DUSP 11.202 | Frank Levy
Gateway: Planning Economics | Session 6

Notes on Net Present Value and Internal Rate of Return

What follows are a short set of note to clarify the meaning of net present value and internal rate of return. Do not hesitate to e-mail me if you still have questions. More material explaining both ideas are contained in the last two tools on Open Courseware Web site.

1) The idea of discounting.

The foundation of all of the ideas in today's lecture is discounting. The rationale for discounting is this: In a world where saved money or loaned money can earn interest, one dollar today is not equivalent in value to one dollar a year or two years from now because the dollar today could be invested at the interest rate and grow into something larger than one dollar in a year.

The implication is this: If we have a project that involves costs and benefits that occur over various years, we need to first convert all the costs and benefits to their values in same year before we can add them. We cannot, for example, just add a dollar today plus a dollar two years from now and call the sum two dollars.

In theory, we could choose any year to be the time by at which we want to measure costs and benefits: today or the year that is halfway through the life of the project or the year the project ends, etc. By convention, we convert all costs and benefits to their value today – i.e. we "bring all future dollars back" to the present.

You can think of this process in the following way. Calculating the present value of a payment of $25.00 that occurs five years from, when the bank interest rate is 5 percent, is the same as asking the question: How large a deposit do I have to put in the bank today if the interest rate is 5 percent and I want the deposit to grow to $25 in five years.

The answer is $\$25.00/(1.05)^5 = \$19.59$

A higher interest rate - say 7 percent – would produce a smaller value = $17.82. No surprise there – at a higher interest rate, you need to put less in the bank today to grow to $25.00 in five years. On the other hand, these numbers also suggest that in this kind of calculation, benefits that occur far in the future are valued very little when we use moderate or high interest rates.

2) Net Present Value

The easiest way to explore this idea is to review the following problem: What is the Net Present Value of a project which has the following stream of costs and payouts.

Year 0	Year 1	Year 2	Year 3
-20	10	10	10

where the interest rate is given as 5 percent.

To be clear on the structure, you put in $20 dollars immediately, never to be seen again. On the other hand, the project gives you three payments of $10 each so unless the interest rate is very high, this should be more than enough to compensate you for your investment of $20.

We calculated the net present value of this project as $7.23 What does this mean?

To get the answer, focus first on the benefits. If we take the discounted value of the benefits, we get:

$$10/(1.05) \quad + 10/(1.05)^2 \quad + 10/(1.05)^3 \quad = \$27.23$$

In terms of our discussion in (1), you can think of this amount as meaning that when the interest rate is 5 percent, you would have to put $27.23 into the bank today to be able to withdraw $10 in one year, another $10 in the second year and another $10.00 in the third year at which time the bank account will have zero balance. Try this yourself to see that the numbers work out – put $27.23 in the bank, let it grow by 5 percent for a year, withdraw $10.00 and let the remainder grow for another year at 5 percent, etc.

On this project, however, you don’t have to put up $27.23 – rather, you only have to put up $20.00. Thus when we say that the project has a net present value of $7.23, we are saying that the project gives you an amount equivalent to your original deposit cost plus $7.23 (where both benefits and costs have been discounted to today’s dollars). This is the profit to which Ian referred.

3) What about the bank account?

The $7.23 also is the margin by which your project beats putting your $20.00 in a bank account at 5 percent (where, again, all costs and benefits have been discounted back to the present).

I will talk this one through and you can work out the numbers to verify it. As we saw, the net present value of the project above, when discounted at 5 percent, is $7.23.

For purposes of comparison, set up a bank account that you think of as a project. The $20 is deposited in year 1, it then grows at the compound rate of 5 percent until it is withdrawn in year 5. If we calculate the net present value of this project discounted at 5 percent, the net present value will equal zero.

At first glance, this seems to say your money isn't growing in the bank account. But that is not what its saying. When we discount by 5 percent, we are asking how fast money in your project is growing **vis-à-vis 5 percent**: does it grow faster than 5 percent (in which case it will produce a positive net present value); does it grow slower than 5 percent (in which case it will produce a negative net present value) or does it grow at exactly 5 percent (in which case it will produce a zero net present value).

4) Internal Rate of Return

What follows is a discussion of Internal Rate of Return that approaches the topic through learning by doing. It will require you to write your own Net Present Value calculator in Excel but the whole lesson shouldn't take more than a half-hour and you should come away with a better feel for the idea.

To set things up, I will begin by constructing a sample "project" based on a bank account that pays an interest rate of 6 percent.

December 3, 2002 - deposit $3,000

December 3, 2003 – balance has grown to $3,180; withdraw $600 leaving remaining balance of $2,580

December 3, 2004 - remaining balance from 2003 has grown to $2,734.80; withdraw $1,275 leaving a remaining balance of $1,459.80

December 3, 2005 - remaining balance from 2004 has grown to $1,547.39; withdraw $1,547.39 leaving balance of zero: Project is now over.

I went through this detail so you could verify that the project really is based on a 6 percent rate of interest - i.e. the balance in the account grows by 6 percent each year.

Now I want you to forget that you know that the underlying interest rate is 6 percent and I want you to think of this as a project with the following properties:

2002: cost = $3,000

2003: benefit = $600

2004: benefit = $1,275

2005: benefit = $1,547.39

View this as an investment project that requires a $3000 initial investment. Your problem is to determine what rate of return the project pays you on your money. If the project's rate of return is higher than the market rate of interest, you will probably do the project.

To determine the project's rate of return on your money, you need to set up a net present value calculator in Excell. One easy way to do it is to set it up in two columns and a couple of other cells:

Use Cell A1 to hold the value of your interest rate

Then set up two columns of data as follows

	Column 1	Column 2	
Row 1	-3000	1	
Row 2	600	= (1 + A1)	(i.e. 1 + the interest rate)
Row 3	1275	= (1 + A1)^2	
Row 4	1547.39	= (1 + A1)^3	

The project's net present value is the sum of:

-3000/1 + 600/(1 +A1) + 1275/(1 +A1)^2 + 1547.39/(1 +A1)^3

You should be able to create this last formula in a cell. To simplify writing this formula, write name of the cell that holds (1 +A1), rather than the expression (1 +A1), etc.

Once this calculator is set up, you can plug in different values of the interest rate to see their effect on the project's net present value.

In particular, you should demonstrate the following ideas:

- Interest rates lower than 6 percent lead to a positive NPV for the project
- Interest rates higher than 6 percent lead to a negative NPV.

- A 6 percent interest rate leads to a zero NPV – i.e. the project just breaks even.

The concept of IRR flips this sequence around. It says that when you find the interest rate that causes the project to break even, that is the rate of return – the interest rate - the project is paying on your investment. In other words, if you had a bank account that paid an interest rate equal to the IRR, you could use that bank account it to generate the same pattern of costs and benefits as the project.

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