Problem 5.19

Spark-ignited spherical combustion

See also Problem 3.7

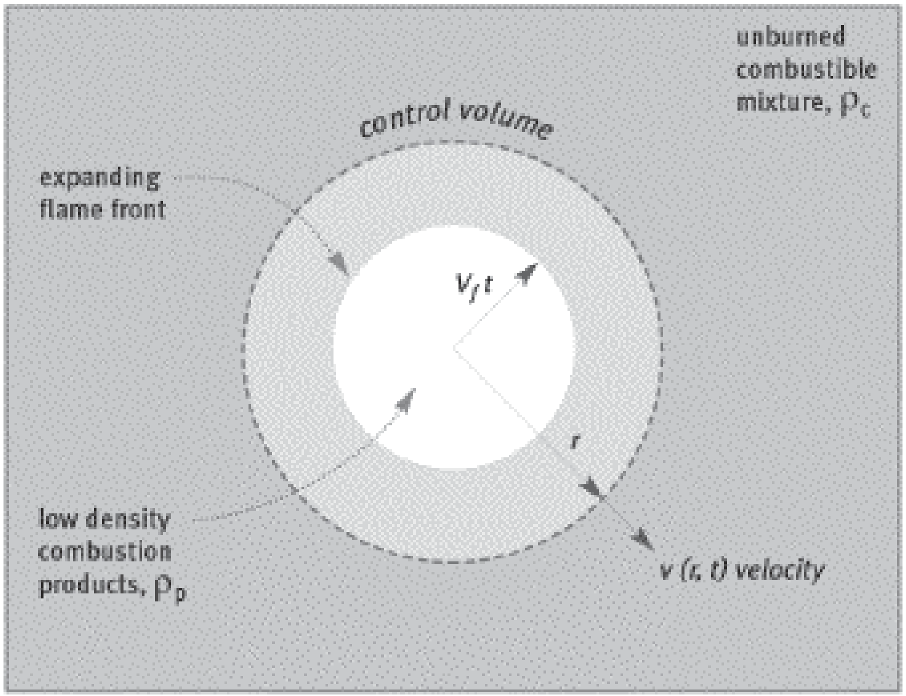

Fig. 1

A combustible mixture of air and fuel is initially at rest at a density ρ_c and uniform pressure p_∞. At t = 0, the mixture is ignited at the origin by a spark and a flame front begins to move radially outward from the origin at a constant speed V_f. As the flame front overtakes a gas particle, it converts the combustible mixture of density ρ_c to a hotter combustion product of lower density ρ_p. This occurs so fast that the flame front can be modeled as an infinitesimally thin density discontinuity. Because the volume of a fluid particle increases as it is enveloped by the flame front, the combustible mixture ahead of the expanding flame front is pushed radially outward by the front. The combustion products of density ρ_p that are left behind the flame must, however, be stationary relative to the reference frame fixed in the origin.

(a) In a reference frame fixed in the origin, determine the radial outflow velocity $u(r,t)$ of the combustible mixture. There will be different expressions for the unburned gas ahead of the flame front ($r > V_f t$) and the burned gas left behind ($r < V_f t$). See also Prob. 3.17. Show that the velocity distribution can be expressed in the dimensionless form $v/V_f = f(s,\gamma)$, where $s = r/V_f t$ is either a dimensionless radial distance at fixed time or a dimensionless inverse time at fixed *r*, and $\gamma = (\rho_c - \rho_p)/\rho_c$ is a dimensionless density difference between the unburned and burned gases.

HINT ANSWER

(b) In a real combustion problem, the quantity that would be specified, as a property of the combustible mixture, would be not the flame speed V_f relative to the origin, but the flame speed U_f relative to the fluid just ahead of it. Show that in this problem the flame front does move at a constant speed relative to the fluid ahead of it if V_f is constant, and find an expression for V_f in terms of U_f and the given densities.

ANSWER

(c) Assuming gravitational and viscous effects are negligible, write an equation for the pressure gradient at *r, t* in the combustible mixture ahead of the flame $(r > V_f t)$ and determine the pressure distribution $p(r,t) - p_\infty$ there. Express the pressure distribution in the dimensionless form $P = f(s,\gamma)$, where $P = (p - p_\infty)/(\rho_c - \rho_p)V_f^2$ is a dimensionless pressure and $s = r/V_f t$ and $\gamma = (\rho_c - \rho_p)/\rho_1$, as in Part (a).

HINT HINT 2 ANSWER

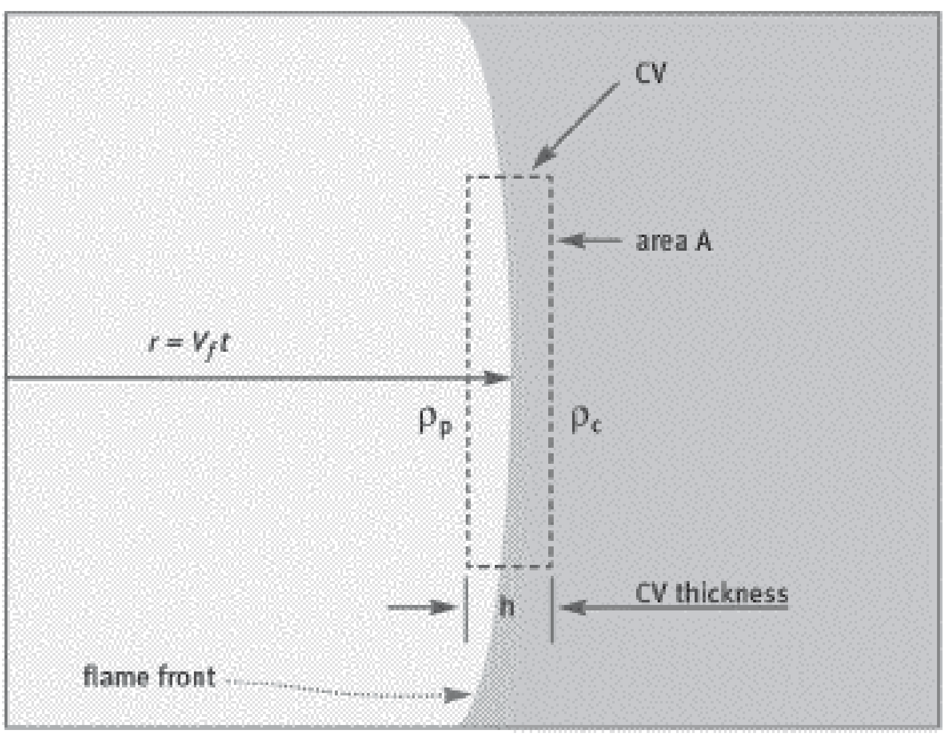

Fig. 2

(d) Show that there must be a pressure jump across the flame front, the pressure on the side of the combustion products being *lower* than in the mixture just ahead of the front. Derive an expression for this pressure discontinuity. (Hint: Use the control volume shown in Fig. 2.) Can you hive a physical explanation of why the pressure is lower behind the flame front than ahead of it? This seems not to make sense at first sight: how can a lower pressure region accelerate a higher pressure region radially outward?

HINT HINT 2 ANSWER

(e) For $\gamma = 0.25$, say, plot the function $P=f(s,\gamma)$ from s=0 to s=10. Also show the limiting cases $\gamma = 0$ and 1.

ANSWER