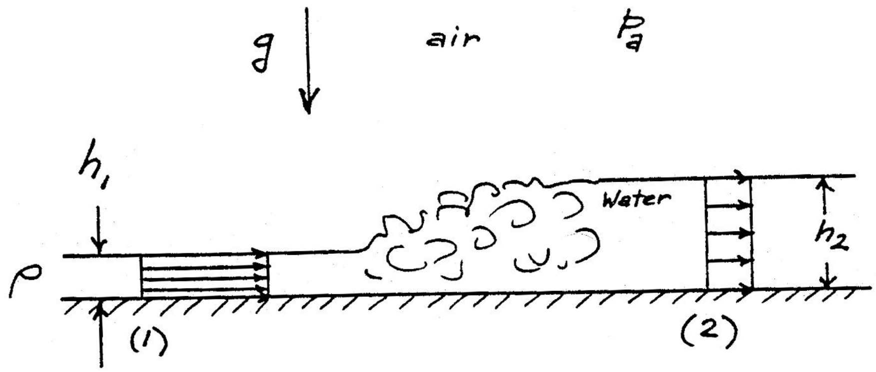
Problem 5.4

Hydraulic jump



The top figure shows a "hydraulic jump" in a steady, two-dimensional water flow with a free surface. The "jump" is a relatively sudden increase in liquid depth in the direction of the flow, and a decrease in velocity, which is induced by a higher downstream water elevation due to flow blockage. In real cases the flow occurs because the bottom slopes downward slightly. As long a the inclination is small, the horizontal approximation of the figure is approximately OK. A kind of hydraulic jump may be observed on the ocean beach at the point where an incoming wave running up over an inclined beach meets the water from the previous wave, which is now flowing back toward it.

Inside the jump shown in the figure, the water tends to flow forward (rightward) near the bottom, carried by its momentum, and backward near the free surface, as a result of the hydrostatic pressure gradient. This results in turbulence and *strong viscous dissipation* inside the water in the jump region. Bernoulli's equation is certainly not applicable through that region. The shear stress at the bottom, however, typically does not play a dominant role in the structure of the jump.

(a) Neglecting the shear stress on the floor entirely, obtain an equation that expresses h_2 in terms of h_1, V_1, the water density ρ, and gravity g, assuming that the velocity is uniform at both stations (1) and (2).

Dimensional analysis shows that the relationship in (a) may be expressed in the form

$$\frac{h_2}{h_1} = f\left(\frac{V_1^2}{gh}\right) \qquad (1)$$

where V_1^2/gh is the square of the Froude Number, a dimensionless quantity that measures the relative importance of kinetic-energy effects and gravitational effects on the flow. Express your answer in the form (1) above.

HINT ANSWER

(b) Show that the *total* pressure (static plus dynamic plus gravitational) drops as one goes along a streamline from station (1) to station (2). Show that you get the same drop regardless of which streamline you pick: along the floor, along the free surface, or any other.

ANSWER

(c) Do you think that the flow in the figure could exist with the velocity directions reversed? Explain.

ANSWER