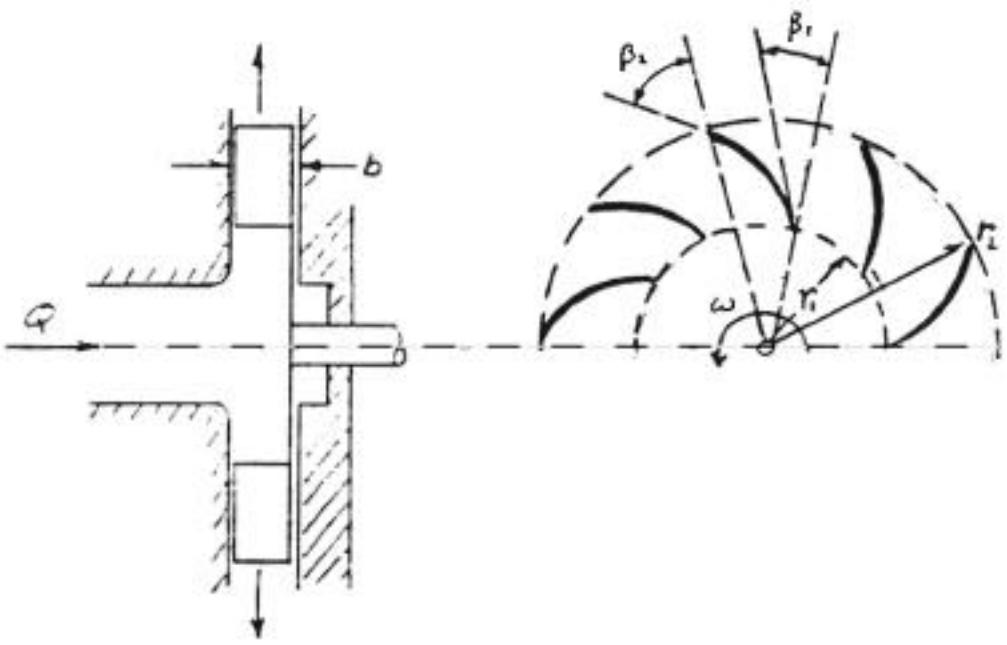
Problem 5.27

Centrifugal pump/turbine



The sketch shows a centrifugal turbine or pump with an impeller in a flat-sided casing of width *b*. A liquid of density ρ flows through the pump at a volume flow rate *Q*. Assuming that (i) the liquid enters with a purely axial velocity and (ii) exits at radius r_2 with a velocity that follows the (moving) blade surfaces, and (iii) neglecting frictional effects,

(a) derive an expression for the c*ounterclockwise* external torque, T_S, exerted on the shaft at an angular speed ω. Express it in terms of *Q*, ρ, *b*, r_1, r_2, β_1,β_2 and ω.

ANSWER

(b) What is the system's run-away speed ω_0, that is, the (counterclockwise) speed of the shaft running unrestrained? What is the torque T_0 at zero rotation?

ANSWER

(c) Show that your result in (a) reduces to the universal form

$$T_S / T_{S0} = f(\omega / \omega_0)$$

and plot this function.

(d) Derive the power output to the shaft,

$$\dot{W}_S = T_S\omega \ ,$$

as a function of ω. At what range of values of β will the power output be positive (that is, the system be a pump), if $\beta>0$? At what value(s) of ω/ω_0 is the power output maximized? Provide expressions for the maximum power and the angular frequency at which it is attained, expressed in terms of Q, ρ, b, r_1, r_2, β_1, and β_2.

ANSWER