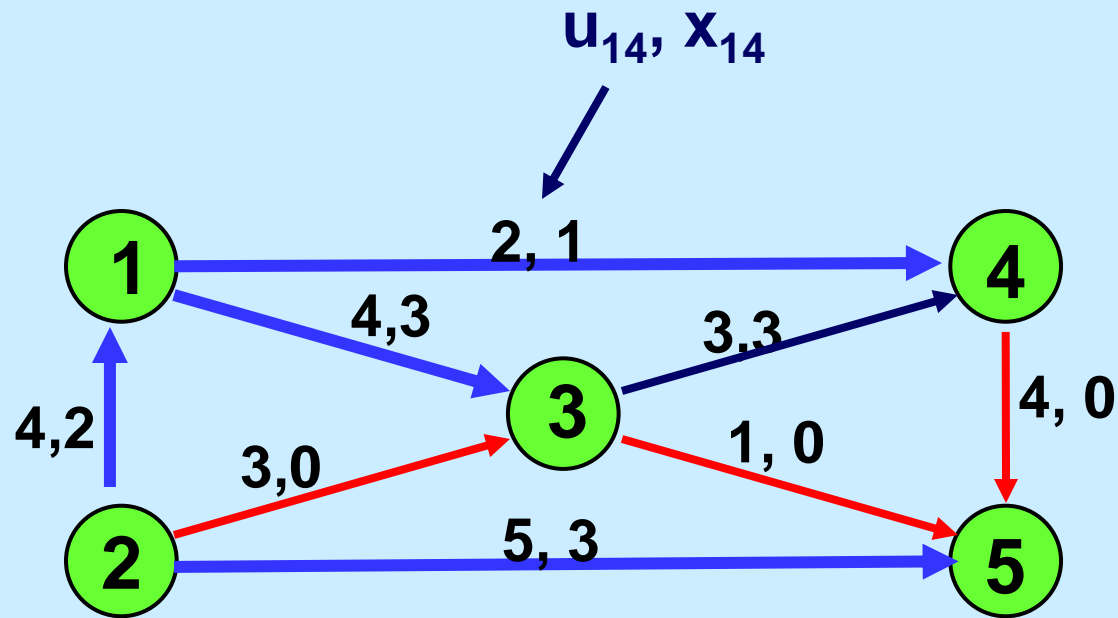
# Send Flow Around the Cycle



2 units of flow were sent along the cycle.

What is the next spanning tree?

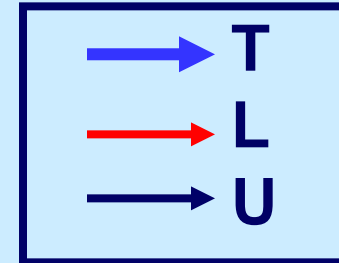
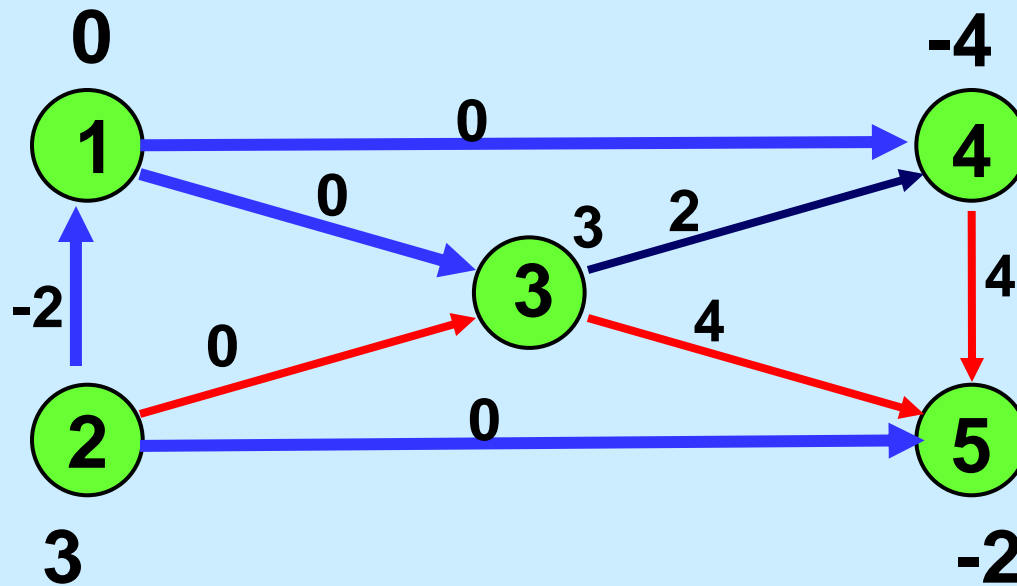
# After a pivot



The Updated Spanning Tree

In a pivot, an arc is added to T and an arc is dropped from T.

