1.050 Engineering Mechanics

II. Stresses and Strength

Examples: Beam Statistics

Program 9th Lecture

TODAY:
1. Review: Beam Stress Model
2. **Formulation of a Beam Boundary Value Problem**
3. **Statically Determined vs. Statically Indetermined Beam Structures**
4. **Closure: Stresses & Equilibrium**

Goal: Appreciate Force-Moment Beam Model for solving beam problems

Review: Beam Model

1. Scales in Structural Mechanics

$$O\left(d\Omega^{1/3}\right) << (h,b) << \ell$$

REV   Section   Beam
Dimension   length

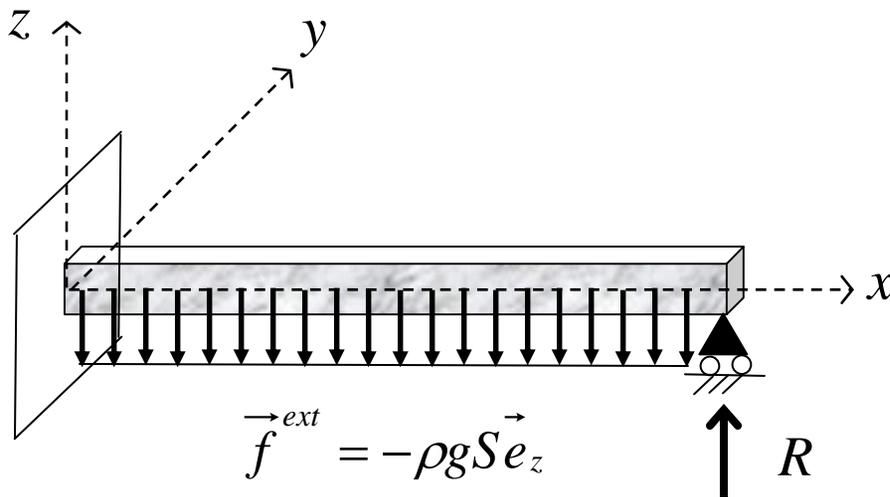2. **Reduction Formulas**: (from stresses to section forces and section moments)

$$\vec{F}_S = \int_S \boldsymbol{\sigma} \cdot \vec{e}_x dS$$

$$\vec{M}_S = \int_S \vec{x} \times (\boldsymbol{\sigma} \cdot \vec{e}_x) dS$$

3. **Equilibrium**: along beam axis, differential equilibrium of forces and moments

$$\frac{d\vec{F}_S}{dx} + \vec{f}^{ext} = 0$$

$$\frac{d\vec{M}_S}{dx} + \vec{e}_x \times \vec{F}_S = 0$$

Formulation of a Beam Boundary Value Problem

- Example

z y x

$$\vec{f}^{\,ext} = -\rho g S \vec{e}_z$$

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- Force and Moment Boundary Conditions
- Sum of all forces and Moments along x is zero
- Differential Equilibrium of
  - Section forces
  - Section moments

Stresses & Equilibrium

	Discrete System	Continuum System	Beam System
Elementary System			
Internal "Stresses"			
Boundary Condition			
Continuity Condition			
Diff. Force Equilibrium			
Diff. Moment Equilibrium			