

Present worth of tomorrow's benefits? Are they worth the (present) cost?

The question is of the sort one confronts when tempted to invest in a photovoltaic system, thinking that, while the initial investment may be large, one anticipates a steady annual return on that investment which will more than justify the initial capital outlay. But how do we compare the accumulated benefit of the steady annual returns with the initial cost of the system?¹

One might be tempted to simply multiply the annual return on the investment by the number of years of system life and compare that with the initial cost; but that won't do. For tomorrow's money is not worth as much as the same amount today. If you thought it was, I would ask you for a loan, say of \$1000 which I promise to pay back over a period of 10 years (at \$100 per year), then invest that \$1000 in a bank that pays me interest, say 2%, a year. After the first year, I have $(1+.02)*1000 = \$1020$. I deliver my annual \$100 payment to you, so I have \$920 to invest the second year. The table summarizes the transactions over the life of the loan:

Year	Your money	My money	My interest earned	My money plus
0		\$1000.00	\$20.00	\$1020.00
1	\$100.00	920.00	18.40	938.40
2	200.00	838.40	16.77	855.17
3	300.00	755.17	15.10	770.27
4	400.00	670.27	13.41	683.68
5	500.00	583.68	11.67	595.35
6	600.00	495.35	9.91	505.26
7	700.00	405.26	8.11	413.36
8	800.00	313.36	6.27	319.63
9	900.00	219.63	4.39	224.02
10	1000.00	124.02		
			Sum = \$124.02	

After 10 years, you can put your money back under your pillow. You haven't lost a cent. I, on the other hand, have earned \$124.02 dollars in interest. This may not strike you as any big deal but think: If I can find others like you to lend me one thousand dollars under the same conditions I can live very well indeed. Say I find 100 souls each year willing to do so². The calculation of 10 years of interest income for this case is left to the reader.

No, today's money in the bank shows not the same value next year, tomorrow. We all know that: Our money in a savings account "earns" interest, albeit not very much interest these days.³ Now consider the future worth of the money you deposit into a savings account. If you are a student, think of the future worth of the future money you deposit once you graduate after you have come up with some innovative scheme to make lots of money - perhaps over the internet.

Let's say you are able to save today, in the present, an amount $P = \$1000.00$ and the bank's interest rate is again, $r = 2\%$ and you want to know what you will have after 20 years. Assume you never withdraw any funds until the end of the 20 years. The calculation of the future value, S , of this deposit goes as follows:

At the end of one year, you will have $S_1 = P \cdot (1+r) = 1000 \cdot (1.02) = 1020.00$ dollars. At the end of the second year $S_2 = (P \cdot (1+r))(1+r) = 1000 \cdot (1.02)^2 = 1040.40$

1. We leave aside the possibility that the initial cost will be spread out over more than an initial year. We also assume the annual benefit is constant over the life of the system.

2. Not so improbable what with the internet and every one on line.

3. We ignore inflation of the cost of goods and services we depend upon. If the rate of inflation exceeds the bank's interest rate, your money, though it appears to be increasing in amount will actually be worth less to you.

Induction leads to the expression $S_N = P(1+r)^N$ for the future value after N years of our initial investment. For our particular numbers

$$S^N = 1000*(1.02)^N = \$1485.95$$

Some people give the factor $(1+r)^N$ its own name i.e., $F = (1+r)^N$. F is the “compound amount factor” in honor of the fact that the bank effectively compounds your interest rate each year.

Now our photovoltaics investment question is of a different sort. We want to know if it is worth investing in photovoltaics, building a system on our rooftop, today anticipating that we will save on electrical energy bills over the life of the system. Does that stream of benefits amount to more than the cost of the system?

One way, and a usual way, is to compute the *present value of a future stream of benefits* and compare that with the *present-day cost*. But what is today’s value of this stream of annual benefits which accumulate over the life of the system - which we take as $N=20$ years?

We start by computing the *future value*, at the end of the life of the system, of this stream of annual benefits. Think of each year investing the benefit you receive, say b , at the end of each year in a savings account which gives an interest rate, again labeled r .

At the end of the first year, you deposit b dollars. You don’t touch this until the end of 20 year life span of the pv system. At that time, the b dollars has earned $N-1$ years of interest compounded at the rate r . Letting S_N be the sum of this stream of annual benefits after N years, we have $S_N = b(1+r)^{N-1}$ for the contribution to the future value of the stream of just the first year’s benefit.

Again, induction leads to the expression $S_N = b(1+r)^{N-1} + b(1+r)^{N-2} + \dots + b(1+r)^0$ where the last years benefit gains no interest being that it arrives at the end of the 20th year.

We can find an expression for this sum, knowing that $1+x + x^2 + x^3 \dots + x^{N-1} = (1-x^N)/(1-x)$ and identifying x with $(1+r)$. Thus:

$$S_N = b \cdot \left[\frac{(1+r)^N - 1}{r} \right]$$

Now to compute the present value, we ask what initial, one-time deposit into our savings account will yield this future sum after N years? Our response is ready; we have already answered this sort of question. We have

$$S_N = P(1+r)^N \quad \text{so} \quad P = \frac{S_N}{(1+r)^N} \quad \text{so finally we have for the present value of our benefit stream}$$

$$P = \frac{b}{(1+r)^N} \cdot \left[\frac{(1+r)^N - 1}{r} \right] = b \cdot \left[\frac{F-1}{Fr} \right]$$

where we have introduced the compound amount factor, F , to give an apparent simplification.

Now consider an example using a “discount rate” - for that’s what it is called - of 0.02, whether that is realistic or not. So $r = .02$ and F , then is $F = (1+r)^N = 1.486$. Then with $N = 20$ years, we find that

$$P = 16.4 b$$

This is significantly less than $20b$, 20 years times the annual anticipated benefit. Note, however, that it is significantly greater than the present value of $20b$ awarded you at the end of the life of the system. The present value of that lump sum award would be just

$$P = 20b/F = 13.5b$$