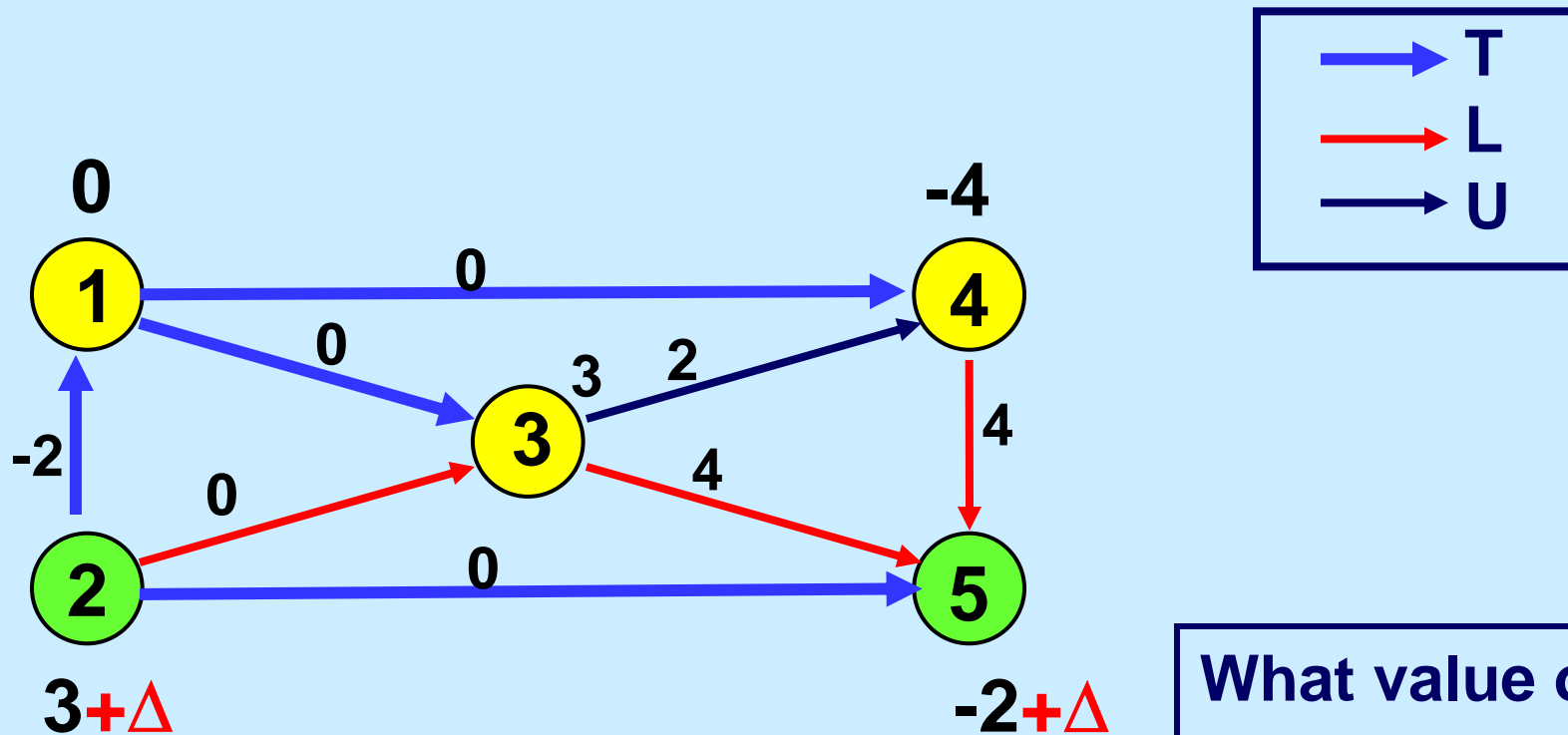
# Updating the Multipliers



The current multipliers and reduced costs

How can we make  $c^{\pi}_{21} = 0$  and have other tree arcs have a 0 reduced cost?

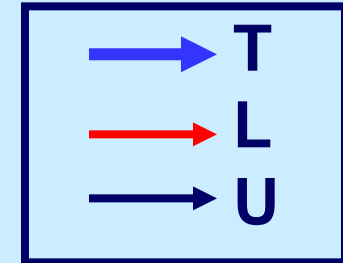
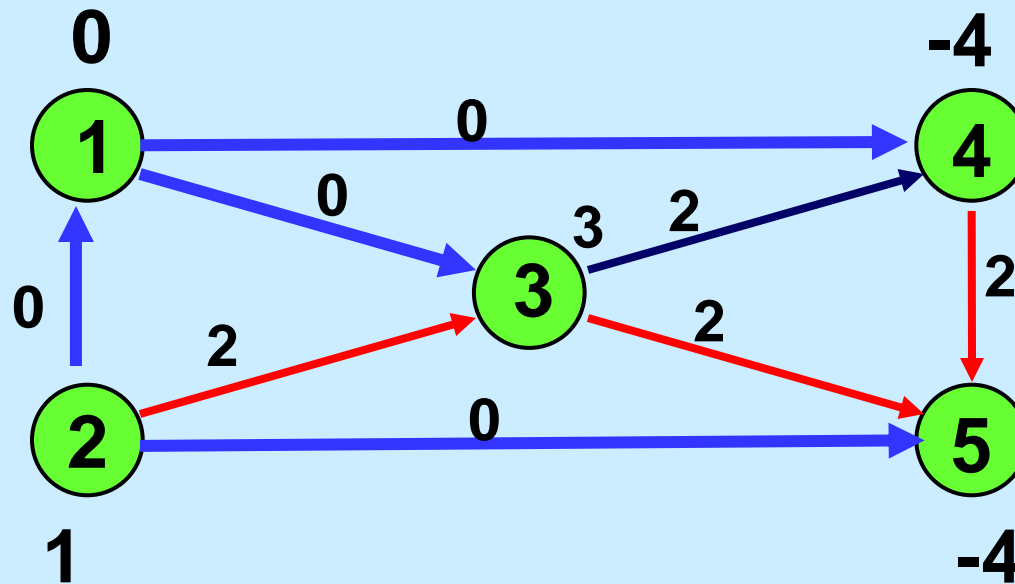
# Deleting (2,1) from T splits T into two parts



Adding  $\Delta$  to nodes on one side of the tree does not effect the reduced costs of any tree arc except (2,1). Why?

What value of  $\Delta$  should be chosen to make the reduced cost of (2,1) = 0?

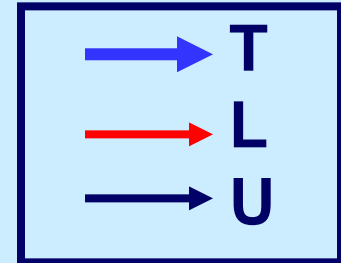
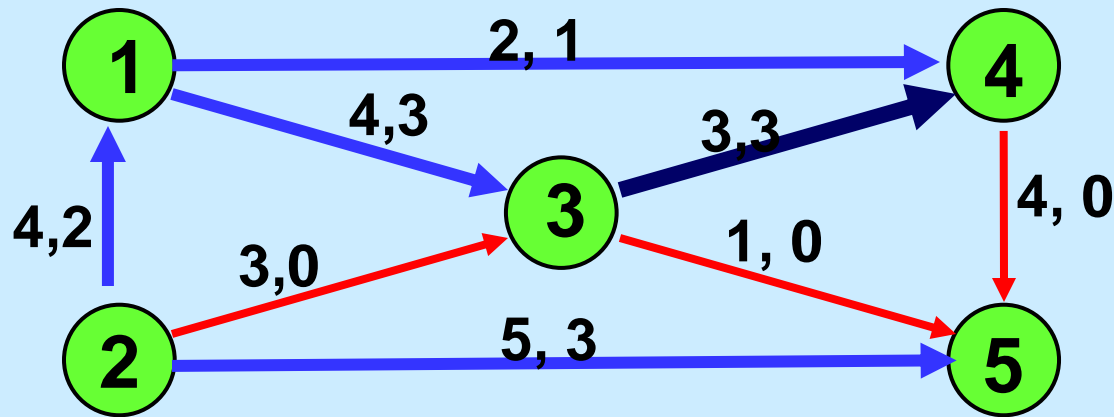
# The updated multipliers and reduced costs



The updated multipliers and reduced costs

Is this tree solution optimal?

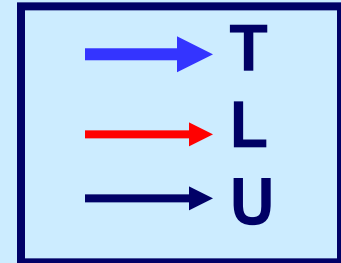
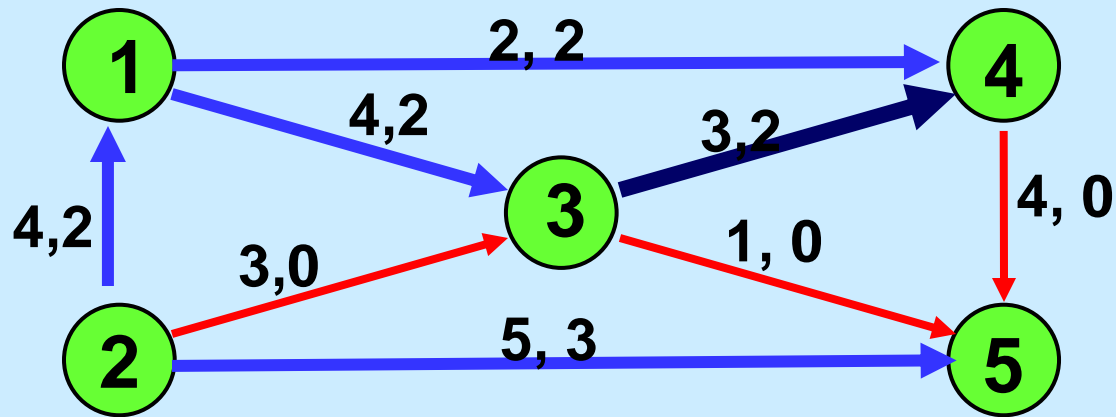
# Add a violating arc to the spanning tree, creating a cycle



Add arc (3,4) to the spanning tree

What is the cycle, and how much flow can be sent?

