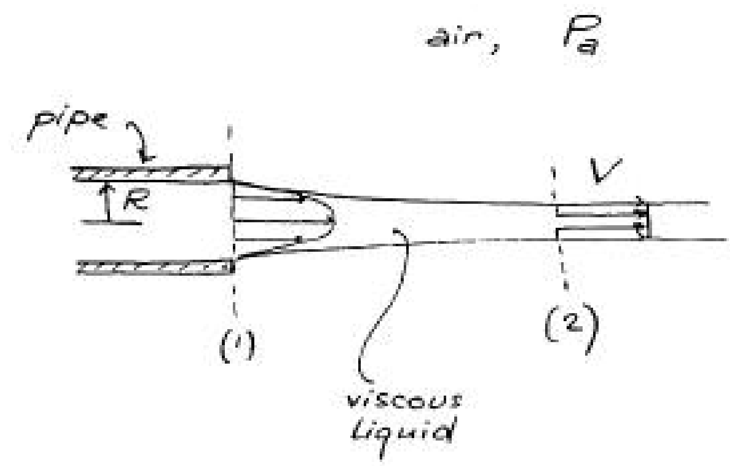
Problem 5.6

Narrowing of water stream from tap



An incompressible viscous liquid flows out of a long, circular tube with a small radius *R*. At the exit plane (1), the velocity profile is the parabolic one characteristic of fully developed, laminar flow in a tube,

$$u(r) = u_m(1 - r^2 / R^2) \qquad (1)$$

Inside the pipe and right up to the exit plane, the velocity at the pipe wall is zero due to viscosity (the no-slip condition in viscous flows). Once the liquid stream emerges into the constant-pressure air environment, however,the air, having very low viscosity, offers little resistance to the liquid's motion and the liquid feels essentially no shear stress at its boundaries. In response to this new boundary condition, internal viscous stresses in the fluid begin to straighten out the velocity profile as the fluid moves along, and eventually, at some station (2), the velocity becomes uniform at the average value *V*.

(a) Does Bernoulli's equation apply in this flow between (1) and (2)? Explain.

ANSWER

(b) Neglecting gravitational and surface tension effects, find the radius contraction ratio R_2/R_1 that occurs in the liquid stream after it emerges from the pipe, and the velocity ratio V/u_m.

HINT ANSWER