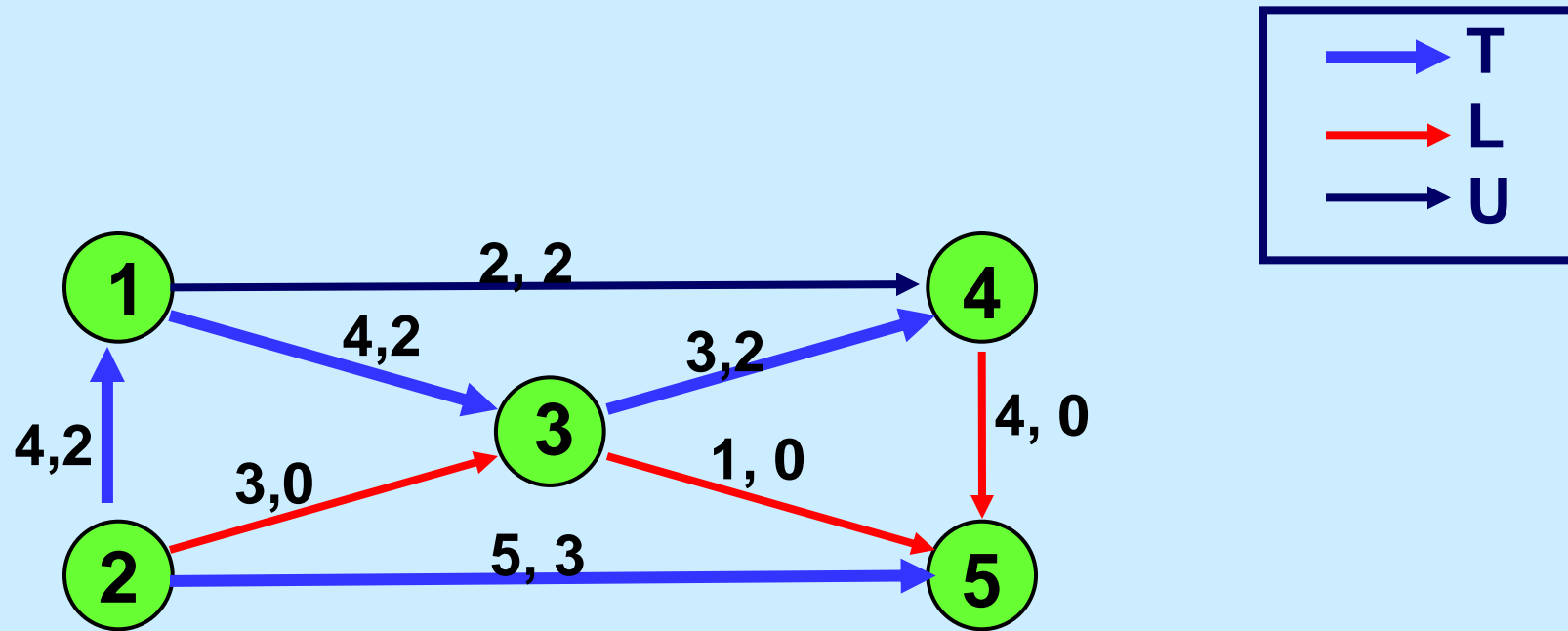
# Send Flow Around the Cycle



1 unit of flow was sent around the cycle.

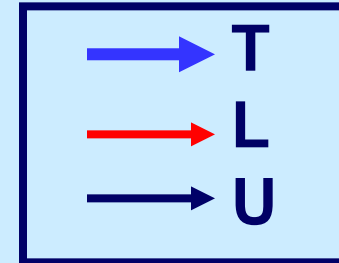
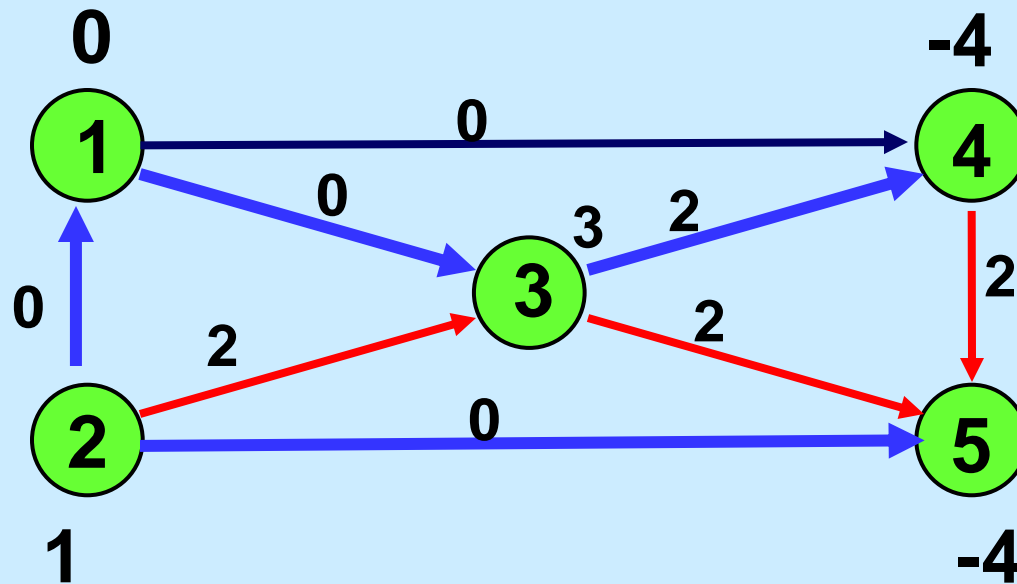
What is the next spanning tree solution?

# The next spanning tree solution



Here is the updated spanning tree solution

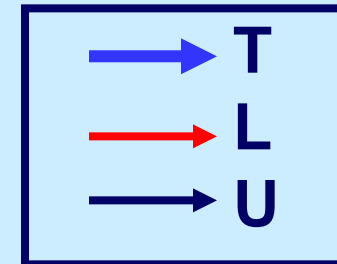
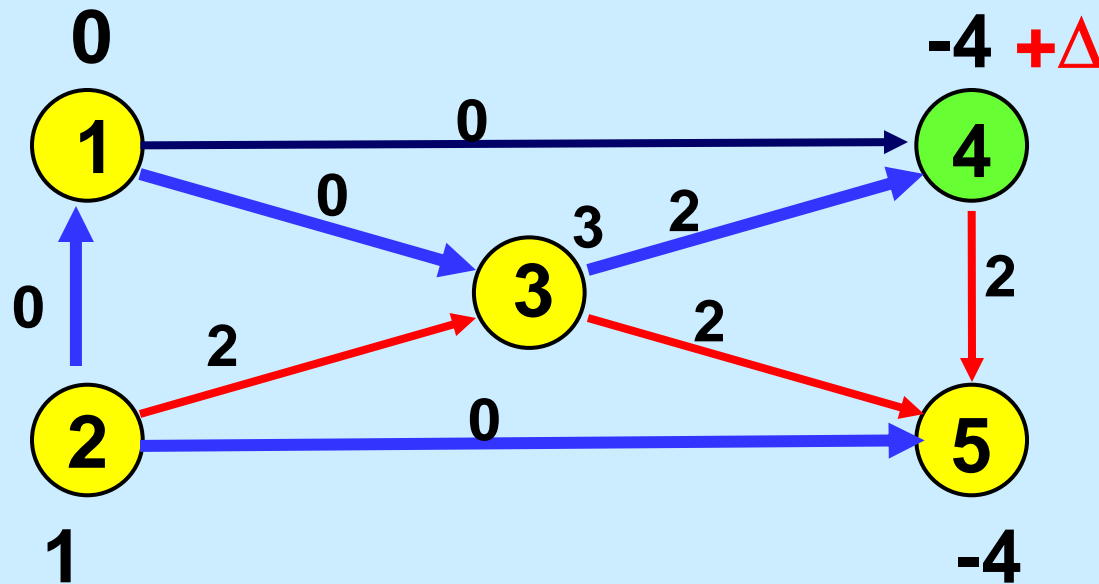
# Updated the multipliers



Here are the current multipliers

How should we modify the multipliers?

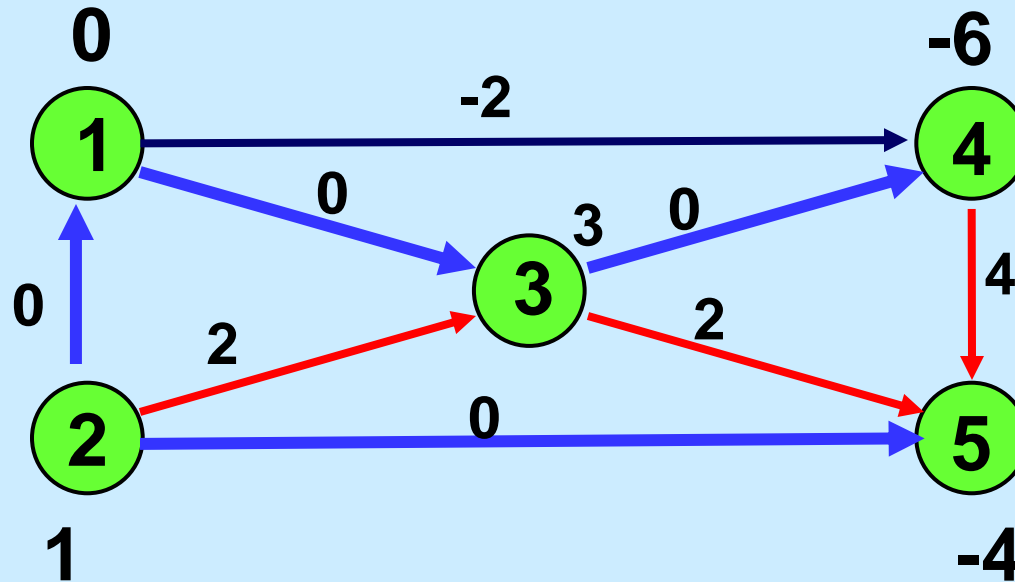
# Updated the multipliers



What value should  $\Delta$  be?

Here are the current multipliers

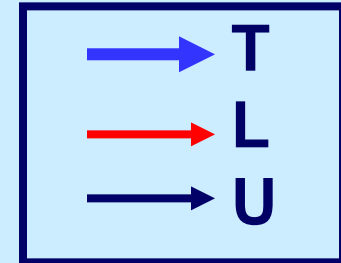
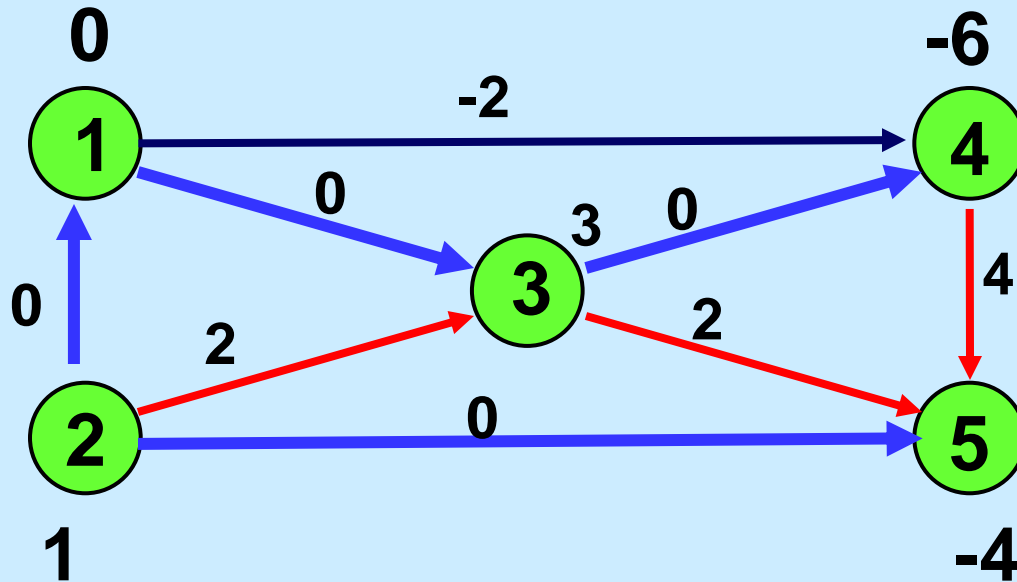
# The updated multipliers



Here are the updated multipliers.

Is the current spanning tree solution optimal?

# The Optimal Solution

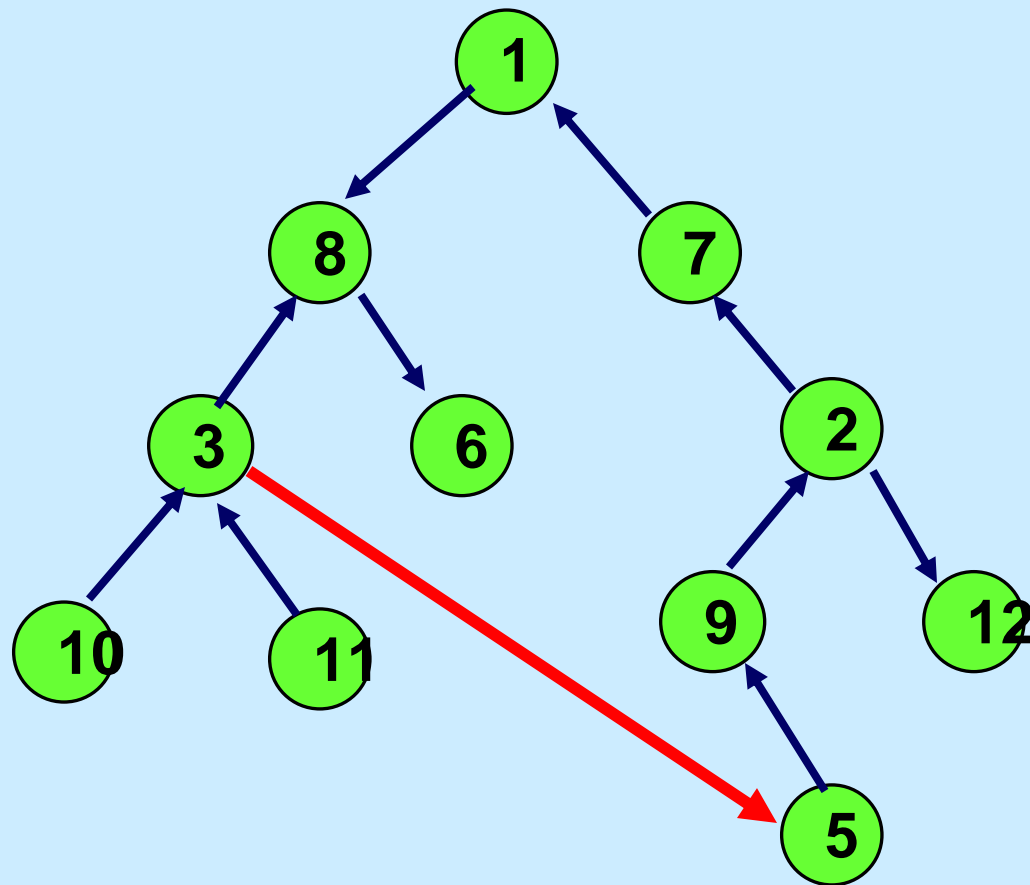


Here is the optimal solution.

No arc violates the optimality conditions.

# Finding the Cycle

---

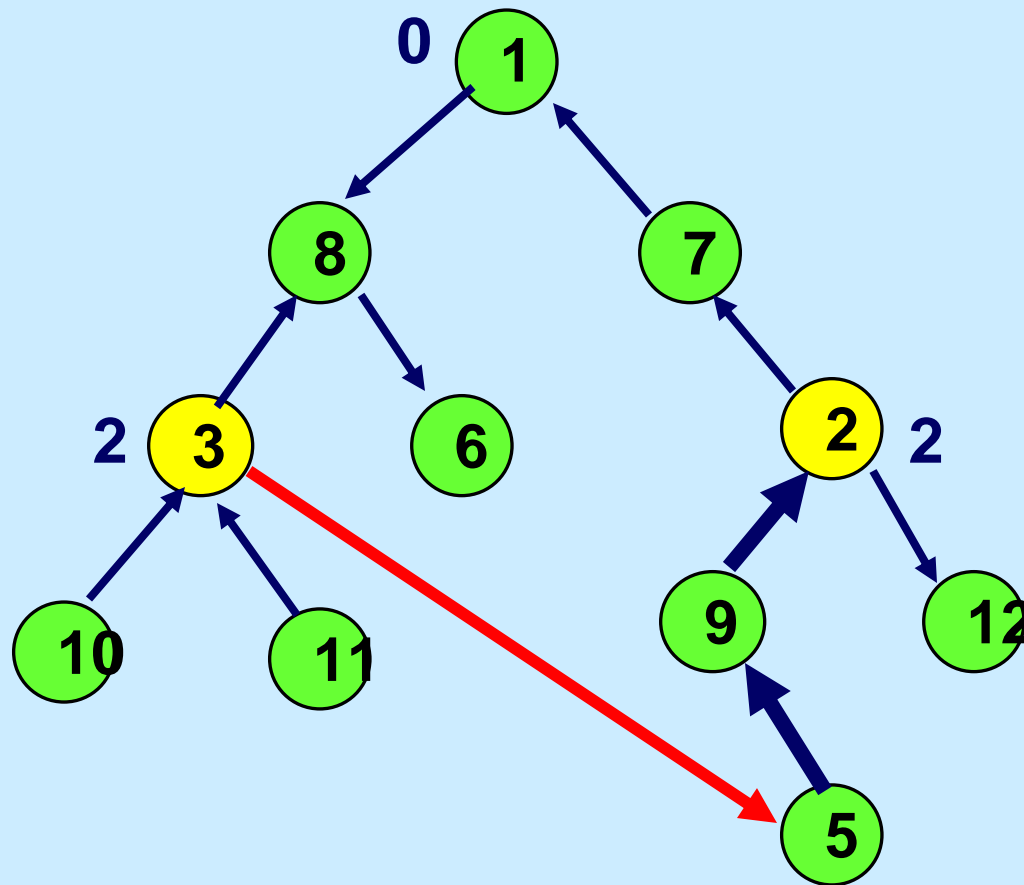






# Use Depth and Predecessor

---



**depth(2) =  
2;**

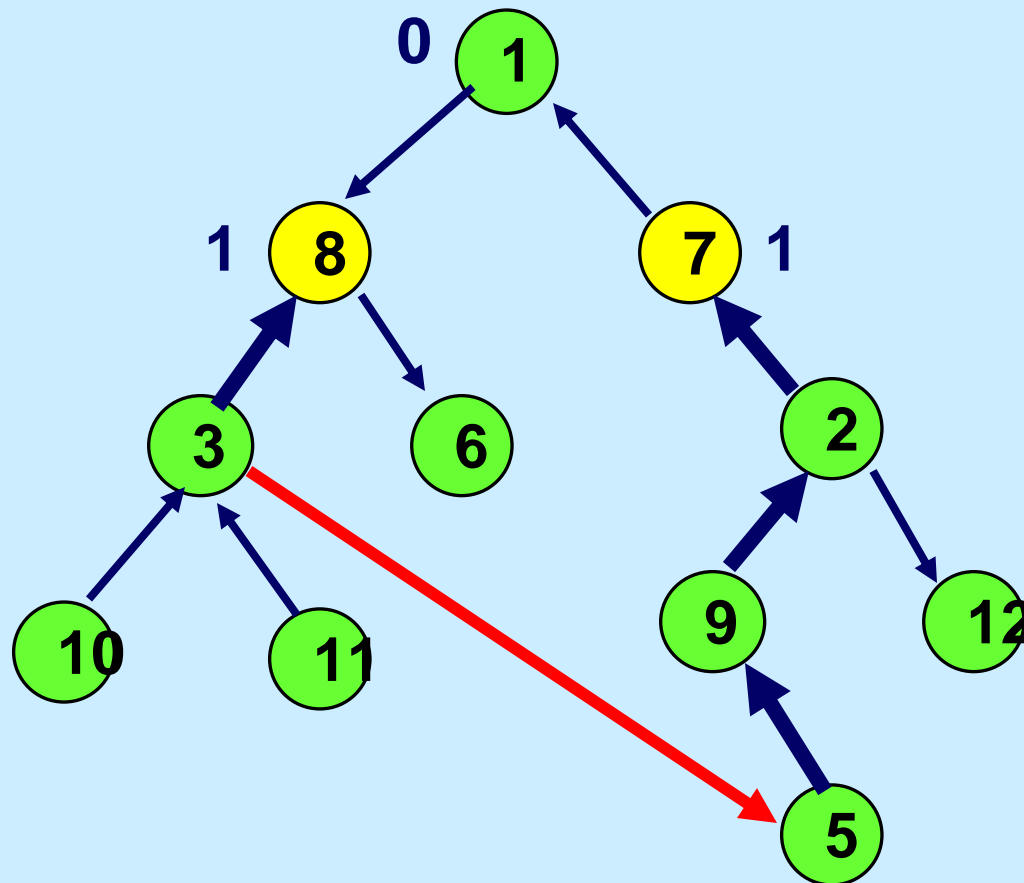
**depth(3) =  
2;**

**replace  
node 2 by  
pred(2);**

**replace  
node 3 by  
pred(3)**

# Use Depth and Predecessor

---



**depth(8) =  
1;**

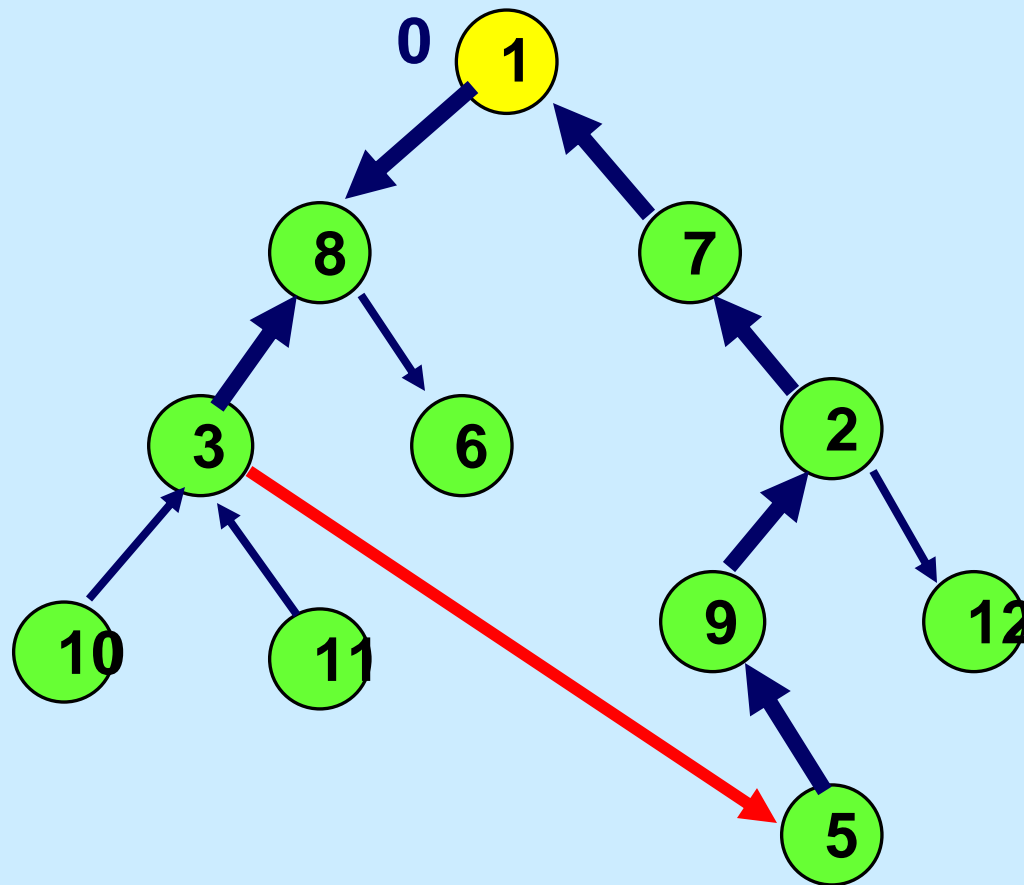
**depth(7) =  
1;**

**replace  
node 8 by  
pred(8);**

**replace  
node 7 by  
pred(1)**

# Use Depth and Predecessor

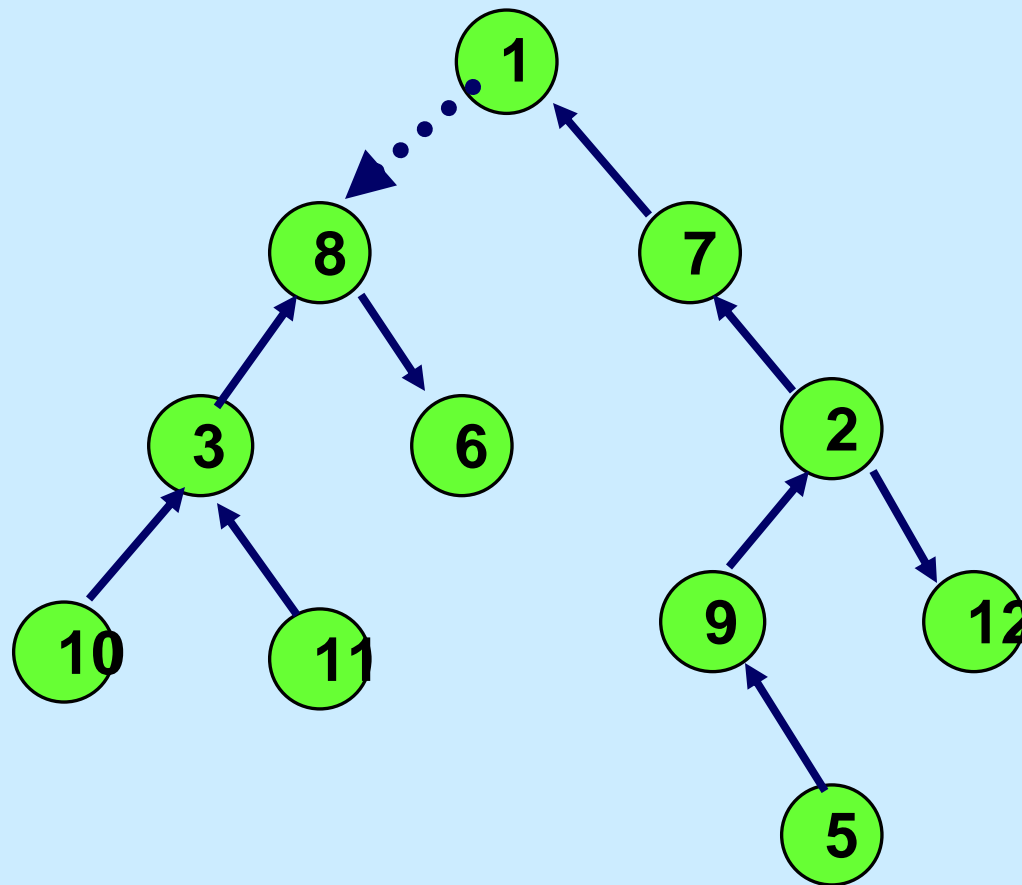
---



The least common ancestor of nodes 3 and 5 has been found.

# Updating the multipliers: use the thread and depth

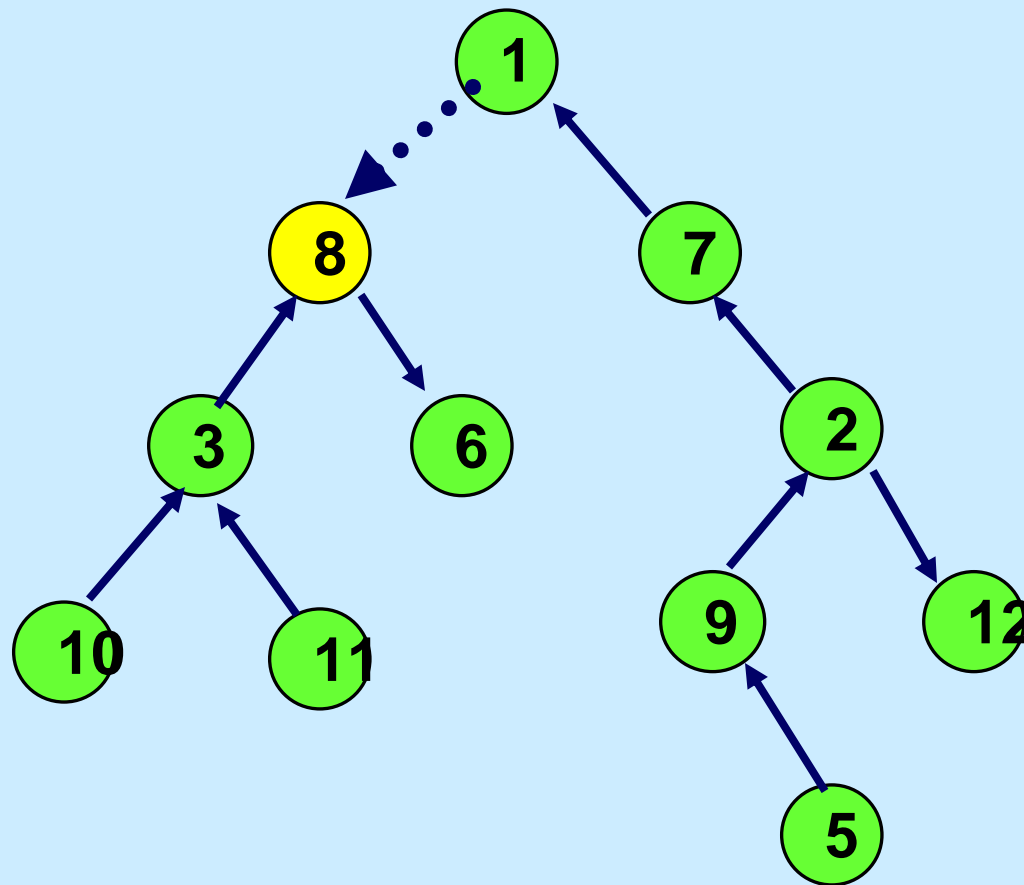
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Suppose that arc (1,8) will drop out of the tree. What is the subtree rooted at node 8?

# Follow the thread starting with node 8

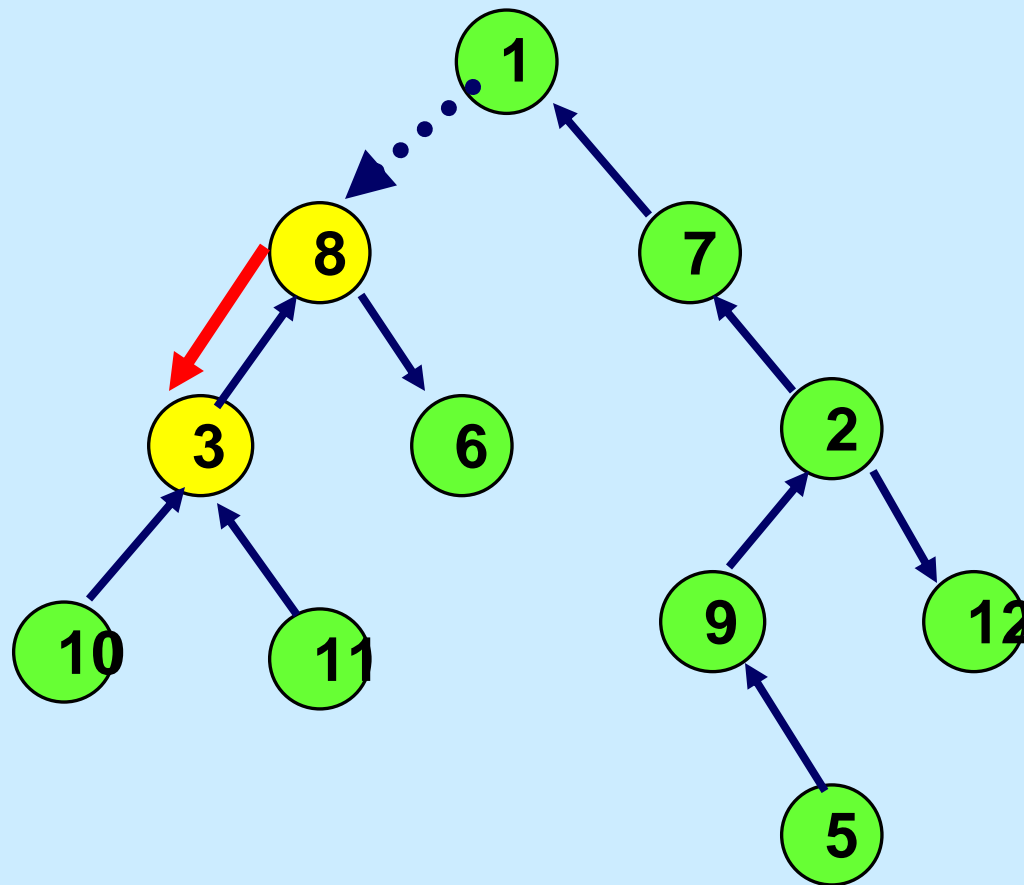
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What is  
thread(8)?

# Follow the thread starting with node 8

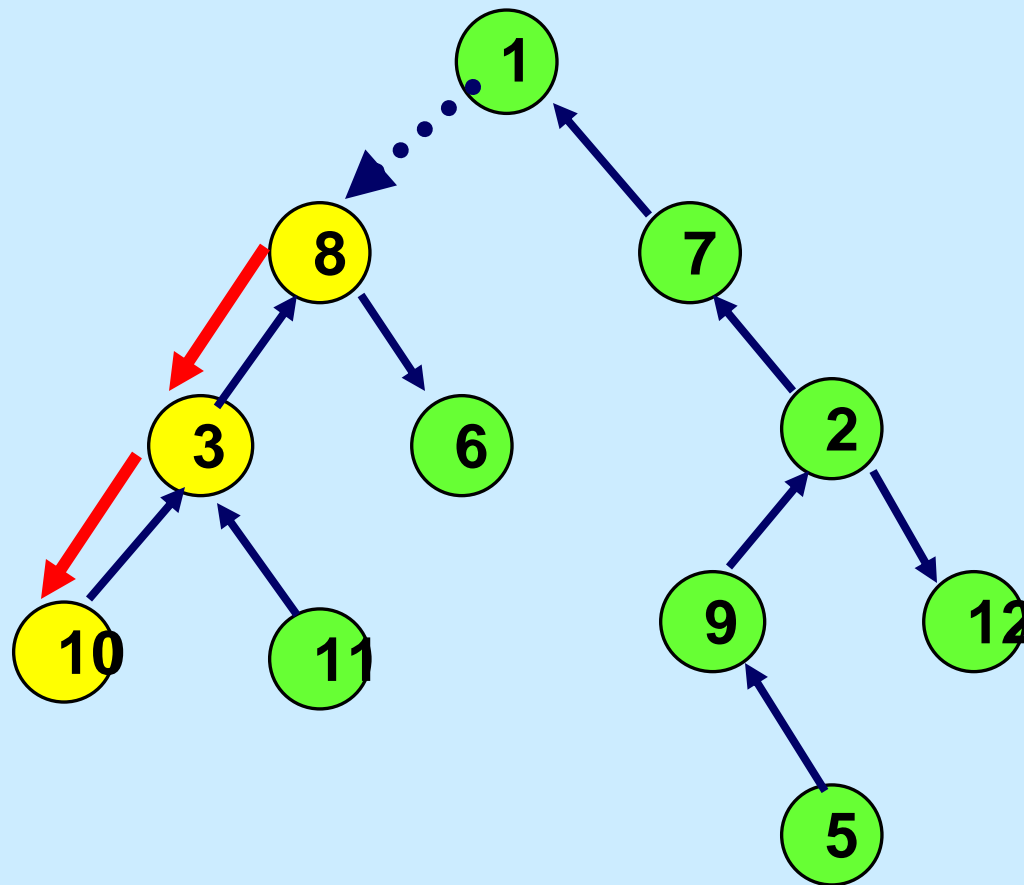
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What is  
thread(3)?

# Follow the thread starting with node 8

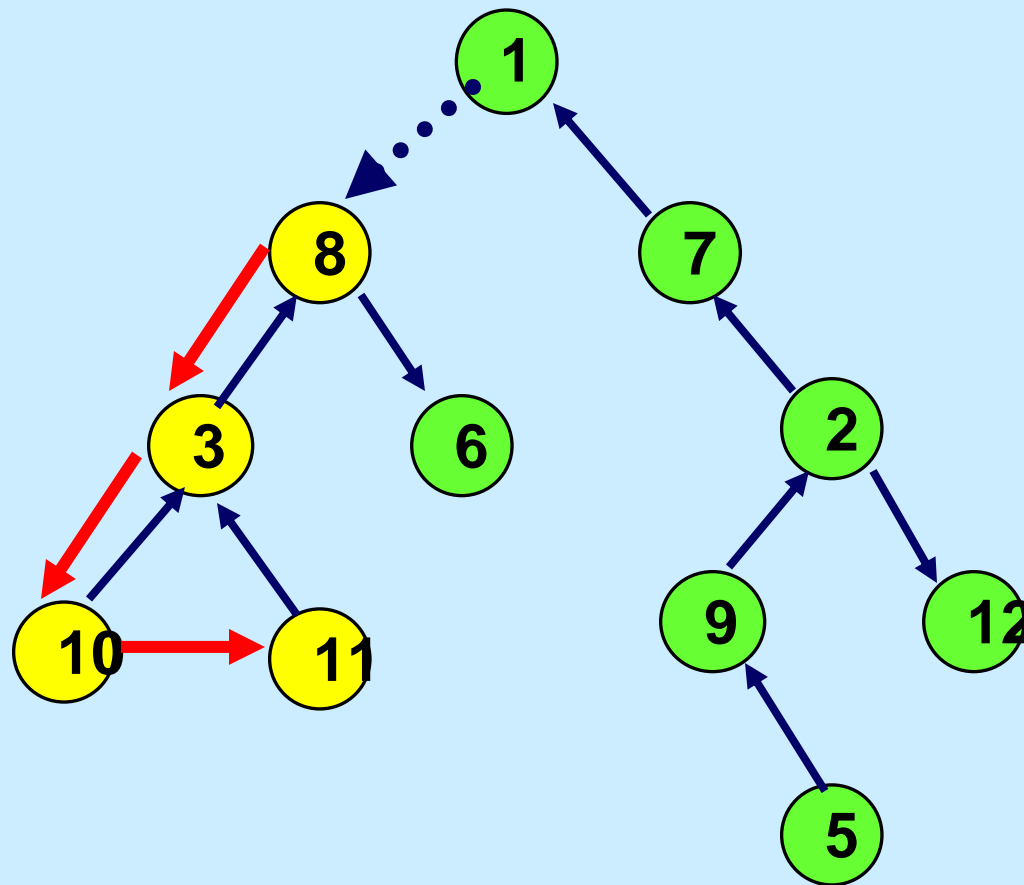
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What is  
thread(10)  
?

# Follow the thread starting with node 8

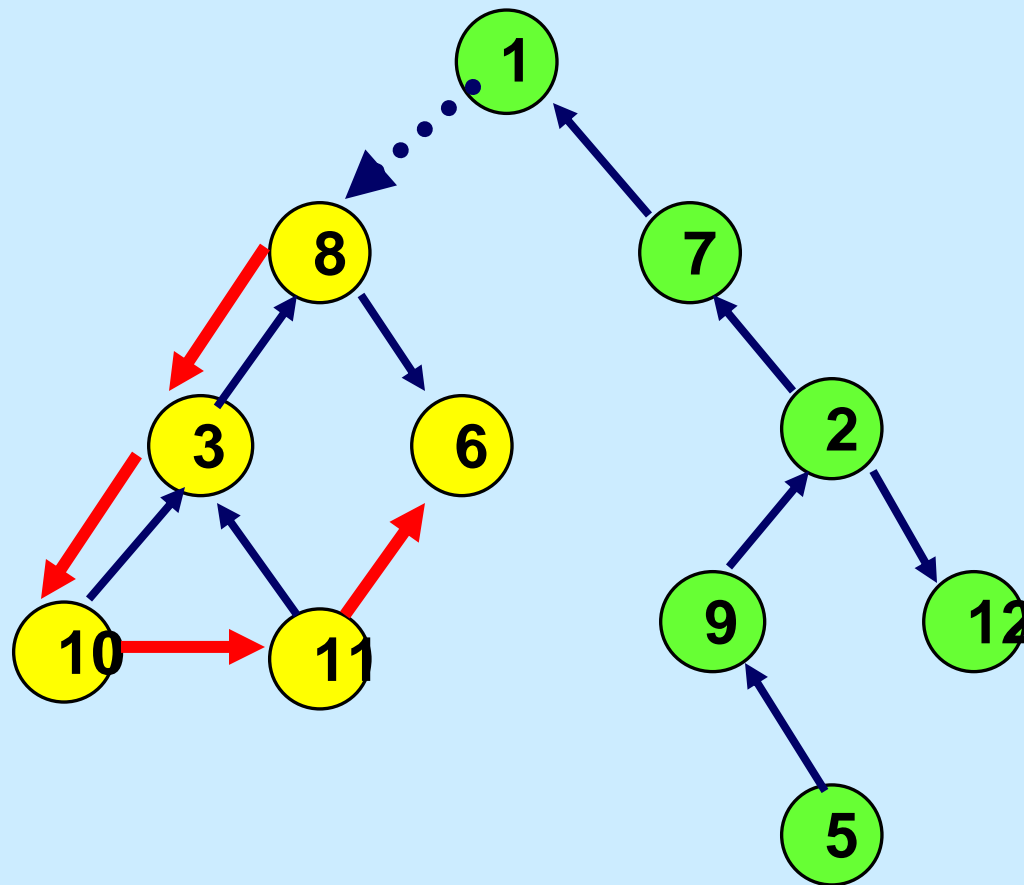
---



What is  
thread(11)  
?

# Follow the thread starting with node 8

---



What is  
thread(6)?

# The stopping rule

Stopping rule: stop when  
 $\text{depth}(\text{current node}) \leq \text{depth}(8)$

